Information Aggregation with Costly Information Acquisition*

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Abstract

We study information aggregation in a dynamic trading model with partially informed traders. Ostrovsky [2012] showed that 'separable' securities aggregate information in all equilibria, however, separability is not robust to small changes in the traders' private information. To remedy this problem, we allow traders to acquire signals with cost κ, in every period. We show that 'κ separable securities' characterize information aggregation and, as the cost decreases, nearly all securities become κ separable, irrespective of the traders' initial private information. Moreover, the switch to κ separability happens not gradually but discontinuously, hence even a small decrease in costs can result in a security aggregating information. We provide a complete classification of securities in terms of how well they aggregate information, which surprisingly depends only on their payoff structure. Finally, even with myopic traders, cheaper information may accelerate or decelerate information aggregation for all but Arrow-Debreu securities.

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1 Introduction

The question of whether financial markets reveal and aggregate the private information of traders has been studied at least since Hayek [1945]. Ostrovsky [2012] provides a strong result, that information gets aggregated in *all Nash equilibria* if the traded securities are *separable* and trading takes place for infinitely many periods. However, separability is not robust to small changes in the composition of the market or to the traders' information structure. As a result, a market designer who does not know who participates or what is their information structure, cannot be sure that the equilibrium price is a good predictor of the security's value.

In this paper, we examine whether the ability to acquire costly signals during trading can make markets more efficient at aggregating information. This question becomes more relevant as the continuous improvements in information technology have created an abundance of available information, which is now cheaper than ever to acquire, analyze, and act upon.[1]

We use the dynamic trading model of Ostrovsky [2012] with infinitely many periods and payoffs given by the Market Scoring Rule (MSR) [Hanson, 2003, 2007]. Each trader's private information is represented by a partition of the state space, and the conjunction of everyone's private information reveals the true value of the security. We first characterize the class of securities which are always separable, irrespective of who trades and what is their information structure. This class is very small and uninformative, as it only contains the Arrow-Debreu (A-D) security, which pays a at some state and b in all other states, and the security that specifies three payoffs: the largest is paid in one state of the world, the lowest in another, and the middle in all other states.[2] For any other security, there is a market (information structure and common prior) at which there is no information aggregation so that the security's price does not converge to its true value. This means that information may not aggregate in 'most' markets.

Does the availability of cheap information alleviate this problem? To study this question, we enhance the model by enabling traders to buy a costly signal structure in each period, before trading. We allow for a large class of information cost functions κ, including the Shannon entropy.

[1] For example, recent advances in generative AI tools such as ChatGPT could add considerable value for investors with information processing constraints [Kim et al., 2023] and assist in picking stocks [Pelster and Val, 2023].

[2] These securities are not very informative because the A-D can only predict whether one state has occurred or not, whereas the other security can only predict whether two states have occurred or not. Note that combining more than one A-D security to construct a composite and more informative security will not solve this issue, because it will be non-separable for some information structures.

We introduce the class of κ separable securities and show (Theorem 2) that they are necessary and sufficient for information aggregation, when the cost of information is κ, thus generalising Ostrovsky [2012]. As information acquisition costs κ decrease, the securities that eventually become κ separable, and therefore aggregate information, in *all* equilibria and for *all* information structures, have a very simple structure: they specify a different payoff at each state. This class of securities with unique values is generic. Hence, the main message of the paper is that the availability of cheap information makes 'most' markets aggregate information.[3]

Surprisingly, there is also a small class of securities, which Theorem 1 characterises, that never become κ separable for *all* information structures, and therefore may fail information aggregation, even when the cost converges (but is not equal) to zero.[4] Such a security specifies the same payoff d in two states and, in two other states, it pays either higher or lower than d. We therefore provide a complete classification of securities which depends only on their payoff structure: always separable, κ separable for some κ, and κ non-separable for all κ.

If we can decrease the cost of information as much as we want, do we even need markets to aggregate information through prices? Each trader could buy the necessary signals and then bid very close to the true value. We argue that this intuition is incorrect, because markets become even more important in an environment with information acquisition. As cost κ decreases, a security switches discontinuously from non-separable to κ separable, hence even a slight reduction can enable a market to aggregate information. This is in contrast to the average of the traders' opinions after receiving the information, or a poll, because its predictive accuracy improves smoothly as costs decrease. Hence, the availability of cheap information leverages the value of the markets, enabling them to aggregate information long before the cost goes to zero. Moreover, we show that a security is κ separable if and only if the market is more accurate than a poll, for all priors.

Finally, we examine whether information acquisition can make markets more efficient by aggregating information faster. We show that this is not true. Even in non-strategic environments, as long as the separable security is not A-D, information aggregation can happen both faster and slower, depending on the parameters. This implies that, as information becomes more affordable, markets could become

[3]This result is also supported by empirical evidence. Farboodi et al. [2022] show, using a structural model, that as the value of a firm's data grows, which is equivalent in our model to a decrease in the cost of information acquisition, the information content of the price of the firm's stock increases as well.

[4]If the cost of information acquisition is zero, then information aggregates trivially.

less efficient if the underlying security is not A-D.

1.1 Literature

Our paper contributes to three strands of the literature. The first studies the inefficiency of information acquisition and its effect on information aggregation in markets.[5] Pavan et al. [2022] show that traders acquire and use information inefficiently. Moreover, as the cost of information declines, traders over-invest in information acquisition and trade too much on their private information. Several experimental studies support these results and find that traders tend to over-acquire information. In addition, while information acquisition is positively correlated with market efficiency, market prices do not aggregate all private information [Kraemer et al., 2006, Page and Siemroth, 2017, 2021, Corgnet et al., 2022]. Mele and Sangiorgi [2015] analyze costly information acquisition in asset markets with ambiguity-averse traders and show that when uncertainty is high enough, information acquisition decisions become strategic complements and lead to multiple equilibria. Our paper complements and differs from this literature. We find that, as the cost of information acquisition decreases, the number of securities (and therefore markets) that aggregate information increases. However, some securities are never able to aggregate information, even if the cost is almost zero. Finally, information aggregation can be delayed when information acquisition is cheap, thus introducing another element of inefficiency.

The second strand looks at the information aggregation properties of financial and, in particular, prediction markets.[6] DeMarzo and Skiadas [1998, 1999] first introduced the notion of separable securities. Ostrovsky [2012] and Chen et al. [2012] show that in a market with dynamically consistent traders, separable securities are both necessary and sufficient for information aggregation. Dimitrov and Sami [2008] and Chen et al. [2010] examine information aggregation by varying the assumptions regarding the traders' information structure. Galanis et al. [2024] study information aggregation with ambiguity-averse traders, whereas Galanis and Kotronis [2021] allow for boundedly rational traders who are unaware of some contingencies.[7] We contribute to this literature by allowing traders to acquire costly signals at every period and we characterize the κ separable securities

[5] See Lim and Brooks [2011] for a survey of the empirical literature on market efficiency.

[6] See Wolfers and Zitzewitz [2004] for an early overview of the literature.

[7] Unawareness and ambiguity aversion generate dynamic inconsistency and negative value of information, which are partly responsible for no information aggregation. See Galanis [2011, 2013] for a model of unawareness and Galanis [2021] for a connection between dynamic inconsistency and the negative value of information.

which aggregate information.

Finally, the paper contributes to the growing literature on the implications of rational inattention, originated by Sims [2003]. We build on the results of [Denti, 2022, Caplin et al., 2019, Matějka and McKay, 2015, Caplin and Dean, 2015] to characterize the traders' optimal behavior in a game with infinitely many periods, where traders have posterior-separable cost functions and can buy signals in every period. We show that, in any Nash equilibrium, almost any security aggregates information for a sufficiently small marginal cost of information.[8] See also Maćkowiak et al. [2023] for a recent survey of the literature on rational inattention.

We conclude by motivating our choice of the MSR model. First, in the MSR model, there are no noise traders and no strategic market makers, hence the issue of information aggregation is not intertwined with that of information revelation, as in Kyle [1985].[9] Unlike the MSR, in Kyle [1985] it is not always the case that the price will converge to the true value of the security, even if there is only one trader and therefore information aggregation is achieved by default. Second, a prediction market with the MSR can be reinterpreted as an inventory-based market with a market maker who continuously adjusts the price of the securities depending on the orders she receives.[10] The advantage of the MSR over more well-known market mechanisms, such as the continuous double auction, is that an agent can make her prediction/trade without waiting for another agent to take the opposite side, or submit a limit order and wait for it to be filled. This feature makes it an attractive mechanism for markets with relatively few participants who do not trade daily, or in markets with automated market makers.[11] MSR-based prediction markets have been used widely, for example, by firms such as Ford, Google, General Electric, and Chevron (see Ostrovsky [2012], Cowgill and Zitzewitz [2015]) as well as governments, for example, in the UK and the Czech Republic (The Economist [2021]).

The paper adheres to the following plan. Section 2 describes the model. Section 3 shows that the class of always separable securities is very small and uninformative, thus prompting Section 4, where we include our main results in an environment with information acquisition. Section 6 concludes.

[8] Atakan and Ekmekci [2023] show that common value auctions with uninformed bidders who can acquire costly signals aggregate information as long as the minimum cost-accuracy ratio is equal to zero.

[9] Note that Ostrovsky [2012] uses both the MSR and the Kyle [1985] model.

[10] See Ostrovsky [2012] and Galanis et al. [2024] for examples.

[11] Automated market makers are widely used in Decentralized Finance, see Schlegel et al. [2022] for an axiomatization of the logarithmic MSR. Frongillo et al. [2023] show the equivalence of prediction markets and constant function market makers that are used overwhelmingly when trading on the blockchain.

2 The Model

2.1 Preliminaries

Uncertainty is described by a finite state space $\Omega = \{\omega_1, ..., \omega_l\}$ and the set of traders is denoted $I = \{1, \ldots, n\}$. Trader i's initial private information is represented by partition Π_i of Ω. Let $\Pi_i(\omega)$ be a partition element of Π_i that contains ω, so that $\omega \in \Pi_i(\omega) \in \Pi_i$. When the true state is $\omega \in \Omega$, Trader i considers all states in $\Pi_i(\omega) \subseteq \Omega$ to be possible. We assume that the join (the coarsest common refinement) of partitions $\Pi = \{\Pi_1, \ldots \Pi_n\}$ consists of singleton sets so that $\bigcap_{i \in I} \Pi_i(\omega) = \omega$ for all $\omega \in \Omega$, which means that the traders' pooled information always reveals the true state.[12] This implies that, for any two states $\omega_1 \neq \omega_2$, there exists Trader i such that $\Pi_i(\omega_1) \neq \Pi_i(\omega_2)$. Let $\mathcal{P}$ be the collection of all information structures Π where Ω has at least three states and $\bigcap_{i \in I} \Pi_i(\omega) = \{\omega\}$ for all $\omega \in \Omega$. Traders have a full-support common prior μ_0 over Ω and they are risk-neutral.

2.2 Trading environment

Trading is organized as follows. At time $t_0 = 0$, nature selects a state $\omega^* \in \Omega$ and the uninformed market maker makes a prediction y_0 about the value of security $X : \Omega \to \mathbb{R}$. At time $t_1 > t_0$, Trader 1 makes a revised prediction y_1, at $t_2 > t_1$ trader 2 makes his prediction, and so on. At time $t_{n+1} > t_n$, Trader 1 makes another prediction y_{n+1}, and the whole process repeats until time $t_\infty \equiv \lim_{k \to \infty} t_k = 1$. All predictions are observed by all traders. Each prediction y_k is required to be within the set $[\min_{\omega \in \Omega} X(\omega), \max_{\omega \in \Omega} X(\omega)]$. At some time $t^* > 1$ the true value $x^* = X(\omega^*)$ of the security is revealed.

The traders' payoffs are computed using a scoring rule, $s(y, x^*)$, where x^* is the true value of the security and y is a prediction. A scoring rule is *proper* if, for any probability measure p and any random variable X, the expectation of s is maximized at $y = E_p[X]$. It is *strictly proper* if y is unique. We focus on continuous strictly proper scoring rules. Examples are the quadratic, where $s(y, x) = -(x - y)^2$, and the logarithmic, where $s(y, x) = (x - a)ln(y - a) + (b - x)ln(b - y)$ with $a < \min_{\omega \in \Omega} X(\omega), b > \max_{\omega \in \Omega} X(\omega)$.

Under the market scoring rule (MSR) (McKelvey and Page [1990], Hanson

[12] This assumption is also made by Ostrovsky [2012] and it is without loss of generality because if the conjunction of the traders' private information does not reveal the state, we cannot expect that trading the security will reveal it.

[2003, 2007]), a trader is paid for each revision he makes. In particular, his payoff, from announcing y_n at t_n, is $s(y_n, x^*) - s(y_{n-1}, x^*)$, where y_{n-1} is the previous announcement and x^* is the true value of the security. For all proper scoring rules, as $E_q[X]$ converges to $X(\omega)$, $s(E_q[X], X(\omega))$ converges to 0. Moreover, if y_{n-1} is further away from $X(\omega)$ than $E_q(X)$ is from $X(\omega)$, then $s(E_q[X], X(\omega)) - s(y_{n-1}, X(\omega))$ is strictly positive. We then say that the trader "buys out" the previous trader's prediction. If he repeats the previous announcement, his period payoff is zero. We say that prior μ is non-degenerate given security X if it does not assign probability 1 to a unique value of X.

2.3 Two trading settings

We examine trading in two settings. In the myopic, or non-strategic, setting, each trader does not care about future payoffs when acquiring information and making an announcement. We denote this setting by $\Gamma^M(\Omega, I, \Pi, X, y_0, \mu, \underline{y}, \overline{y}, s)$, where I is the set of n players, s is a strictly proper scoring rule, y_0 is the market maker's initial announcement at time t_0, μ is the common prior, $[\underline{y}, \overline{y}]$ is the set of possible announcements.

The strategic setting is studied in Section 4.5. Following Dimitrov and Sami [2008], we focus on the discounted MSR, which specifies that the payment at t_k is $\beta^k(s(y_{t_k}, x^*) - s(y_{t_{k-1}}, x^*))$, where $0 < \beta \leq 1$ is the common discount factor. The total payoff of each trader is the sum of all payments for revisions. We denote this setting by $\Gamma^S(\Omega, I, \Pi, X, y_0, \mu, \underline{y}, \overline{y}, s, \beta)$. In Section 4, we will add function h in the specification of the game, which denotes the cost of statistical experiments.

2.4 Information aggregation

We say that information is aggregated if the traders' predictions converge to the intrinsic value $X(\omega)$ of security X, for all $\omega \in \Omega$. For every $\omega \in \Omega$, let $y_k(\omega)$ be the announcement of the trader who moves in period t_k. The announcement $y_k(\omega)$ depends on ω because traders have different private information across states. Because $\{y_k\}_{k=1}^{\infty}$ is a sequence of random variables, we need a probabilistic version of convergence.

Definition 1. *Under a profile of strategies in Γ^M or Γ^S, information aggregates if sequence $\{y_k\}_{k=1}^{\infty}$ converges in probability to random variable X.*

2.5 Example

The following Example illustrates how we incorporate costly information acquisition in the model of Ostrovsky [2012].

Example 1. *The state space is* $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4\}$*, the security is* $X = (0, \frac{10}{7}, 0, 1)$*, and there are two myopic traders with common prior* $\mu = (\frac{1}{4}, \frac{1}{6}, \frac{1}{4}, \frac{1}{3})$*. Trader 1's partition is* $\{\{\omega_1, \omega_2\}, \{\omega_3, \omega_4\}\}$ *and Trader 2's is* $\{\{\omega_1, \omega_4\}, \{\omega_2, \omega_3\}\}$*.*

As traders are myopic, under the MSR they take turns in announcing their expected value of X. Suppose that the true state is ω_1 and traders cannot acquire costly signals. In period 1, Trader 1 with conditional beliefs $(\frac{3}{5}, \frac{2}{5}, 0, 0)$ will announce $\frac{4}{7}$. The same announcement would be made by Trader 1 in all states and, thus, no information is transmitted to Trader 2. In period 2, Trader 2 also makes the same announcement at ω_1 and, furthermore, the same announcement would be made in all states. Hence, no information is transmitted to Trader 1. Because the two traders agree on the announcement, there is no information updating and the process ends. We say that there is no information aggregation at ω_1 because the final announcement $\frac{4}{7}$ is not equal to the intrinsic value of X at ω_1, which is 0.[13]

Suppose now that traders are able to acquire a noisy signal about the value of the security before making their announcement. For instance, Trader 1 can acquire a statistical experiment $\mathcal{R}$ that generates a signal z, given each state, with probabilities $\mathcal{R}_{\omega_1}(z) = 1$ and $\mathcal{R}_{\omega_2}(z) = \mathcal{R}_{\omega_3}(z) = \mathcal{R}_{\omega_4}(z) = 0.5$.

At state ω_1, Trader 1's prior belief is $(\frac{3}{5}, \frac{2}{5}, 0, 0)$ and, after receiving signal z, his posterior belief is $(\frac{3}{4}, \frac{1}{4}, 0, 0)$. He then announces the expected value of X, which is $\frac{5}{14}$. Trader 2 considers states ω_1 and ω_4 to be possible. The public announcement of $\frac{5}{14}$ reveals to Trader 2 that the true state is ω_1. The reason is that, irrespective of whether Trader 1 received signal z or not, his posteriors at ω_4 are $(0, 0, \frac{3}{7}, \frac{4}{7})$ and he would have announced $\frac{4}{7}$. As a result, Trader 2 announces 0 in the second round and the game ends. Note that the final price is equal to the intrinsic value of X at ω_1, hence information aggregation does occur. In summary, the ability to acquire extra signals transforms X from non-separable to separable, enabling information aggregation.

We make two observations. First, we have abstracted from the cost of acquiring an experiment. Each trader will acquire the signal structure only if the expected

[13] Information aggregation fails in the first round in this example and no-one updates from μ. In general, traders could start from a different common prior μ', and after several rounds of updating they could update to some other posterior μ'', at which there is no information aggregation.

benefit from making a better prediction outweighs the cost. See Section 4 for the formal treatment. Second, Trader 2 free-rides on Trader 1 buying the signal. By moving the price from $\frac{5}{14}$ to 0, he books a profit, without paying the cost of a signal. This example illustrates that the ability to acquire information can turn a non-separable security into a κ separable.

Security X is κ separable if whenever there is agreement about the expected value of X, given a prior that does not put probability 1 to only one value of X, then at least one (myopic) trader finds it profitable to acquire information. It is therefore a generalization of separability, which requires that such a prior does not exist. In Section 4.3, we formally define the notion of κ separability and Theorem 2 shows that this class of securities characterizes information aggregation when the cost is κ. A natural question is whether all securities eventually become κ separable, for sufficiently low cost of information. Surprisingly, Theorem 1 shows that there is a small class of securities that never become κ separable, even if the cost is negligible but strictly positive. We therefore have a discontinuity as costs converge to zero.

3 No Information Acquisition

In an environment without information acquisition, Ostrovsky [2012] shows that separable securities are necessary and sufficient for aggregating information. In this section, we first define the class of separable securities. We then characterize the class of always separable securities, which are separable for all information structures, and show that it is small and uninformative.

3.1 Separable Securities

Consider the following example, which appears in similar form in Geanakoplos and Polemarchakis [1982] and in Example 1 of Ostrovsky [2012].

Example 2. *The state space is* $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4\}$*, the security is* $X = (0, 1, 0, 1)$*, and there are two traders with common prior* $\mu = (1/4, 1/4, 1/4, 1/4)$*. Trader 1's partition is* $\{\{\omega_1, \omega_2\}, \{\omega_3, \omega_4\}\}$ *and Trader 2's is* $\{\{\omega_1, \omega_4\}, \{\omega_2, \omega_3\}\}$*.*

At all states, it is common knowledge that both traders agree that the expected value of X is 0.5, hence there can be no more learning from further announcements.[14] However, it is also common knowledge that the intrinsic value of X is

[14] Note that agreement about the value of the security will eventually be reached at some period, as shown by Geanakoplos and Polemarchakis [1982].

not 0.5; it is either 0 or 1, which implies that there is no information aggregation. When both these conditions are satisfied for some prior and some partitions, the security is non-separable. If there is not such prior, the security is separable.

Definition 2. *A security X is called non-separable under information structure Π if there exists probability μ and value $v \in \mathbb{R}$ such that:*

(i) $X(\omega) \neq v$ *for some* $\omega \in Supp(\mu)$*,*

(ii) $E_\mu[X|\Pi_i(\omega)] = v$ *for all* $i = 1, ..., n$ *and* $\omega \in Supp(\mu)$*.*

We then say that security X is non-separable at μ. Otherwise, it is called separable.

A security X is non-separable if, for some prior μ and at all states in its support, all traders' expected value of X is v, yet there is uncertainty about the value of X. If for any prior at least one condition is violated, then the security is separable. Note that separability (and non-separability) is a property that depends on the information structure Π. For a different Π, a security may switch from separable to non-separable and vice-versa. The following theorem shows that separability characterizes information aggregation, in an environment without information acquisition.

Theorem (Ostrovsky [2012])**.** *Fix information structure Π. Then:*

- *If security X is separable under Π, then in any Nash equilibrium of game $\Gamma^S(\Omega, I, \Pi, X, y_0, \mu, \underline{y}, \overline{y}, s, \beta)$, information gets aggregated.*

- *If security X is non-separable under Π, then there exists prior μ such that for all s, y_0, $\underline{y}$, $\overline{y}$, and β, there exists a Perfect Bayesian equilibrium of the corresponding game Γ^S in which information does not get aggregated.*

This is a powerful result because it applies to all equilibria and irrespective of the market power of traders. Separable securities can be very useful for a market designer because they always aggregate information, hence the price can predict whether an event has occurred or not. However, separability depends on the information structure Π. If the market maker does not know Π, he cannot be certain that the equilibrium price is a good predictor of the intrinsic value of the security.

3.2 Always Separable Securities

The dependence of separability on Π is not a problem if the security is separable *for all* information structures. In this section, we characterize the securities that have this property, so that information aggregation does not depend on who trades or what is their private information.

Unfortunately, the following Proposition shows that this class is very small and uninformative. It consists of just three types of securities. The first is the constant, which pays the same at all states. The second is the Arrow-Debreu (A-D), which pays a at some state ω and b at all other states. The third pays a at some state ω, d at ω', and b at all other states, where $a < b < d$.

Proposition 1. *The only non-constant securities that are separable for all information structures in $\mathcal{P}$ are the A-D and the security that is of the following form. There are values $a < b < d$ such that $X(\omega_a) = a$ and $X(\omega_d) = d$ for two states ω_a, ω_d, and $X(\omega) = b$ for all $\omega \neq \omega_a, \omega_d$.*

See proof on page 31.

All three types of securities are 'uninformative'. Even when there is information aggregation and the price of X always converges to the true value $X(\omega)$ at state ω, this only reveals that either ω, ω', or neither, have occurred. In other words, the price of X does not reveal information about most events in Ω. In contrast, the most informative security X pays differently across all states. If there is information aggregation, then X reveals whether any event in Ω has occurred or not. However, because X is not always separable, we know that for some information structure Π, information does not aggregate.

4 Information Acquisition

In this section, we show how the problem of non-separability and no information aggregation can be alleviated if we allow traders to acquire information in every period where they make an announcement. Note that the join of the partitions of all traders reveals the true value of security X, hence buying costly signals is 'wasteful', in the sense that a trader learns something that other traders already know. However, there is value in allowing information acquisition, because it eliminates the bad equilibria where there is no information aggregation, as shown in Theorem 2.

We define a new class of securities, called κ separable, where κ is the cost of acquiring information. Our first main result, Theorem 1, characterizes the securities that are κ non-separable for some information structure Π but for all κ. Our second main result, Theorem 2, shows that κ separable securities characterize information aggregation, in both strategic and non-strategic settings, thus generalizing Ostrovsky [2012] in an environment with information acquisition.

4.1 Cost of Information

We first formalise the cost of acquiring information. For ease of exposition, many of the technical details are relegated to Section A.1 in the Appendix.

Let T be a Polish space of possible signals with Borel σ-algebra $\mathcal{T}$. Let $\Delta(T)$ be the set of Borel probabilities on T. Following Blackwell [1951], we model the acquisition of information using statistical experiments. A statistical experiment is a function from states into probabilities on signals, $\mathcal{R} : \Omega \to \Delta(T)$. An experiment is bounded if it does not definitely rule out any state. Let $\mathcal{E}$ be the collection of all experiments and $\mathcal{E}_b$ be the collection of all bounded experiments. An experiment is bounded if no signal can reveal, with probability one, that a state in Ω has not occurred.

Given a prior belief $\mu \in \Delta(\Omega)$, a statistical experiment $\mathcal{R}$ induces via Bayesian updating a probability distribution over posteriors, or random posterior, $B(\mu, \mathcal{R}) = Q \in \Delta(\Delta(\Omega))$. Let $Q(\gamma)$ be the probability of posterior $\gamma \in \Delta(\Omega)$, $\mathcal{Q}(\mu) = \{B(\mu, \mathcal{R}) : \mathcal{R} \in \mathcal{E}\}$ be the set of all random posteriors that can be generated by some experiment $\mathcal{R} \in \mathcal{E}$, and $\mathcal{Q} = \bigcup_{\mu \in \Delta(\Omega)} \mathcal{Q}(\mu)$. Note that $Q \in \mathcal{Q}(\mu)$ if and only if $\int_{\Delta(\Omega)} \gamma Q(d\gamma) = \mu$.

A cost on experiments is a map $h : \Delta(\Omega) \times \mathcal{E} \to [0, \infty]$, where $h(\mu, \cdot)$ is Borel measurable for each prior $\mu \in \Delta(\Omega)$. The cost h generates a cost structure $\kappa = (K, c)$ on random posteriors, where $c > 0$ is the unit cost of information and $K : \mathcal{F} \to [0, \infty]$ maps elements of $\mathcal{F} = \{(\mu, Q) : \mu \in \Delta(\Omega), Q \in \mathcal{Q}(\mu)\}$, the set of all priors μ and all posterior distributions Q that can be generated by some experiment $\mathcal{R} \in \mathcal{E}$, to the extended real line.

For Theorem 1 and Proposition 5, we assume that it is prohibitively costly to acquire an unbounded experiment.

Assumption 1. *If $\mathcal{R} \notin \mathcal{E}_b$, then $h(\mu, \mathcal{R}) = \infty$.*

If all unbounded experiments have infinite cost, then generating a random posterior with support on a posterior δ_ω, which assigns probability 0 to a state $\omega \in Supp(\mu)$, is infinitely costly.

A widely used class of functions is the posterior-separable cost functions (Caplin et al. [2022]).

Definition 3. *A cost of information function K is posterior-separable if, given $\mu \in \Delta(\Omega)$ and any Bayes-consistent posteriors $Q \in \mathcal{Q}(\mu)$,*

$$K(\mu, Q) = \sum_{\gamma \in Supp(Q)} Q(\gamma) W_\mu(\gamma)$$

for some function $W_\mu : \Delta(Supp(\mu)) \to \overline{\mathbb{R}}$ *which is strictly convex and continuous in* γ, $W_\mu(\gamma) < \infty$ *on* $int\Delta(Supp(\mu))$ *and* $W_\mu(\gamma) \geq 0$.

An example of a posterior-separable cost function that satisfies Assumption 1 is the Shannon cost function,

$$K^S(\mu, Q) = \sum_{\gamma \in Supp(Q)} Q(\gamma) \sum_{\omega \in Supp(\gamma)} \gamma(\omega) \ln \gamma(\omega) - \sum_{\omega \in Supp(\mu)} \mu(\omega) \ln \mu(\omega).$$ [15]

We have described two ways of representing the cost of information acquisition. The first is in terms of costly statistical experiments, whereas the second is in terms of costly random posteriors. There is a growing literature that examines the connection between the two approaches. Bloedel and Zhong [2020] and Hébert and Woodford [2021, 2023] show how costly statistical experiments, with a function h that depends both on the agent's prior and the experiment, can generate uniformly posterior-separable cost functions for random posteriors. Denti et al. [2022], however, restrict h to depend only on the experiment and show that uniformly posterior-separable cost functions cannot be generated.

We do not take a stance on which representation is the most suitable. Our results do not require any specific functional forms, such as posterior separability, however, we adopt this framework for most of the paper because it is the most well-known. When we define the game in Section 4.5, we use the standard framework of costly statistical experiments with function h.

4.2 The myopic problem

Suppose that at time t it is Trader i's turn to make an announcement. Having observed all previous announcements and using the public information that is revealed, an outside observer updates the common prior μ_0 to a belief μ over Ω. If the true state is ω, then Trader i's private information is $\Pi_i(\omega)$ and his posterior belief is the Bayesian update of μ, denoted $\mu_{\Pi_i(\omega)}$. In other words, he updates using both his private information and the public information revealed by previous announcements.

His myopic problem consists of buying a random posterior $Q \in \mathcal{Q}(\mu_{\Pi_i(\omega)})$ at cost $cK(\mu_{\Pi_i(\omega)}, Q)$, so that when his posterior beliefs are $\gamma \in Supp(Q)$, he

[15] Another example is expected Tsallis entropy (Tsallis [1988]). Both are examples of weakly uniformly posterior-separable cost functions, where K depends on μ only through the support of μ (Caplin et al. [2022]). Denti [2022] characterizes posterior-separable and uniformly posterior-separable cost functions. Caplin and Martin [2015] and Caplin and Dean [2015] provide necessary and sufficient conditions for a data set to be represented by optimal choice subject to an information cost.

optimally announces $E_\gamma[X]$ because the scoring rule is proper. The optimal Q solves the problem

$$\sup_{Q\in\mathcal{Q}(\mu_{\Pi_i(\omega)})}\left(\sum_{\gamma\in Supp(Q)} Q(\gamma)\sum_{\omega'\in\Omega}\gamma(\omega')\Big[s(E_\gamma[X],X(\omega'))-s(z,X(\omega'))\Big]-cK(\mu_{\Pi_i(\omega)},Q)\right), \tag{1}$$

where $c>0$ is the unit cost of information and z is the previous announcement.

In the standard model of Ostrovsky [2012], without information acquisition, the previous announcement z does not influence the myopic best announcement because the scoring rule is proper. The same is true here, where we allow for information acquisition.[16] To see this, note that we can rewrite the expression in the parenthesis as

$$\sum_{\gamma\in Supp(Q)} Q(\gamma)\sum_{\omega'\in\Omega}\gamma(\omega')\Big[s(E_\gamma[X],X(\omega')))\Big]-\sum_{\omega'\in Supp(\mu)}\mu(\omega')s(z,X(\omega'))-cK(\mu_{\Pi_i(\omega)},Q),$$

hence the previous announcement z does not influence the choice of Q or the announcement.

Note that the trader can always receive a payoff of 0 by just repeating the previous announcement z and not acquiring any new information. If his payoff at (1) is smaller or equal to 0 for all $Q\in\mathcal{Q}(\mu_{\Pi_i(\omega)})$, we say that he does not prefer to acquire information at ω. If, given a common prior μ, no Trader i prefers to acquire information at any state $\omega\in Supp(\mu)$, we say that there is no information acquisition for security X.

Definition 4. *There is no information acquisition for security X, given prior μ and previous announcement z, cost structure $\kappa=(K,c)$ and information structure $\Pi\in\mathcal{P}$, if for all states $\omega\in Supp(\mu)$, all traders i, and all $Q\in\mathcal{Q}(\mu_{\Pi_i(\omega)})$,*

$$\sum_{\gamma\in Supp(Q)} Q(\gamma)\sum_{\omega'\in\Omega}\gamma(\omega')\Big[s(E_\gamma[X],X(\omega'))-s(z,X(\omega'))\Big]-cK(\mu_{\Pi_i(\omega)},Q)\leq 0.$$

Otherwise, there is information acquisition.

4.3 κ Separable Securities

The result of Proposition 1, that very few securities are always separable, indicates that 'most' markets may not aggregate information. A natural question is whether

[16] Interestingly, with ambiguity averse preferences, the myopic best depends on the previous announcement, as shown in Galanis et al. [2024].

the possibility of acquiring information fixes this problem. Intuitively, a non-separable security may still aggregate information if, whenever everyone makes the same announcement, at least one trader finds it profitable to acquire more information, thus changing his posterior and his announcement.

In this section, we define the class of κ separable securities, where $\kappa = (K, c)$ is the cost of information acquisition. In the next section, we show that they are necessary and sufficient for information aggregation, thus generalising the main result of Ostrovsky [2012].

We say that security X is κ non-separable given an information structure Π and a cost structure κ if there is a prior μ such that no one is acquiring any information, yet the security is non-separable at μ.

Definition 5. *Security X is κ non-separable given an information structure $\Pi \in \mathcal{P}$ and cost structure $\kappa \in \mathcal{K}$, if there exists prior μ such that*

- *Security X is non-separable at μ,*
- *There is no information acquisition for X given μ.*

Otherwise, security X is κ separable.

Recall that a non-separable security is non-separable for at least one μ, whereas a separable security is separable for all μ. If security X is κ separable, then for each μ there are two cases. Either X is separable at μ, or X is non-separable at μ but there is information acquisition.

We make the following remarks. First, separability implies κ separability, for all κ, because there does not exist a prior at which the security is non-separable. Second, non-separability implies κ non-separability for some κ, because for a high enough marginal cost c, no trader will acquire any information.

Remark 1. *If security X is separable given an information structure $\Pi \in \mathcal{P}$, then it is also κ separable for any cost structure $\kappa \in \mathcal{K}$.*

Remark 2. *If security X is non-separable given an information structure $\Pi \in \mathcal{P}$, then for any cost function K it is $\kappa = (K, c)$ non-separable for some marginal cost of information c.*

Note that if X is $\kappa = (K, c)$ non-separable, then there is a μ for which there is no information acquisition and X is non-separable at μ. If we increase the marginal cost to $c' > c$, then there is still no information acquisition for μ, hence X is $\kappa' = (K, c')$ non-separable. Conversely, if X is $\kappa = (K, c)$ separable, it is separable for all μ for which there is no information acquisition. If we decrease the marginal cost to $c' < c$, then the set of priors for which there is no information acquisition will shrink and therefore X will be $\kappa' = (K, c')$ separable for all $c' < c$. We, therefore, have the following remark.

Remark 3. *If X is $\kappa = (K, c)$ non-separable (separable), then it is $\kappa' = (K, c')$ non-separable (separable) for all $c' > c$ ($c' < c$).*

It would be natural to expect that, for a sufficiently low marginal cost c, any security eventually becomes κ separable. But this is not true. Recall Example 2 with state space $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4\}$, security $X = (0, 1, 0, 1)$, and two traders with the following information structure. Trader 1's partition is $\{\{\omega_1, \omega_2\}, \{\omega_3, \omega_4\}\}$ and Trader 2's is $\{\{\omega_1, \omega_4\}, \{\omega_2, \omega_3\}\}$. With a uniform prior, all traders would agree that the expected value of X is 0.5, at all states, hence the security is non-separable.

Consider now prior $\mu = (\frac{1-2m}{2}, m, \frac{1-2m}{2}, m)$, where $0 < m < 0.5$. Given any m, each trader's expected value of X at all states is $2m$. As c converges to 0, we can always specify m that is sufficiently close to 0.5, so that no trader is willing to buy a signal, because this will move their expected value of X from $1 - 2\epsilon$ to $1 - \epsilon$, for small enough $\epsilon > 0$, hence the expected benefit is lower than c. Because there is no information acquisition and all traders agree on the expected value, security X is $\kappa = (K, c)$ non-separable for all $c > 0$.

Note that this result requires Assumption 1, which specifies that it is impossible to buy an unbounded experiment that reveals the true state with certainty.[17] Moreover, all traders agree, at some state ω, that at least two values are possible, 0 and 1. Proposition 5 in the Appendix shows that this condition is necessary for a security to be κ non-separable for all κ and a specific information structure. Additionally, it specifies a second, independent condition, which is an extension of Theorem 7 in Ostrovsky [2012], a dual characterisation of separability.

However, what if we do not know the information structure of traders? The following Theorem shows that security X is κ non-separable given *some* Π and *for all* κ, if and only if there are four states at which X pays (a, d, b, d), where either $a, b < d$ or $a, b > d$.

[17] It is interesting to note that the incompatibility of the $(0, 1, 0, 1)$ vector with information aggregation is met in other settings as well. In the model of DeGroot [1974], agents update their beliefs naively, by looking at their immediate neighbours, according to a fixed network. If the network is represented by a periodic matrix, such as $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, then beliefs do not converge (Golub and Jackson [2010]).

Theorem 1. *Suppose that Ω has at least four states. Under Assumption 1, the following are equivalent:*

- *X is κ non-separable given some Π and for all κ,*
- *X pays (a, b, d, d) in four states, where either $a, b < d$ or $a, b > d$.*

See proof on page 33.

A Corollary is that if security X pays differently across all states, then it is κ separable for some κ, given *any* information structure Π. In other words, if information is sufficiently cheap to acquire, there are easily describable securities where information aggregation is achieved and this is robust to changes in who participates in the market and what is their private information.

Corollary 1. *If $X(\omega) \neq X(\omega')$ for all $\omega, \omega' \in \Omega$, then, given any Π, X is κ separable for some κ.*

This Corollary echoes the following result within the context of perfectly competitive markets, as pointed out in Laffont and Maskin [1990]. As long as there are "enough prices", so that trade can be made contingent on sufficiently many events, then the competitive equilibrium is generically separating and the function relating information to prices is invertible (Grossman [1976], Radner [1979], Allen [1981]). We could interpret Corollary 1 as a generalisation of this result, because our setting applies to all Nash equilibria, not just competitive equilibria.

4.4 Classification of securities

We provide a complete classification of securities in terms of how well they aggregate information, which surprisingly depends only on their payoff structure. There are three types of securities: always separable, κ non separable for all κ and *some* information structure, and κ separable for some κ and *all* information structures. This classification does not depend on who is trading or what is their information structure. Moreover, it is very simple to classify each security, which is not true when determining whether a security is separable or not, given an information structure.

Let X be a security defined on Ω that has at least four states. If there is m such that $X(\omega') = X(\omega'') = m$ for at least two states $\omega', \omega'' \in \Omega$, let M be the set of states ω with $X(\omega) = m$. Otherwise, set $M = \emptyset$.

Case 0. $M = \Omega$. Security X is constant and therefore (trivially) always separable, for all information structures Π.

Case 1. There is either a unique $\omega \notin M$, so that the security is A-D, or there are exactly two states $\omega, \omega' \notin M$, such that $X(\omega) > m > X(\omega')$. From Proposition 1, X is always separable, for all information structures Π.

Case 2. There exist $\omega, \omega' \notin M$ with either $X(\omega), X(\omega') < m$, or $X(\omega), X(\omega') > m$. From Theorem 1, X is κ non-separable for all κ, for some information structure Π.

Case 3. $M = \emptyset$, hence $X(\omega) \neq X(\omega')$, for all $\omega, \omega' \in \Omega$. From Corollary 1, X is κ separable for some κ, for all information structures Π.

It is interesting to note that, within our framework that has continuous values and finitely many states, all cases except Case 3 are of measure zero. Hence, if we were to randomly pick a security, generically we would pick one with unique values which aggregates information in *all* equilibria and given *any* information structure, as long as the cost of information is sufficiently low.

4.5 Strategic traders

We now show that κ separable securities characterize information aggregation in strategic settings where there cost of information acquisition is κ. Consider game $\Gamma^S(\Omega, I, \Pi, X, h, y_0, \mu, \underline{y}, \overline{y}, s, \beta)$, where I is the set of n players, s is a strictly proper scoring rule, y_0 is the market maker's initial announcement at time t_0, μ is the common prior, h is the cost of statistical experiments, $[\underline{y}, \overline{y}]$ is the set of possible announcements, and β is the common discount rate. Let $(y_1, \ldots, y_k) \in [\underline{y}, \overline{y}]^k$ be a history of announcements and $(\tau_1, \ldots, \tau_k) \in T^k$ be a history of signals up to time t_k. Denote by T_i^k the collection of all histories about i's signals up to time t_k.

At time t_k, Trader i with belief μ can choose to acquire information in the form of a statistical experiment $\mathcal{R} \in \mathcal{E}$, with cost $h(\mu, R)$. Each experiment $\mathcal{R}$ and belief μ uniquely induce a Bayesian plausible belief Q over posteriors, which costs $cK(\mu, Q) = h(\mu, R)$. We can therefore analyse the game in terms of an information cost structure $\kappa = (K, c)$, which is generated by h.

After receiving a signal τ of experiment $\mathcal{R}$ and updating to a posterior γ in the support of Q, Trader i makes an announcement y_k. Let $[\underline{y}, \overline{y}]^T$ be the set of all functions from signals to announcements in $[\underline{y}, \overline{y}]$. His mixed strategy at time t_k is a measurable function

$$\sigma_{i,k} : \Pi_i \times T_i^{k-1} \times [\underline{y}, \overline{y}]^{k-1} \times [0,1] \longrightarrow \mathcal{E} \times [\underline{y}, \overline{y}]^T.$$

It specifies a statistical experiment and an announcement for each signal that is drawn from that experiment, given the element of his partition, the history of his past signals and everyone's announcements up to time t_k, and the realization of random variable $\iota_k \in [0,1]$, which is drawn from the uniform distribution. One such draw takes place at each time t_k and the draws are independent of each other and of the true state ω.[18]

The full state is $\phi = (\omega, \iota_1, \iota_2, \ldots)$ and describes the initial uncertainty and the randomizations of the players. Let $X(\phi) \equiv X(\omega)$ be the true value of X at $\phi = (\omega, \iota_1, \iota_2, \ldots)$ and $\Phi = \Omega \times [0,1]^{\mathbb{N}}$ be the full state space. Denote by σ_i the collection $\sigma_{i,k}$ of i's strategies, at all times t_k where it is his turn to make an announcement. Let $\sigma = (\sigma_1, \ldots, \sigma_n)$ be a profile of strategies. Given a strategy σ and state ϕ, let $y_{i+nk}(\sigma, \phi)$ be the announcement of Trader i, $\mu_{i+nk}(\sigma, \phi)$ his belief, and $cK(\mu_{i+nk}(\sigma, \phi), Q_{i+nk}(\sigma, \phi))$ his cost of information acquisition in period t_{i+nk} which is generated by the experiment he has chosen and his current beliefs.

Definition 6. *A strategy profile σ is a Nash equilibrium if, for every Trader i and every alternative strategy $\sigma' = (\sigma_{-i}, \sigma'_i)$, we have*

$$E_\mu\left[\sum_{k=0}^{\infty} \beta^{i+nk}\left(s\Big(y_{i+nk}(\sigma,\phi), X(\phi)\Big) - s\Big(y_{i+nk-1}(\sigma,\phi), X(\phi)\Big) - cK\Big(\mu_{i+nk}(\sigma,\phi), Q_{i+nk}(\sigma,\phi)\Big)\right)\right] \geq$$

$$E_\mu\left[\sum_{k=0}^{\infty} \beta^{i+nk}\left(s\Big(y_{i+nk}(\sigma',\phi), X(\phi)\Big) - s\Big(y_{i+nk-1}(\sigma',\phi), X(\phi)\Big) - cK\Big(\mu_{i+nk}(\sigma',\phi), Q_{i+nk}(\sigma',\phi)\Big)\right)\right],$$

where the expectation is taken with respect to the common prior μ.

We now show that κ separable securities characterize information aggregation when the cost structure is $\kappa = (K, c)$. Note that Assumption 1 is not needed for this result.

Theorem 2. *Fix information structure Π and cost of experiments h, which generates cost structure κ. Then:*

- *If security X is κ separable under Π, then in any Nash equilibrium of game $\Gamma^S(\Omega, I, \Pi, X, h, y_0, \mu, \underline{y}, \overline{y}, s, \beta)$, information gets aggregated.*
- *If security X is κ non-separable under Π, then there exists prior μ such that for all s, y_0, $\underline{y}$, $\overline{y}$, and β, there exists a Perfect Bayesian equilibrium of the corresponding game Γ^S in which information does not get aggregated.*

See proof on page 35.

[18] This formulation, with $[0,1]$ denoting the trader's randomisation, is taken from Ostrovsky [2012].

Although Theorem 2 ensures that information aggregates in all Nash equilibria, it does not guarantee that an equilibrium exists. This is true also in Ostrovsky [2012], and the main reason is that the action spaces are infinite. If we descretize the action spaces, so that traders can pick from a finite set of announcements and signals, then a Perfect Bayesian Equilibrium exists whenever the time horizon is finite. Because $\beta < 1$, game Γ^S is continuous at infinity. This means that we can adapt the proofs of Fudenberg and Levine [1983, 1986] to approximate the infinite horizon game with a sequence of finite horizon games and show that the sequence of Perfect Bayesian equilibria in the finite games converges to a Perfect Bayesian equilibrium in the infinite game.[19]

4.6 The value of the market

If the cost of information drops significantly, do we even need markets to aggregate information through prices? Each trader could buy the necessary signals and then trade. In this section, we argue that this intuition is not correct, because markets become even more important in an environment with information acquisition. The reason is that markets can aggregate information before it becomes economically viable for each trader to acquire the required information on their own.

To make this point, we compare the prediction accuracy of the market with that of a poll, where traders simultaneously make only one announcement and we compute the average.[20] Recall from Section 4.5 that the full state $\phi = (\omega, \iota_1, \iota_2, \dots)$ describes the initial uncertainty $\omega \in \Omega$ and the randomizations of the players, as well as the signal realisations.

Definition 7. *For each $\phi \in \Phi$ and common prior μ, the prediction of the poll is the average of the myopic predictions, y_i, where each Trader i optimally obtains a*

[19] Ostrovsky [2012] describes such an approach but does not provide any details. The full construction is provided in the Supplementary Appendix of Galanis et al. [2024], for the identical game without information acquisition but with ambiguity aversion.

[20] Note that there are many ways of improving the accuracy of a poll by aggregating announcements differently [Baron et al., 2014]. In our framework, markets will always be more accurate than polls because more information is disseminated through multiple rounds of announcements, and the value of information is positive. Several papers have examined the two settings in experiments and real-life settings, and the results are mixed. Snowberg et al. [2013] argue that prediction markets are better. Berg et al. [2008] show that the Iowa Electronic Markets were more accurate than 964 polls in predicting the outcomes of five presidential elections between 1988 and 2004. Cowgill and Zitzewitz [2015] show that internal prediction markets in Google and Ford were more accurate than the predictions of professional forecasters. On the other hand, Atanasov et al. [2017], Dana et al. [2019] argue that while prediction markets are more accurate than the simple mean of forecasts from polls, the latter outperform prediction markets when forecasts are aggregated with transformation algorithms or made in teams. Camerer et al. [2016] show that markets are equally accurate with a survey in predicting the replicability of economic experiments.

random posterior Q and then they all announce simultaneously in the first period of the corresponding game Γ^M:

$$p^p(\phi,\mu) = \frac{\sum_{i=1}^{n} y_i}{n}.$$

The prediction $p^m(\phi,\mu)$ of the market is the last price of Γ^S.

We define the accuracy of the market given ϕ and μ as $A^m(\phi,\mu) = 1 - |p^m(\phi,\mu) - X(\phi)|$, and similarly for the poll, $A^p(\phi,\mu) = 1 - |p^p(\phi,\mu) - X(\phi)|$. The highest possible accuracy is 1, when $p^m(\phi,\mu) = X(\phi)$. The expected accuracy of the market given μ is defined as $A^m(\mu) = E_\mu A^m(\phi,\mu)$, whereas for the poll it is $A^p(\mu) = E_\mu A^p(\phi,\mu)$.

Recall that a non-degenerate prior μ given X does not assign probability 1 to a unique value of security X. If the security is κ separable for some κ, then the information gets aggregated for some positive marginal cost of information, which is not true for polls. We, therefore, have the following remark.

Remark 4. *Under Assumption 1, fix an information structure $\Pi \in \mathcal{P}$ and a cost of information acquisition $\kappa = (K, c)$. If security X is κ separable, then for any non-degenerate μ given X and for all $0 < c' \leq c$, information gets aggregated by a market with cost $\kappa' = (K, c')$, so that $A^m(\phi,\mu) = 1$ for all $\phi \in Supp(\mu)$, but it is not aggregated by the poll, so that $A^p(\phi,\mu) < 1$.*

Proof. Remark 3 shows that X is κ' separable, hence the first part follows directly from Theorem 2, which shows that information gets aggregated. Assumption 1 implies that individual traders would never acquire full information in the first round. Because μ is non-degenerate given X, we have that $p^p \neq X(\phi)$. □

We now show that κ separability is equivalent to the market being strictly more accurate than the poll, for all non-degenerate priors given X.

Proposition 2. *Suppose Assumption 1 and cost of information κ. Security X is κ separable given $\Pi \in \mathcal{P}$ if and only if $A^m(\mu) > A^p(\mu)$ for all non-degenerate priors μ given X.*

See proof on page 37.

To interpret this result, suppose we define the value of the market (with security X) to be $V(X) = \min_{\mu \in \Delta_0} A^m(\mu) - A^p(\mu)$, the minimum improvement in accuracy given all non-degenerate priors given X. Then, Proposition 2 implies that X is κ separable if and only if $V(X) > 0$.

We conclude with the following example, which shows how the market compares in prediction accuracy with the poll, if we fix a prior μ for which the security is non-separable and we vary the marginal cost of information c.

Example 3. *The state space is* $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4\}$*, the security is* $X = (0, 1, 2, 3)$ *and the common prior is* $\mu = (\frac{1}{8}, \frac{3}{8}, \frac{3}{8}, \frac{1}{8})$*. Trader 1's partition is* $\{\{\omega_1, \omega_3\}, \{\omega_2, \omega_4\}\}$ *and Trader 2's is* $\{\{\omega_1, \omega_4\}, \{\omega_2, \omega_3\}\}$*.*

Security X is non-separable μ because the expected value of X for both traders is $v = \frac{3}{2}$ at all states, yet there is uncertainty about the value of the security. Traders can acquire information, where K is the Shannon cost of information. We assume that traders are myopic and proper scoring rules are used in both settings, hence each announcement is the expected value of the security given the acquired information.

Figure 4.6 shows the expected accuracy of markets and polls for prior μ. When the marginal cost of information is high (c>4.5), no trader acquires any information and the accuracy of the market is equal to that of the poll. As the marginal cost decreases below 4.5, Trader 2 starts acquiring information if the state is either 1 or 4. This implies that his announcement differs across partitions, revealing whether event $\{\omega_1, \omega_4\}$ or $\{\omega_2, \omega_3\}$ is true. Trader 1 combines this with his private information and learns which state is true, thus announcing $v = X(\omega)$. Therefore, a small change in the marginal cost of information allows the market to aggregate information, with a prediction accuracy of 1. In contrast, the poll's accuracy improves gradually as information gets cheaper. This means that, as the cost of information decreases, the prediction accuracy of the market suddenly jumps to 1, whereas the accuracy of the poll gradually increases. Equivalently, the value $V(X)$ of the market is 0 for $c > 4.5$, at $c = 4.5$ it becomes positive and it decreases as $c < 4.5$ decreases, converging to 0 as $c \to 0$.

We conclude by observing that, in a strategic setting, markets can incentivize traders to acquire and utilize information more efficiently when the cost is close to the threshold of 4.5, because a small information acquisition could enable the aggregation of all available information, whereas in polls it can only marginally improve individual predictions.

5 The speed of information aggregation

Another interesting question about information acquisition is whether it makes the process of information aggregation conclude faster. If this were the case, the

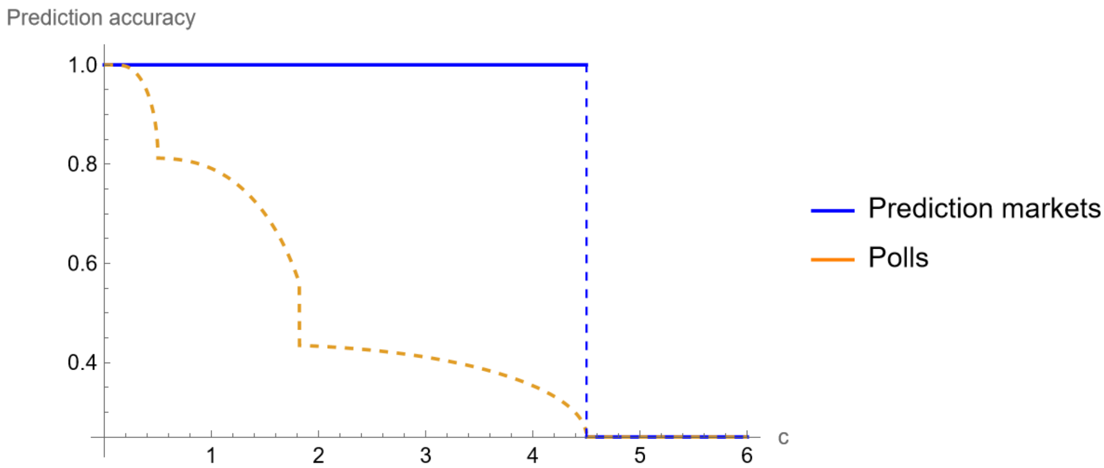

Figure 1: Prediction accuracy $(A(\mu) = E_\mu A(\omega, \mu))$ for markets and polls.

market would benefit from learning the intrinsic value of X, and the corresponding event, faster.

Unfortunately, there are no good news. Even when traders are myopic, we show that information acquisition can make the process of information aggregation both faster and slower, depending on the parameters. This is true for all securities except for the Arrow-Debreu (A-D), where information aggregates at the same period, irrespective of whether there is information acquisition or not. If information aggregation does not occur faster with information acquisition, the competition of traders could be wasteful, because they pay a cost to learn something that other traders already know. However, it can also be the case that, even though 'complete' information aggregation occurs in the same period, the price approaches the true value of the security faster and this can be beneficial for the market.

We begin with an example, which shows that information acquisition can make information aggregation both faster and slower.

Example 4. *The state space is* $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4\}$, *the security is* $X = (0, 2, 1, 1)$, *and there are two myopic traders with common prior* $\mu = (\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4})$. *Trader 1's partition is* $\{\{\omega_1, \omega_2\}, \{\omega_3, \omega_4\}\}$ *and Trader 2's is* $\{\{\omega_1, \omega_4\}, \{\omega_2, \omega_3\}\}$.

Suppose that the true state is ω_1 and traders cannot acquire any information. In the first period, Trader 1 announces 1. This reveals no information to Trader 2 because his private information is $\{\omega_1, \omega_4\}$, and the same announcement would be made by Trader 1 in both states. In period 2, Trader 2 announces 0.5. This reveals to Trader 1 that the true state is ω_1, hence, in period 3 he announces 0 and the game ends.

Suppose that traders are able to acquire a statistical experiment $\mathcal{R}$ that generates a signal z, given each state, with probabilities $\mathcal{R}_{\omega_1}(z) = 1$ and $\mathcal{R}_{\omega_2}(z) = \mathcal{R}_{\omega_3}(z) = \mathcal{R}_{\omega_4}(z) = 0.5$. When the true state is ω_1 and Trader 1 acquires signal z, he updates his beliefs to $(\frac{2}{3}, \frac{1}{3}, 0, 0)$ and announces $\frac{2}{3}$. The announcement reveals to Trader 2 that the state is ω_1, because at ω_4 Trader 1 would have announced 1. As a result, Trader 2 announces 0 in period 2 and the game ends, hence information aggregation happens faster with information acquisition.

We now show that the converse is also true, so that information acquisition may delay information aggregation. Suppose that the common prior is $\mu = (\frac{3}{16}, \frac{6}{16}, \frac{7}{32}, \frac{7}{32})$. At ω_1, if there is no information acquisition, Trader 1 announces $\frac{4}{3}$. The announcement reveals to Trader 2 that the state is ω_1, hence Trader 2 announces 0 and the game ends in the second round. If there is information acquisition and Trader 1 acquires signal z, he updates his beliefs to $(\frac{1}{2}, \frac{1}{2}, 0, 0)$ and announces 1. Trader 2 gains no new information and announces 0.5. This reveals to Trader 1 that the true state is ω_1, hence in the third round he announces 0 and the game ends.

Interestingly, if the security is the Arrow-Debreu (A-D), then information acquisition has no impact on the speed of information aggregation. To understand why, consider an A-D security that pays 1 at ω and 0 otherwise. At any period, there are two cases. First, the trader who announces knows that either ω is true, or that it is not true, accordingly announcing 1 or 0. All other traders learn the true value of X and the game ends. Second, he is unsure about whether ω is true or not. Irrespective of whether he buys a signal structure, he will announce some value $0 < v < 1$. Crucially, other traders already know that his partition cell includes ω, hence his announcement does not reveal any public information and the speed of information aggregation is unaffected.

We now provide the formal results. Proposition 3 shows that for any security that is not A-D, we can find information structures for which X is separable, and depending on the prior the process can be faster or slower. Proposition 4 shows that for an A-D security, allowing for information acquisition neither speeds up nor delays the information aggregation process.

Recall from Section 4.5 that the full state $\phi = (\omega, \iota_1, \iota_2, \ldots)$ describes the initial uncertainty ω and the randomizations of the players. In this section, traders are myopic, so they have a pure strategy of an announcement and the choice of the random posterior, hence ι_n denotes the realisation of the posterior in period t_n.

Let $t_I^*(\phi)$ be the period where information aggregation is achieved, given a separable security and state ϕ, in an environment where traders can get information before they make an announcement. Similarly, let $t^*_{NI}(\phi)$ be the period where information aggregation is achieved, in an environment where there is no information acquisition.

For Proposition 3, we make the following mild assumption.

Assumption 2. *Given a state space Ω with two states and for any proper scoring rule s and non-constant security X, there is a posterior separable cost function K such that, for all c, the solution to (1) is unique.*

The assumption is true for a quadratic scoring rule and entropy cost function, as shown in Appendix 11 in Ilinov et al. [2024]. More generally, consider cost K and the payoff that is generated by making the myopic best announcement, both as functions of the posterior $\gamma \in [\mu, 1]$, where μ is the prior on one of the two states. Both these functions are strictly convex and a sufficient condition for a unique solution is that they do not have the same first-order condition for more than one γ. As we have complete freedom in choosing K, it can be that we achieve this for any s.[21]

Proposition 3. *Suppose Assumption 2 and that non-constant security X is not A-D. Then, there is information structure $\Pi \in \mathcal{P}$ for which X is separable, such that for some information cost $\kappa = (K, c)$ and $\phi \in \Phi$,*

(i) $t^*_{NI}(\phi) < t^*_I(\phi)$ *for some common prior* μ,

(ii) $t^*_{NI}(\phi) > t^*_I(\phi)$ *for some common prior* μ'.

See proof on page 38.

Finally, we show that the speed of aggregation is unchanged if the security is A-D.

Proposition 4. *Suppose Assumption 1. If X is A-D, then $t^*_{NI}(\omega) = t^*_I(\omega)$ for any state $\omega \in \Omega$, cost κ, information structure $\Pi \in \mathcal{P}$, and common prior μ.*

See proof on page 39.

The property that speed is unaffected is related to the fact that the A-D security is not very informative. A trader will buy a posterior only in the partition cell where 1 is possible, as in all others he knows that the value is 0. This action does not have the positive externality of revealing to other traders what is his partition cell and therefore it does not provide any public information about what the true state is. Note, however, that some positive externality persists. The other

[21] See Tsakas [2020] for an analysis of this decision problem.

traders can solve the announcer's problem and therefore know the optimal signal structure he has purchased. By hearing the announcement, they also update their posteriors and they can benefit as long as it is their turn to announce and there is still some surplus to be obtained. For example, if Trader 1 buys a signal structure and moves the price from 0.5 to 0.99 (when the correct price is 1), then all other traders can only benefit by moving it from 0.99 to 1, hence the remaining surplus is very small. Finally, even though the number of periods for full aggregation remains the same, the price will converge to the true value faster, hence the market will still benefit by attaching faster a higher probability to the true value of the security.

6 Concluding Remarks

The paper provides a thorough examination of the interplay between information aggregation and information acquisition. We show that κ separability characterizes information aggregation when the cost of information acquisition is κ and we classify securities into three classes. First, the 'always separable' securities aggregate information irrespective of who trades or what is their information structure. This class is very small and uninformative. Second, the securities which are κ separable for some κ and all information structures, aggregate information if the cost is sufficiently low. This is the most generic and informative class of securities. Finally, there is a small class of securities such that for any κ, each is κ non-separable for some information structure. This means that even if the cost is very close to zero, information may not aggregate. Surprisingly, these three classes are easily distinguishable just by looking at the payoff structure of each security.

An interesting question is whether information aggregates if the security is non-separable but the prior is generic. By perturbing the common prior slightly, a non-separable security could become separable, and therefore information could aggregate. Ostrovsky [2012] shows that information gets aggregates in all pure-strategy equilibria with a generic prior, even for non-separable securities. However, in his setting, it is an open question what happens with mixed-strategy equilibria. Intuitively, even if we perturb the initial prior, and given that agents have infinite action spaces, it is not clear whether they will converge to some belief at which the security is separable.

The current paper provides a novel perspective on this issue of genericity, by endogenizing the perturbation of beliefs. If information is not aggregated at time t and all traders agree on the price, a trader could buy an additional signal, if the cost is not too high, and profit from changing his belief. By examining what happens when the cost is very low, we effectively allow for arbitrarily small perturbations of beliefs to be feasible for traders. A "generic" security pays differently across states and Corollary 1 shows that if the cost is sufficiently low, such a security is κ separable, for all information structures. Therefore, using Theorem 2 we can say that generically (for almost all securities) information gets aggregated, in all mixed-strategy equilibria, thus providing an answer to the open question of Ostrovsky [2012].

A Appendix

A.1 Cost of Information

Let T be a Polish space of possible signals with Borel σ-algebra $\mathcal{T}$. Let $\Delta(T)$ be the set of Borel probabilities on T, with generic element ξ. We endow $\Delta(T)$ with the weak* topology: a sequence of probabilities $\{\xi_n\}$ converges to a probability ξ if for every bounded continuous function $f : T \to \mathbb{R}$, we have $\int f d\xi_n \to \int f d\xi$. Following Blackwell [1951], we model the acquisition of information using statistical experiments. A statistical experiment is a function from states into probabilities on signals, $\mathcal{R} : \Omega \to \Delta(T)$.

Following Bloedel and Zhong [2020], we say that an experiment $\mathcal{R}$ is bounded if (i) the conditional signal distributions $\{\mathcal{R}_\omega\}_{\omega\in\Omega}$ are mutually absolutely continuous, and (ii) there exists a constant $B > 0$ such that the Radon-Nikodym derivatives $\frac{d\mathcal{R}_\omega}{d\mathcal{R}_{\omega'}} \in [1/B, B]$ for all $\omega, \omega' \in \Omega$. In other words, a bounded experiment does not definitively rule out any state and, moreover, has uniformly bounded likelihood ratios. Let $\mathcal{E}$ be the set of all experiments and $\mathcal{E}_b \subseteq \mathcal{E}$ be the collection of all bounded experiments. Note that $\mathcal{E}$ contains experiments that are not bounded. We say that $\mathcal{R}' : \Omega \to \Delta(T')$ is a garbling of $\mathcal{R} : \Omega \to \Delta(T)$ given $E \subseteq \Omega$ if there exists $\psi : T \to \Delta(T')$ such that $\mathcal{R}'_{\omega'}(t') = \int_{t\in T} \psi(t'|t)\mathcal{R}_\omega(t)dt$.

Given a prior belief $\mu \in \Delta(\Omega)$, a statistical experiment $\mathcal{R}$ induces via Bayesian updating a probability distribution over posteriors, or random posterior, $B(\mu, \mathcal{R}) = Q \in \Delta(\Delta(\Omega))$. Let $Q(\gamma)$ be the probability of posterior $\gamma \in \Delta(\Omega)$, $\mathcal{Q}(\mu) = \{B(\mu, \mathcal{R}) : \mathcal{R} \in \mathcal{E}\}$ be the set of all random posteriors that can be generated by some experiment $\mathcal{R} \in \mathcal{E}$, and $\mathcal{Q} = \bigcup_{\mu\in\Delta(\Omega)} \mathcal{Q}(\mu)$. Note that $Q \in \mathcal{Q}(\mu)$ if and only if $\int_{\Delta(\Omega)} \gamma Q(d\gamma) = \mu$. We endow $\mathcal{Q}$ with the weak* topology, so that it is a compact and separable topological space. The subsets of $\mathcal{Q}$ are endowed with the appropriate relative topologies.

Given a full support prior μ, experiment $\mathcal{R}$ is bounded if and only if the induced random posterior $Q = B(\mu, \mathcal{R})$ satisfies $Supp(Q) \subseteq \Delta_\epsilon = \{q \in \Delta(\Omega) : q(\omega) > \epsilon \text{ for all } \omega \in \Omega\}$ for some $\epsilon > 0$. Let $\mathcal{D}_\epsilon = \{Q \in \mathcal{Q} : Supp(Q) \subseteq \Delta_\epsilon\}$. Then, $\mathcal{D}_b = \bigcup_{\epsilon>0} \mathcal{D}_\epsilon$ denotes the set of random posteriors that are induced by some full support prior and bounded experiment. Denote by δ_μ the degenerate random posterior that puts probability 1 on μ.

Definition 8. *A cost on experiments is a map $h : \Delta(\Omega) \times \mathcal{E} \to [0, \infty]$, where $h(\mu, \cdot)$ is Borel measurable for each prior $\mu \in \Delta(\Omega)$ and has the following properties.*

(i) If $B(\mu, \mathcal{R}) = B(\mu, \mathcal{R}')$ and $\mathcal{R}, \mathcal{R}' \in \mathcal{E}_b$, then $h(\mu, \mathcal{R}) = h(\mu, \mathcal{R}')$,

(ii) *If* $B(\mu, \mathcal{R}) = \delta_\mu$ *and* $\mathcal{R} \in \mathcal{E}_b$*, then* $h(\mu, \mathcal{R}) = 0$*,*

(iii) *Let* $\{\mu_j, \mathcal{R}_j\}_{j\in\mathbb{N}}$ *be a sequence of experiment-prior pairs inducing random posteriors* $Q_j = B(\mu_j, \mathcal{R}_j)$*. If* $Q_j \to Q^*$ *and there exists some* $\delta > 0$ *such that* $\{Q_j\}_{j\in\mathbb{N}} \subseteq \mathcal{D}_\delta$*, then* $h(\mu_j, \mathcal{R}_j) \to h(\mu^*, \mathcal{R}^*)$*, where* $Q^* = B(\mu^*, \mathcal{R}^*)$*,*

(iv) *If* $\mathcal{R}'$ *is a garbling of* $\mathcal{R}$ *then* $h(\mu, \mathcal{R}) \geq h(\mu, \mathcal{R}')$*.*

Point (i) specifies that if two bounded experiments generate the same random posterior given μ, then they have the same cost. Point (ii) says that a bounded experiment that is completely uninformative has a cost of zero. Point (iii) is a continuity condition that is weaker than weak* continuity, in order to allow for important classes of unbounded cost functions.[22] Finally, point (iv) specifies that more informative experiments are more costly.

The cost on experiments h generates a cost structure $\kappa = (K, c)$ on random posteriors, where $c > 0$ is the unit cost of information and $K : \mathcal{F} \to [0, \infty]$ maps elements of $\mathcal{F} = \{(\mu, Q) : \mu \in \Delta(\Omega), Q \in \mathcal{Q}(\mu)\}$, the set of all priors μ and all posterior distributions Q that can be generated by some experiment $\mathcal{R} \in \mathcal{E}$, to the extended real line.

We assume that the cost $cK(\mu, Q)$ of acquiring random posterior Q, given belief μ, is determined by optimally acquiring the relevant experiment $\mathcal{R}$, so that

$$cK(\mu, Q) \equiv \min\{h(\mu, \mathcal{R}) : B(\mu, \mathcal{R}) = Q\}.$$

Note that Point (i) in Definition 8 ensures that all bounded experiments $\mathcal{R}$ which induce Q given μ have the same cost, hence in that case $cK(\mu, Q) = h(\mu, \mathcal{R})$. If only unbounded experiments generate Q given μ, we have $cK(\mu, Q) = \infty$. Moreover, the cost of learning nothing is zero, so that $K(\mu, Q) = 0$ whenever $Supp(Q) = \{\mu\}$. A cost structure $\kappa = (K, c)$ consists of a cost function K and a unit cost of information $c > 0$. Let $\mathcal{K}$ be the collection of cost structures $\kappa = (K, c)$ that are generated from the experiments.

If all unbounded experiments have infinite cost, then generating a random posterior with support on a posterior δ_ω, which assigns probability 0 to a state $\omega \in Supp(\mu)$, is infinitely costly.

Corollary 2. *Suppose Assumption 1. Given any* $\omega \in Supp(\mu)$ *and sequences* $\{\mathcal{R}_j\}_{j\in\mathbb{N}}$ *with* $Q_j = B(\mu, \mathcal{R}_j)$ *and* $\{\gamma_j\}_{j\in\mathbb{N}}$ *with* $Q_j(\gamma_j) > \epsilon$ *for all* j *and some* $\epsilon > 0$*, if* $\gamma_j \to \delta_\omega$ *then* $h(\mu, \mathcal{R}_j) \to \infty$*.*

[22] See Bloedel and Zhong [2020] for the motivation behind this continuity condition.

The Corollary implies that for any induced sequences $\{Q_j\}_{j\in\mathbb{N}}$ and $\{\gamma_j\}_{j\in\mathbb{N}}$ with $Q_j(\gamma_j) > \epsilon$ for all j and some $\epsilon > 0$, if $\gamma_j \to \delta_\omega$ then $K(\mu, Q_j) \to \infty$.

B Proofs

Proposition 5 provides two independent conditions which are necessary for a security to be κ non-separable for all κ. The first condition specifies that, at some state ω, all traders agree that some value $m \neq X(\omega)$ is possible. The second is an extension of Theorem 7 in Ostrovsky [2012] (see Proposition 6), which provides a characterization of separable securities. It says that for every v' value of X and within all intervals around v', there exists value v for which we cannot pick multipliers $\lambda_i(\pi)$, for all elements π of partitions Π_i and for all traders, such that the sign of $X(\omega) - v$ is the same as the sign of $\sum_{i\in I}\lambda_i(\Pi_i(\omega))$, for all ω with $X(\omega) \neq v$.

Proposition 5. *Suppose X is κ non-separable given an information structure Π and for all $\kappa \in \mathcal{K}$. Then, the following independent conditions are true.*

- *There exists ω with $X(\omega) \neq m$ and $m \in \bigcap_{i\in I} X(\Pi_i(\omega))$.*
- *For some $v' \in X(\Omega)$, for all $\epsilon > 0$, there exists $v \in (v' - \epsilon, v' + \epsilon)$ for which there are no functions $\lambda_i : \Pi_i \to \mathbb{R}$ such that for all states ω with $X(\omega) \neq v$,*

$$(X(\omega) - v)(\sum_{i\in I}\lambda_i(\Pi_i(\omega)) > 0, \tag{2}$$

Proof. For the first condition, we prove the contrapositive. Suppose that for all $m \in X(\Omega)$, for all ω with $X(\omega) \neq m$ we have $m \notin \bigcap_{i\in I} X(\Pi_i(\omega))$. We will show that X is κ separable for some $\kappa = (K, c)$.

For any K and sufficiently low marginal cost c, Assumption 1 implies that the only priors μ for which there is no information acquisition are the ones where the resulting posterior assigns probability close to 1 to some state $\omega' \in \Pi_i(\omega)$, for all $i \in I$ and $\omega \in Supp(\mu)$. Suppose that there exists such μ, with $E_\mu[X|\Pi_i(\omega)] = v$ for all $i = 1, ..., n$ and $\omega \in Supp(\mu)$, where v is arbitrarily close to some m with $X(\omega) = m$.

For the security to be κ non-separable, there is also should be uncertainty about its value. This means that for some $\omega' \in Supp(\mu)$, $X(\omega') \neq v$. From our initial hypothesis, there exists trader i such that $m \notin X(\Pi_i(\omega'))$, so that he considers m to be impossible. Because he assigns probability close to 1 to some state, from continuity it must be that $E_\mu[X|\Pi_i(\omega')] \neq v$. Therefore, X is separable at μ. As this is true for all μ that assign probability close to 1 to some state $\omega' \in \Pi_i(\omega)$, we have that the the security X is κ separable for some κ.

For the second condition, suppose X is κ non-separable for all κ. Take κ with very small marginal cost c, so that for any prior μ for which there is no information acquisition, at each state $\omega \in Supp(\mu)$, and for each $i \in I$, $\mu_{\Pi_i(\omega)}$ assigns probability which is arbitrarily close to 1 to some state $\omega' \in \Pi_i(\omega)$. This means that $E_{\mu_{\Pi_i(\omega)}}[X] \in (v' - \epsilon, v' + \epsilon)$, for small enough $\epsilon \in \mathbb{R}$ and $X(\omega') = v'$.

Because X is κ non-separable, there exists a prior μ with no information acquisition and value $v \in (v' - \epsilon, v' + \epsilon)$ for which X is non-separable. This implies that, for any $\lambda_i : \Pi_i \to \mathbb{R}$,

$$E_\mu\left[(X(\omega) - v)\sum_{i\in I}\lambda_i(\Pi_i(\omega))\right] =$$

$$\sum_{i\in I} E_\mu\left[(X(\omega) - v)\lambda_i(\Pi_i(\omega))\right] =$$

$$\sum_{i\in I}\sum_{\omega\in Supp(\mu)} \mu(\Pi(\omega))\lambda_i(\Pi_i(\omega))E_{\mu_{\Pi_i(\omega)}}\left[(X(\omega) - v)\right] = 0,$$

where the last equality derives from the definition of non-separability. But then we cannot have $(X(\omega) - v)\sum_{i\in I}\lambda_i(\Pi_i(\omega)) > 0$ for all ω, hence there are no such λ_i functions.

To show that the first two conditions are independent, consider first security $(1, 0, 0, -1, 2)$. Let the partition of Trader 1 be $\{\{\omega_1, \omega_2\}, \{\omega_3, \omega_4\}, \{\omega_5\}\}$ and for Trader 2 be $\{\{\omega_1, \omega_3\}, \{\omega_2, \omega_5\}, \{\omega_4\}\}$, which satisfies the first condition. We show that it fails the second condition. First, if $v \le -1$, then condition (4) is satisfied by setting $\lambda_i(\Pi_i(\omega)) = 1$ for all $i \in I$ and $\omega \in \Omega$. Similarly, if $v \ge 2$, we set $\lambda_i(\Pi_i(\omega)) = -1$ for all $i \in I$ and $\omega \in \Omega$. Then, suppose there exist functions such that $\lambda_1(\{\omega_1, \omega_2\}) = a$, $\lambda_1(\{\omega_3, \omega_4\}) = b$, $\lambda_1(\{\omega_5\}) = c$, $\lambda_2(\{\omega_1, \omega_3\}) = d$, $\lambda_2(\{\omega_2, \omega_5\}) = e$ and $\lambda_2(\{\omega_4\}) = f$. Consider $0 \ge v \ge -1$. We then have that $X(\omega) - v < 0$ for ω_4, and $X(\omega) - v > 0$ for $\omega_1, \omega_2, \omega_3, \omega_5$. Set $f = -2$ and $a = b = c = d = e = 1$ and (2) is satisfied. Consider $2 \ge v \ge 1$. We then have that $X(\omega) - v > 0$ for ω_5, and $X(\omega) - v < 0$ for $\omega_1, \omega_2, \omega_3, \omega_4$. Set $c = 2$ and $a = b = c = d = e = -1$ and (2) is satisfied. Finally, consider $1 \ge v \ge 0$. We then have that $X(\omega) - v < 0$ for $\omega_2, \omega_3, \omega_4$, and $X(\omega) - v > 0$ for ω_1, ω_5. Set $c = 3$, $f = b = e = -2$ and $a = d = 1$ and (2) is satisfied. Therefore, the second condition is not satisfied.

Consider now security $(-2, -1, 0, 1, 2)$. Suppose there are three traders. The partition of Trader 1 is $\{\{\omega_2, \omega_4\}, \{\omega_1, \omega_3, \omega_5\}\}$, for Trader 2 it is $\{\{\omega_1, \omega_5\}, \{\omega_2, \omega_3, \omega_4\}\}$ and for Trader 3 it is $\{\{\omega_1, \omega_4\}, \{\omega_2, \omega_3, \omega_5\}\}$. Therefore, such security violates the first condition. We will show that it satisfies the second. Let $v' = 0$ and suppose that $v \in (0, \epsilon)$, for any $\epsilon > 0$. We then have that $X(\omega) - v < 0$ for $\omega_1, \omega_2, \omega_3$, and $X(\omega) - v > 0$ for ω_4, ω_5.

Suppose there exist functions such that $\lambda_1(\{\omega_2, \omega_4\}) = a$, $\lambda_1(\{\omega_1, \omega_3, \omega_5\}) = b$, $\lambda_2(\{\omega_2, \omega_3, \omega_4\}) = c$, $\lambda_2(\{\omega_1, \omega_5\}) = d$, $\lambda_3(\{\omega_2, \omega_3, \omega_5\}) = e$ and $\lambda_3(\{\omega_1, \omega_4\}) = f$. For (2) to be satisfied, we have the following inequalities.

$$\begin{cases} b + d + f < 0 \\ a + c + e < 0 \\ b + c + e < 0 \\ a + c + f > 0 \\ b + d + e > 0 \end{cases} \tag{3}$$

If we add the first with the fifth, we have $e > f$. If we add the second with the fourth, we have $f > e$. This is a contradiction and such equations do not exist, which means that the second condition is satisfied.

□

Proof of Proposition 1. We start by restating a useful characterization of separable securities by Ostrovsky [2012]. It specifies that X is separable if and only if, for any possible announcement v, we can find numbers $\lambda_i(\Pi_i(\omega))$, for each i and ω, such that the sum over all traders has the same sign as the difference of $X(\omega) - v$. Intuitively, for any v and at each ω, all traders "vote" and the sign of the sum of the votes has to agree with the sign of the difference between the value of the security and v.

Proposition 6 (Ostrovsky [2012])**.** *Security X is separable under partition structure Π if and only if, for every $v \in \mathbb{R}$, there exist functions $\lambda_i : \Pi_i \to \mathbb{R}$ for $i = 1, \ldots, n$ such that, for every state ω with $X(\omega) \neq v$,*

$$(X(\omega) - v) \sum_{i \in I} \lambda_i(\Pi_i(\omega)) > 0.$$

If Ω has up to three states, then all securities are of the two types that we have described or the uninteresting case of a constant security. Hence, without loss of generality, we fix a state space Ω with at least four states and a security X. If X is constant, it is trivially separable. Ostrovsky [2012] shows that an A-D security is always separable.

We now show that X is always separable if it is of the following form. Suppose there are $a < b < c$ such that $X(\omega_a) = a$ and $X(\omega_c) = c$ for two states ω_a, ω_c, whereas $X(\omega) = b$ for all $\omega \neq \omega_a, \omega_c$.[23] Using Proposition 6, we need to show that for every $v \in \mathbb{R}$, there exist functions $\lambda_i : \Pi_i \to \mathbb{R}$ for $i = 1, \ldots, n$ such that, for every state ω with $X(\omega) \neq v$,

$$(X(\omega) - v) \sum_{i \in I} \lambda_i(\Pi_i(\omega)) > 0. \tag{4}$$

If $v \leq a$, then condition (4) is satisfied by setting $\lambda_i(\Pi_i(\omega)) = 1$ for all $i \in I$ and $\omega \in \Omega$. Similarly, if $v \geq c$, we set $\lambda_i(\Pi_i(\omega)) = -1$ for all $i \in I$ and $\omega \in \Omega$. Suppose that $a < v \leq b < c$. For all $i \in I$, set $\lambda_i(\Pi_i(\omega_a)) = -1$ and (4) is satisfied for ω_a. For all ω with $\Pi_i(\omega) \neq \Pi_i(\omega_a)$, set $\lambda_i(\Pi_i(\omega)) = k$, where $k = |I|$ is the number of agents. Because of our assumption that the join of all partitions consists of singleton sets, we have that for each $\omega \neq \omega_a$, there exists i such that $\Pi_i(\omega) \neq \Pi_i(\omega_a)$. This implies that if $X(\omega) - v > 0$, we also have $\sum_{i \in I} \lambda_i(\Pi_i(\omega)) \geq k - (k-1) > 0$ and (4) is satisfied for ω. Using a symmetric argument, we can show that (4) is satisfied for $a < b \leq v < c$, by setting $\lambda_i(\Pi_i(\omega_c)) = 1$ and $\lambda_i(\Pi_i(\omega)) = -k$ for ω with $\Pi_i(\omega) \neq \Pi_i(\omega_c)$, for all $i \in I$. By applying Proposition 6, security X is always separable.

Suppose that X is not of the three aforementioned types. Then, we can find four distinct states where X assigns values $a \leq b < c \leq d$. For simplicity, we refer to the state with value a as state a and similarly for b, c, and d.

We will show that X is non-separable for an information structure in $\mathcal{P}$ with two agents. The partition of Trader 1 is $\{\{a, d\}, \{b, c\}\}$ for these four states, whereas for any other state, we have $\Pi_1(\omega) = \{\omega\}$. For Trader 2 it is $\{\{a, c\}, \{b, d\}\}$ and for any other state we have $\Pi_2(\omega) = \{\omega\}$. Hence, the information structure is in $\mathcal{P}$.

To show that X is non-separable, it is enough to find a prior p with support on $\{a, b, c, d\}$ such that, for some v,

(i) $X(\omega) \neq v$ for some $\omega \in Supp(p)$,

(ii) $E_p[X|\Pi_i(\omega)] = v$ for all $i = 1, 2$ and $\omega \in Supp(p)$.

Let p_1 be 1's probability of state a conditional on $\{a, d\}$, whereas q_1 is 1's

[23] Our proof for this type of security also applies to an A-D security. Ostrovsky [2012] used Corollary 1 to show that an A-D security is always separable, however, we cannot use it for this type of security.

probability of state b conditional on $\{b, c\}$. Let p_2 be 2's probability of state a conditional on $\{a, c\}$, whereas q_2 is 2's probability of state b conditional on $\{b, d\}$. Condition (ii) then translates to the following equations

$$\begin{cases} ap_1 + d(1-p_1) = bq_1 + c(1-q_1) \\ ap_1 + d(1-p_1) = ap_2 + c(1-p_2) \\ ap_1 + d(1-p_1) = bq_2 + d(1-q_2) \\ bq_1 + c(1-q_1) = ap_2 + c(1-p_2) \\ ap_1 + d(1-p_1) = bq_2 + d(1-q_2) \\ ap_2 + c(1-p_2) = bq_2 + d(1-q_2) \end{cases} \tag{5}$$

The posteriors of the two agents can be derived by a common prior p if the following conditions hold:

$$\begin{cases} xp_1 = yp_2 \\ (1-x)q_1 = (1-y)q_2 \\ (1-x)(1-q_1) = y(1-p_2) \\ x(1-p_1) = (1-y)(1-q_2) \end{cases} \tag{6}$$

where x is the prior probability of (a, d) and y is the prior probability of (a, c). When $a \leq b < c \leq d$, the system (5 - 6) has the following solution:

$$\begin{aligned} q_1 &= \frac{a-c}{a+b-c-d}, \\ p_1 &= \frac{b-d}{a+b-c-d}, \\ p_2 &= \frac{b-c}{a+b-c-d}, \\ q_2 &= \frac{a-d}{a+b-c-d}, \\ x &= p_2, \\ y &= p_1. \end{aligned}$$

These posteriors uniquely define the respective prior probabilities. □

Proof of Theorem 1. Suppose Ω has at least four states and let $E = \{\omega_1, \omega_2, \omega_3, \omega_4\}$, where X pays (a, d, b, d), with either $a, b < d$ or $a, b > d$. What X pays outside E is irrelevant, because we will only consider priors that have full support on E, in order to show that X is κ non-separable for all κ. Consider two traders with the following information structure on E. Trader 1's partition is $\{\{\omega_1, \omega_2\}, \{\omega_3, \omega_4\}\}$ and Trader 2's is $\{\{\omega_1, \omega_4\}, \{\omega_2, \omega_3\}\}$. We need to show that for any $\kappa = (K, c)$, X is κ non-separable. It is enough to show that, as the marginal cost c converges to 0, there is always a common prior μ on E such that no trader acquires any information and X is non-separable at μ.

Consider prior $\mu = (p, m, 1 - p - 2m, m)$, where $0 < m < 0.5$. We specify p such that, given any m, each trader's expected value of X is the same at all states. In the following equation, the left hand-side computes Trader 1's expected value of X at $\{\omega_1, \omega_2\}$ and Trader 2's at $\{\omega_1, \omega_4\}$, whereas the right hand-side computes Trader 1's expected value at $\{\omega_3, \omega_4\}$ Trader 2's at $\{\omega_2, \omega_3\}$:

$$a\frac{p}{p+m} + d\frac{m}{p+m} = b\frac{1-2m-p}{1-p-m} + d\frac{m}{1-p-m}.$$

There is always a solution to this equation. For example, if $a + b \neq 0$, we can normalise to $p + m = 1/2$ and we have that

$$p = \frac{b(1-2m)}{a+b}.$$ [24]

Given this normalisation, the expected value of X at all states and for all traders is $u = 2pa + 2dm$. Fix any marginal cost c, which may be very close to 0. We can then choose m very close to 0.5 (and corresponding μ) such that u is very close to d, say $d - \epsilon$, assuming, without loss of generality, that $a, b < d$, otherwise we have $d + \epsilon$ and the rest of the proof proceeds accordingly. By repeating the consensus announcement of $u = d - \epsilon$ and not acquiring any signals, both traders get 0 utility. Suppose that one trader wants to buy a signal structure Q, which, in the case of $X(\omega) = d$, will move his posterior expected value of X from $d - \epsilon$ to $d - \epsilon + \nu$, for some $0 < \nu < \epsilon$. The upper bound of his utility, net of the cost of the signal, is realised at $X(\omega) = d$ and it is $s(d - \epsilon + \nu, d) - s(d - \epsilon, d)$. This is positive because $d - \epsilon + \nu$ is closer to d than the previous announcement, $1 - \epsilon$. However, by decreasing ϵ appropriately it can be made as small as needed, and therefore smaller than $cK(\mu, Q)$ for all Q. This is a direct consequence of Assumption 1, which specifies that the cost gets arbitrarily high for posteriors which assign probability that is arbitrarily close to one for some state. For this result, we only need that the cost is non-decreasing. Hence, no trader will buy any signal structure and X is κ non-separable.

For the converse, suppose X is κ non-separable given some Π and for all κ.

[24] If $a + b = 0$, we can choose normalisation $p + m = 1/3$ and the proof is identical.

The second condition of Proposition 5 and the assumption in Section 2.1 that $\bigcap_{i \in I} \Pi_i(\omega) = \{\omega\}$, for all $\omega \in \Omega$, imply that there exist two states $\omega_a \neq \omega_b$ such that $X(\omega_a) = X(\omega_b) = m$. Let M be the set of states ω with $X(\omega) = m$. Given that Ω has at least four states, we have the following cases.

Case 0. $M = \Omega$. Security X is constant and therefore (trivially) always separable, for all information structures Π. This is a contradiction because we have assumed X is κ non-separable given some Π and for all κ.

Case 1. There is either a unique $\omega \notin M$, or there are exactly two states $\omega, \omega' \notin M$, such that $X(\omega) > m > X(\omega')$. From Proposition 1, X is always separable, for all information structures Π. As with Case 0, this is a contradiction.

The only other remaining case is that There exist $\omega, \omega' \notin M$ with either $X(\omega), X(\omega') < m$, or $X(\omega), X(\omega') > m$, which concludes the proof.

□

Proof of Theorem 2. We follow the proof of Theorem 1 in Ostrovsky [2012], which proceeds in four steps. In the first step, we show that there is a uniform lower bound on the expected profits that at least one trader can make by improving the forecast.

Let r be a distribution over Ω and z be the previous announcement. Following Ostrovsky [2012], we define the instant opportunity of Trader i as the highest expected payoff that he can achieve by changing the forecast from z, if the state is drawn according to r. The difference from Ostrovsky [2012] is that we allow the trader to acquire information before making an announcement. Let Q_ω be the optimal experiment that Trader i acquires at $\Pi_i(\omega)$, given his beliefs $r_{\Pi_i(\omega)}$. Trader i's instant opportunity is

$$\sum_{\omega \in \Omega} r(\omega) \left(\sum_{\gamma \in Supp(Q_\omega)} Q_\omega(\gamma) \sum_{\omega' \in \Omega} \gamma(\omega') \Big[s(E_\gamma[X], X(\omega')) - s(z, X(\omega')) \Big] - cK(r_{\Pi_i(\omega)}, Q_\omega) \right).$$

Let Δ be the set of probability distributions $r \in \Delta(\Omega)$ for which there is uncertainty about security X, so that there are states a, b with $r(a) > 0$, $r(b) > 0$ and $X(a) \neq X(b)$.

Lemma 1. *If security X is κ separable, then for every $r \in \Delta$, there exist $\chi > 0$ and Trader i such that, for every $z \in \mathbb{R}$, the instant opportunity of Trader i given r and z is greater than χ.*

Proof. Fix $r \in \Delta$. There are two cases. First, X is separable with respect to r. Then, the proof of Lemma 1 in Ostrovsky [2012] applies and we have the result.

Second, X is non-separable at r and value v, where $v = E_r[X|\Pi_i(\omega)]$ for all $\omega \in Supp(r)$ and $i \in I$. Because X is κ separable, there is information acquisition. This implies that for some $\omega \in Supp(r)$, Trader i receives a strictly positive payoff by acquiring information and changing the previous announcement $v = E_r[X|\Pi_i(\omega)]$. Note that the new announcement is not deterministic but depends on the optimal Q, so he announces $E_\gamma[X]$ with probability $Q(\gamma)$. From continuity, there is a small enough $\epsilon > 0$, so that for all previous announcements $z \in [v - \epsilon, v + \epsilon]$, Trader i receives a strictly positive payoff of at least $\chi_1 > 0$ by acquiring information and changing the announcement. If $z \notin [v - \epsilon, v + \epsilon]$, then Trader i receives a strictly positive payoff of at least $\chi_2 > 0$, by not acquiring any information and announcing the myopically best $E_r[X|\Pi_i(\omega)] = v$.[25] By setting $\chi_3 = \min\{\chi_1, \chi_2\}$, we have that at $\Pi_i(\omega)$, Trader i receives a strictly positive payoff of at least χ_3, for all previous announcements z. For any $\omega' \in Supp(r) \setminus \Pi_i(\omega)$, Trader i can repeat v and receive 0. Hence, the instant opportunity of i given r, which is the ex-ante expectation over all $\omega \in Supp(r)$, is at least $\chi = r(\Pi_i(\omega))\chi_3 > 0$ for all previous announcements.

□

Steps 2-3 are identical to the proof of Ostrovsky [2012] and establish that there is Trader i and lower bound $\nu^* > 0$ such that, for infinitely many periods t_{nk+i}, the expected instant opportunity of Trader i is greater than ν^*. The reason they are identical is that, once we fix an equilibrium strategy, the only difference from Ostrovsky [2012] is that traders buy an extra, payoff-irrelevant, signal in each period where they make an announcement.

In Step 4, we show that the presence of a "non-vanishing arbitrage opportunity" is impossible in equilibrium. The proof is very similar to that of Ostrovsky [2012], but we write it here for completeness.

Let $\overline{s}_k$ be the expected score of prediction y_k, where the expectation is over all ϕ and the moves of players according to the mixed equilibrium. The expected payoff to the trader who moves in period t_k is $\beta^k(\overline{s}_k - \overline{s}_{k-1} - \overline{c}_k)$, where $\overline{c}_k \geq 0$ is the expected cost of information acquisition. It is zero if and only if there is no information acquisition.

Take any period t_k. Let Ψ_k be the sum of all players' expected continuation

[25] This is true because a proper scoring rule is 'order-sensitive' so that the further away the previous announcement is from the true expected value $E_r[X|\Pi_i(\omega)] = v$, the higher the payoff is for Trader i. The lowest payoff from the myopically best announcement is 0 and it is achieved when it is equal to the previous announcement (see p. 2618 in Ostrovsky [2012]).

payoffs at t_k, divided by β^k:

$$\Psi_k = (\bar{s}_k - \bar{s}_{k-1} - \bar{c}_k) + \beta(\bar{s}_{k+1} - \bar{s}_k - \bar{c}_{k+1}) + \beta^2(\bar{s}_{k+2} - \bar{s}_{k+1} - \bar{c}_{k+2}) + \dots$$

Note that Ψ_k is weakly positive, because each trader can guarantee a zero payoff by repeating the previous announcement and buying any costly information. Additionally, it is strictly positive if i's expected instant opportunity is strictly positive and it is i's turn to make an announcement. That is, with some probability, some history H^{k-1} occurs and i's instant opportunity is strictly positive.

Consider now $\lim_{K\to\infty} \sum_{k=1}^{K} \Psi_k$. On the one hand, this limit must be infinite, because each Ψ_k is non-negative and an infinite number of them are greater than ν^*, from Step 3. On the other hand, for any K, we have

$$\begin{aligned}
\sum_{k=1}^{K} \Psi_k &= (\bar{s}_1 - \bar{s}_0 - \bar{c}_1) + \beta(\bar{s}_2 - \bar{s}_1 - \bar{c}_2) + \beta^2(\bar{s}_3 - \bar{s}_2 - \bar{c}_3) + \dots \\
&+ (\bar{s}_2 - \bar{s}_1 - \bar{c}_2) + \beta(\bar{s}_3 - \bar{s}_2 - \bar{c}_3) + \beta^2(\bar{s}_4 - \bar{s}_3 - \bar{c}_4) + \dots \\
&+ \quad \vdots \\
&+ (\bar{s}_K - \bar{s}_{K-1} - \bar{c}_K) + \beta(\bar{s}_{K+1} - \bar{s}_K - \bar{c}_{K+1}) + \beta^2(\bar{s}_{K+2} - \bar{s}_{K+1} - \bar{c}_{K+2}) + \dots \\
&= (\bar{s}_K - \bar{s}_0 - \sum_{k=1}^{K} \bar{c}_k) + \beta(\bar{s}_{K+1} - \bar{s}_1 - \sum_{k=2}^{K+1} \bar{c}_k) + \beta^2(\bar{s}_{K+2} - \bar{s}_2 - \sum_{k=3}^{K+2} \bar{c}_k) + \dots \\
&\leq (\bar{s}_K - \bar{s}_0) + \beta(\bar{s}_{K+1} - \bar{s}_1) + \beta^2(\bar{s}_{K+2} - \bar{s}_2) + \dots \\
&\leq 2M/(1-\beta),
\end{aligned}$$

where costs drop out as they are negative and $M = \underset{y\in[\underline{y},\overline{y}],\omega\in\Omega}{max} |s(y, X(\omega))|$. But this is impossible, hence y_k must converge in probability to the true value of X.

The proof of the second part of the Theorem is similar to that of Ostrovsky [2012]. Suppose that X is κ non-separable. Then, there exist μ and v at which there is no information acquisition and X is non-separable at μ. In the corresponding game Γ^S where the initial announcement is v, no trader will find it profitable to acquire any information and their best response is to repeat v, in every period and after every history, hence it is a Perfect Bayesian equilibrium.

□

Proof of Proposition 2. If X is κ separable, then from Theorem 2 we have that information aggregates at all states and therefore $A^m(\mu) = 1$. Assumption 1 implies that no trader would acquire full information, hence $p^p(\phi, \mu) \neq X(\phi)$ and $A^p(\mu) < 1$.

If X is κ non-separable, then we can find non-degenerate μ given X for which there is no information acquisition and X is non-separable at μ. This means that everyone agrees on the announcement at all states in the support of μ and the game ends in the first round. The announcement is the same for everyone, so the poll gives the same prediction as the market and $A^m(\mu) = A^p(\mu)$. □

Proof of Proposition 3. Consider non-constant security X which is not A-D. Then, X maps to at least three values, $a < b < d$, and we denote the respective payoff-relevant states as a, b, and d. Note that each payoff-relevant state $a, b, d \in \Omega$ is associated with several full states $\phi \in \Phi$, which resolve any uncertainty about which posteriors a trader receives at each period. To ease the notation, we omit mentioning ϕ when it is straightforward how statements about Ω are translated to statements about Φ.

Let the partition of Trader 1 be $\Pi_1 = \{\{a, d\}, \{b\}\}$ for these three states, whereas for any other state, we have $\Pi_1(\omega) = \{\omega\}$. For Trader 2 it is $\Pi_2 = \{\{a, b\}, \{d\}\}$ and for any other state we have $\Pi_2(\omega) = \{\omega\}$. Hence, the information structure is in $\mathcal{P}$. For Trader 1, let p_1 be the probability of state a conditional on $\{a, d\}$. For Trader 2, let p_2 be the probability of state a conditional on $\{a, d\}$.

We first show that X is separable with respect to Π, so that t^*_I, t^*_{NI} are well-defined for all priors. If Ω has more than three states, then for all $\omega' \in \Omega \setminus \{a, b, d\}$ and $i \in I$ we have $\Pi_i(\omega') = \{\omega'\}$ and $E_\mu[X|\Pi_i(\omega')] = X(\omega')$. Therefore, separability only depends on the three states, a, b, and d. From Proposition 1, security X is separable with respect to the restriction of Π on $\{a, b, d\}$. But then, X is separable with respect to Π.

Second, we show that (i) is true at state a, given Π and some $\kappa = (K, c)$. Take a full support prior μ such that $E_\mu[X|\Pi_1(a)] = v \neq b$. This is possible because $X(a) < X(b) < X(d)$. At state a, Trader 1 announces v, so Trader 2 learns that b is not true and announces a. We therefore have $t^*_{NI}(a) = 2$.

Because $X(a) < X(b) < X(d)$, there exists prior μ^* with full support on states $\{a, b, d\}$ and posterior probability p^*_1 on a, such that $E_{\mu^*}[X|\Pi_1(\omega)] = v = b$ for all $\omega \in Supp(\mu^*)$. We argue that for some cost structure K, the optimal posterior bought by Trader 1 will be p^*_1, resulting in an announcement of v.

Let K be a posterior separable cost function which satisfies Assumption 2, so that the optimal random posterior is unique. Let $g(c)$ map each cost c to the set of optimal random posteriors. Applying the Maximum Theorem, g is an upper semicontinuous correspondence. From Assumption 2, g is a function and therefore continuous. Let $g_1(c)$ be the restriction of $g(c)$ on one of the two posteriors that comprise the optimal random posterior, and in particular the one that takes values in $[\mu, 1]$.[26] For sufficiently high c, $g_1(c)$ will choose a posterior very close to the prior μ, whereas for sufficiently low c, $g(c)$ will choose a posterior very close to 1. From the Intermediate Value Theorem, there exists c such that the optimal solution is p_1^*.

This implies that Trader 1 optimally acquires information given c and updates his beliefs from p_1 to p_1^* at some ϕ, announcing b. Trader 2 does not gain any new information from the announcement, because the same announcement would be made at state b. The game proceeds to the next period, so $t_I^*(\phi) > 2$. However, without information acquisition, p_1 is such that $E_p[X|\Pi_i(\omega)] \neq b$ for all $i = 1, 2$ and $\omega \in Supp(p)$. Therefore, without information acquisition, after Trader 1 announces his prediction, Trader 2 will know the state and $t_{NI}^*(\phi) \leq 2$.

Finally, we show that (ii) is true at state a given the same Π and some K. Following the same argument as before, because $X(a) < X(b) < X(d)$, there exists μ' with full support on the first three states and resulting unique p_1 such that $E_{\mu'}[X|\Pi_1(\omega)] = v = b$ for all $\omega \in Supp(p)$. Therefore, after Trader 1 announces his prediction, Trader 2 does not know the state and $t_{NI}^*(\phi) > 2$. With information acquisition, we can find sufficiently low c such that Trader 1 acquires information, updates his beliefs to $p_1^* \neq p_1$, and announces $E_{p^*}^*[X|\Pi_i(\omega)] \neq v$ at state ϕ which projects to a. Therefore, Trader 2 realises that the state is not b and $t_I^*(\phi) = 2$. □

Proof of Proposition 4. Without loss of generality we assume that $X(\omega^*) = 1$ and $X(\omega) = 0$ for all $\omega \neq \omega^*$. Suppose that the true state is ω. In every period t, there are two cases. First, the trader announces 0 and the process ends. In this case, the trader does not acquire any information, because he knows that the value of X is 0. Second, the trader considers ω^* with $X(\omega^*) = 1$ to be possible, acquires a random posterior and after the signal realization forms a new posterior. From Assumption 1, the posterior cannot exclude any state with certainty, hence his private information $\Pi_i(\omega) \cap E$, where E is the public event revealed by previous announcements, stays the same, irrespective of whether he acquired any information, but his posterior might change. Hence, no information is revealed to other traders, because he does not make a different announcement based on which partition cell he is in. Since no information is revealed to other traders conditional on the game continuing, the common knowledge event that is created by each announcement is the same with and without information acquisition, hence, the process ends in the same number of periods. Note, however, that announcements may differ across the two environments. The same argument applies if the true state is ω^*. □

[26]From Bayesian plausibility, the other posterior will take values in $[0, \mu]$.

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