Alternative Types of Ambiguity and their Effects on Climate Change Regulation

Phoebe Koundouri* Nikitas Pittis† Panagiotis Samartzis‡
Nikolaos Englezos§ Andreas Papandreou¶

Abstract

This paper focuses on different types of ambiguity that affect climate change regulation. In particular, we analyze the effect of the interactions among three types of agents, namely, the decision maker (DM), the experts and the society, on the probabilistic properties of green-house gas (GHG) emissions and the formation of environmental policy, under two types of ambiguity: "deferential ambiguity" and "preferential ambiguity". Deferential ambiguity refers to the uncertainty that DM faces concerning to which expert's forecast (scenario) to defer. Preferential ambiguity stems from the potential inability of DM to correctly discern the society's preferences about the desired change of GHG emissions. This paper shows that the existence of deferential and preferential ambiguities have significant effects on GHG emissions regulation.

Keywords: decision making on climate change, ambiguity, deep uncertainty, deferential ambiguity, preferential ambiguity, tail risks of environmental-policy variables.

JEL Classification: D8, D80, D81, D83, D84, D89

*School of Economics, Athens University of Economics and Business, Greece; Grantham Research Institute, London Schoopl of Economics and Political Science, UK; International Center for Research on the Environment and the Economy, Greece; e-mail: pkoundouri@aueb.gr (corresponding author)

†Department of Banking and Financial Management, University of Piraeus, Greece; International Center for Research on the Environment and the Economy, Greece; e-mail: npittis@unipi.gr

‡Department of Banking and Financial Management, University of Piraeus, Greece; School of Economics, Athens University of Economics and Business, Greece; International Center for Research on the Environment and the Economy, Greece; e-mail: psamartzis@unipi.gr

§Department of Banking and Financial Management, University of Piraeus, Greece; International Center for Research on the Environment and the Economy, Greece; e-mail: englezos@unipi.gr

¶School of Economics, National and Kapodistrian University of Athens, Greece; International Center for Research on the Environment and the Economy, Greece; e-mail: aap@econ.uoa.gr

1 Introduction and Policy Motivation

It is often claimed that decision making on climate change is characterized by *ambiguity* (or deep uncertainty). Building on the work by Von Neumann and Morgenstern (1943) and Savage (1954), economic decision theory under uncertainty has been dominated, since the middle of the last century, by the expected utility theory and the Bayesian paradigm. A fundamental assumption in this tradition is that any source of uncertainty can be quantified in probabilistic terms, but in real life situations, probabilities of random events are often unknown. Thus, a distinction between risk (characterizing situations in which the probabilities of an uncertain event are perfectly known), and uncertainty or ambiguity (which exists when a random event cannot be described by a probability assessment), has been increasingly used. For climate change issues it is now widely accepted that most sources of uncertainty cannot be characterized as risk, and as a consequence the expected utility theory need to be modified to include ambiguity.

The Special Report on Emissions Scenarios (SRES) (Nakicenovic et al., 2000) generated 40 scenarios of 21st century anthropogenic greengouse gas (GHG) emissions for the IPCC's Third Assessment Report (Houghton et al., 2001) using six different computer models and a wide range of assumptions about the values of key driving forces. Each of the 40 scenarios is based on a basic qualitative storyline that describes a future state of the world and breaks down into a number of sub-scenarios. For example, the A1 storyline (scenario family) describes a future world of very rapid economic growth, global population that peaks in mid-century and declines thereafter and the rapid introduction of new and more efficient technologies. The A2 storyline describes a very heterogeneous world with an underlying theme of self-reliance and preservation of local identities where fertility patterns across regions converge very slowly, resulting in continuously increasing global population. In effect, we have various scenarios built under some general hypotheses for alternative future states of the world. None of these scenarios, however, includes assumptions about the behavior of the decision maker (DM) in response to the predictions to these scenarios (e.g. whether DM is environmentally friendly or hostile or indifferent).

Let us have a closer look at the interaction between scenarios and DM. The experts present DM with the description of the scenarios and their implications on the GHG emissions. Then, DM decides whether she will adopt a policy that is consistent with one of the scenarios. What is the probability of each possible future state of the world? Lempert et al. (2003, 2006) in their editorial essay argue that in SRES "... it is not possible to assign a likelihood to any of the emissions scenarios and that the associated uncertainties are best characterized by the full range of scenarios." The experts' weakness to assign a probability to every possible future state of the world is a central concern in this literature. Moss and Schneider (2000) have published a guideline which aims at helping experts decide the probability of each scenario: "In addition, all authors—whether in Working Group I, II or III—should be as specific as possible throughout the report about the kinds of uncertainties affecting their conclusions and the nature of any probabilities given."

In the case of climate chnage, there are two different sources of uncertainty that emerge: i) Uncertainty about which future state of the world will actually materialize, called "ambiguity" and ii) Given that a state of the world (scenario) materializes, uncertainty regarding the model's predictions of GHG emissions, called "risk". The second type of uncertainty is called risk and derives from uncertainty about "the distribution of values that a parameter, variable, or outcome may take" (Moss and Schneider 2000) and *not* uncertainty about the distribution of GHG emissions itself. In this paper, we focus on the first type of uncertainty (i.e. ambiguity) and argue that when attempting to resolve it, one has to account for the reaction of DM. For example, a sub-scenario in SRES (e.g. A1F1: very rapid economic growth, etc) assumes a "fossil intensive" development path. Let us assume that the experts assign equal probabilities to all 40 scenarios, $\frac{1}{40}$, and that DM starts with some scenario and realizes that GHG emissions will be huge. This alarms the DM and makes him adopt a fully renewable energy strategy, which corresponds to scenario A1T1 in SRES. By adopting this environmental friendly policy the DM effectively increases the probability of the non-fossil energy scenario. Hence her own policy action cancels the original $\frac{1}{40}$ probability of the fossil intensive scenario and effectively makes it smaller.

The preceding discussion highlights the fact that we cannot estimate the probability of a scenario without taking into consideration DM's reaction to that scenario. That is, we need to endogeneize DM's reaction to the arrival of new information, which in the case of climate change takes the form of alternative scenarios provided by the experts. The main argument of this paper is that the statement by SRES "it is not possible to assign a likelihood to any of the emissions scenarios" may derive from the difficulty of endogeneizing DM's reaction function. To the best of our knowledge there is no literature that attempts to model this endogeneity issue. Heal and Milner point to this issue: "Even if all scientific uncertainty were resolved, we would still face major uncertainties stemming from the socioeconomic dimensions of climate change. Suppose that all scientists agreed on both a climate model and the impacts of climate change on sea level rise, fresh water availability, and so on. Even in this case they would not be able to forecast the future climate because this requires knowing future emissions. We are far from being able to forecast future emissions because they depend on whether technological change provides us with new ways of reducing GHG emissions, and the policies chosen, which are themselves difficult to forecast. Even if future emissions were known, the future would still be unknown in many important ways. Take the case of sea level rise, for example. Let's suppose we had an accurate forecast of sea levels for the next century. How would societies react to rising seas? By protecting settlements or moving them? Would movements occur in a peaceful and organized manner, or would there be strife and dislocation? ... Clearly the answers to these types of questions will affect the welfare costs and distributional impacts of climate change. Thus even the full resolution of scientific uncertainties would leave huge residual uncertainties about the costs of climate change. "

In our model, the climate change experts provide DM with different GHG emissions forecasts (scenarios) while DM has to decide on which of these forecasts she will base her policy decision. The DM does not possess the epistemic status to select among the different expets' forecasts, which creates what we call *Deferential Ambiguity*, that is, the DM does not know to which expert's forecast (scenario) to defer. A second source of ambiguity (present even in the case of a single expert) stems from the potential inability of DM to correctly discern the society's preferences about the desired change of GHG emissions at each point in time. Hereafter, this type of ambiguity will be referred to as *Preferential Ambiguity.* An interesting question that emerges, is whether these two sources of ambiguity affect the future realization of GHG emissions. In this paper we argue that they do and thus, it is crucial to incorporate them in a general framework that will provide insights with strong policy impications. To summarize, our work focuses on: first, characterizing the different types of ambiguity faced by the DM and the experts, second examining their interaction and third analyzing the effects of this interaction on the probabilistic properties of GHG emissions.

2 Introducing the Conceptual Framework

Let us first describe the involved agents, namely DM, the society and the expert(s). Consider a DM who at time t is about to form her system of probabilistic beliefs, that is, her subjective probability function, P_t^{DM}, defined on a field of propositions/events Σ. DM is assumed to be rational, which amounts to saying that (i) DM's subjective probability function obeys the basic probability rules for every t, (ii) DM updates her probabilistic beliefs in the light of new evidence by Bayesian conditionalization[1] (BC) and (iii) DM obeys the Principal Principle (see, Lewis 1980), which states that if DM knows the objective probability (chance), $Ch(A)$, of $A \in \Sigma$, then she sets her subjective probability of A equal to the corresponding objective probability. The DM is interested in the climate variable Y (GHG emissions in our case), in the sense that her objective is to take the necessary policy actions to drive Y towards a desired level, set by the society which can be thought of as a group of agents, who have no scientific background and form ad-hoc estimations

[1] Conditionalization is a diachronic rule, requiring an agent's degrees of belief to line up in particular ways across times.

about the desired level of the future value of GHG emissions.

An expert is defined to be the agent who knows the objective probabilities (chances), $Ch(A)$, $A \in \Sigma$ of the events of interest. The expert's objective is to provide the DM with the necessary guiding information (i.e. his forecast/scenarios concerning the future evolution of Y) that will allow DM to implement the senario that the society prefers. In the case of a unique expert, DM is most likely to perceive this expert as the true bearer of the objective probabilities, which in turn implies that DM will have a strong incentive (since DM is benevolent) to defer to him at each point in time. As a result, DM's subjective probability distribution always coincides with the corresponding (unique) objective probability distribution. Hence, DM always knows the true probabilities of the events of interest, which in turn implies that she always operates under an environment of "risk" (known probabilities) rather than "ambiguity" (unknown probabilities).

What happens when there are more than one experts, say n, who disagree with each other about the chances of the events in Σ (or equivalently, when there are n competing scenarios for the same phenomenon)? Each of these experts has his own belief about the objective probability function on Σ (his own model). Hence, DM is faced with n experts' subjective probability functions, P_t^i, $i = 1, ..., n$, instead of one, which in turn complicates her attempts to form her subjective probability function P_t^{DM}. How can one interpret and model these complications? A method for combining or aggregating experts' probabilistic input is the so called *axiomatic method*, which is based on (i) setting a number of desirable axioms that the combined distribution should satisfy and (ii) finding the functional form that satisfies most (if not all) of these axioms. One of the most widely used functional form is the so called linear opinion pool, according to which $P_t^{DM} = \sum_{i=1}^{n} w_i P_t^i$, where the weights w_i are non-negative and sum to one. As Clemen and Winkler (1999) remark, the weights w_i may be interpreted as representing the relative quality of the n experts. In the case that all the experts are regarded as equivalent (by DM) linear opinion pooling reduces to a simple arithmetic average.[2] Under the linear opinion pooling, there is an implicit assumption concerning the relationship among (i) the input about the phenomenon Y that DM receives from the experts (ii) DMs' actions based on this input and (iii) the actual probabilistic properties of Y : DM's actions (informed by the views of the experts) do not affect the actual probability distribution of Y, or put differently, DM's actions are exogenous to Y. This assumption does not seem to be realistic concerning issues of climate change.

Heel and Millner (2015) refer to the endogeneity of emissions as follows: "Emissions uncertainty

[2] The linear opinion pool satisfies the axioms of *unanimity* and *marginalization* (eg. Clemen and Winkler 1999). However, it fails to satisfy the principle of *External Bayesianity*, which is one of the reasons why an alternative combination scheme, the so-called logarithmic opinion pool, is occasionally employed. For tractability, the linear opinion pool is used more oftenly.

arises because anthropogenic greenhouse gas emissions drive climate change projections in all models, and future emissions pathways are unknown, as they depend on our own future policy choices." (pp. 5). This is a case in which forecasts result in an adaptive change which in turn affects the forecasted quantity. Consequently, an expert who tries to produce a forecast for the change in Y between t and $t+1$, ΔY_{t+1}, should integrate in his forecast the DM's forecast for Y_{t+1}. This feature, however, produces a two way causality between the probabilistic views of DM and those of the experts: Experts' forecasts of ΔY_{t+1} affect DM's forecasts of ΔY_{t+1}, but at the same time, DM's forecasts, being causal factors in experts' models for ΔY_{t+1}, affect experts' forecasts. As a result, a model that allows DM to affect, via her actions, the determination of Y, must, at the same time, allow for a two way causality between DM's and the experts' views.

The main aim of the paper is to investigate the effects from the interaction between DM's and experts' forecasts for ΔY_{t+1} on the actual (objective) distribution, $F_{\Delta Y}$, of the change in GHG emissions. Various forms of such interactions are analyzed, with each one generating Deferential Ambiguity (Def) and/or Preferential Ambiguity ($Pref$). Def is defined as follows: At each point in time, each expert $i, i = 1, ..., n$ faces the following possibilities: (a) DM defers to his own forecasts (forecasts of expert i), (b) DM defers to the forecasts of expert $j \neq i$, (c) DM defers to a combination (e.g. linear pooling) of the two experts' forecasts and (d) DM defers to none of the two. These possibilities raise for each of the experts the following "specification issue": how should DM's deferential attitude be introduced in each of the experts' models? $Pref$ stems from the potential inability of DM to correctly discern the society's preferences about the desired change in GHG emissions at each point in time. How should the aforementioned DM's inability be introduced in each of the experts' models? The way that each expert answers these questions bears different implications for the actual generation mechanism of ΔY_{t+1}.

Concerning $Pref$, assume that the society's preference for GHG emissions formed at t for the value of Y at $t+1$ is denoted by $Y^*_{t,t+1}$ which is the sum of the current value, Y_t, of Y plus a quantity Z_t which represents the desired change in Y between t and $t+1$, that is, $Y^*_{t,t+1} = Y_t + Z_t$. This rule of dynamic determination of social preferences may be justified by assuming that in forming its desired level of Y for next period, the society takes into account the current level of Y. That is, tomorrow's level of desired Y is in the neighborhood of the current level of Y due to physical and technological limitations. Let us further assume, that despite her best efforts, DM fails to diagnose Z_t, and instead she believes that society's preferences are best captured by W_t. Alternatively, she might be able to identify Z_t, but she believes that society is currently wrong in focusing on Z_t and should focus on W_t. In Tunney and Ziegler's (2015) taxonomy, such a DM exhibits "benevolent attitude", in the sense that her actions are driven not by identifying "what the society would do" but rather by "what the society should do". This possibility is akin to the so-called "centralism" thesis (Press 1994) according to which "ecological problems can be solved only by strong centralized control of human behavior, thus making common resource decisions by central authorities and replacing democratic rule by 'ecological mandarins' with the 'esoteric' knowledge and public spirit required" (Coenen et. al 1998, pp5)).

Irrespective of the reasons that make DM to adopt W_t instead of Z_t, the crucial question is the following: Does the expert know that DM does not act upon Z_t but upon W_t? To this end there are three possibilities: (1) The expert (being a true expert) knows DM's preferential error right from the start. In such a case, the effects of DM's error on the actual generation process of ΔY_{t+1} are relatively simple to analyze: The probabilistic properties of ΔY_{t+1} do not depend on the probabilistic properties of the stochastic process $\{Z_t\}$, but rather on those of $\{W_t\}$. (2) The expert never realizes that the DM adopts W_t and erroneously believes that DM acts on Z_t. In this case, the actual distribution of ΔY_{t+1} will be different than the one in the expert's mind, which in turn implies that the expert is never the bearer of the objective chance. (3) The expert initially believes that DM acts on Z_t, but he endorses a learning process, in the context of which he repeatedly compares the realized values of ΔY_{t+1} with those implied by his model. Interestingly, we show that the resulting asymptotic distribution of ΔY_{t+1} does not coincide to that in the expert's mind, since there is a non-zero bias that survives even asymptotically, which has important policy implications.

The paper is organized as follows: Section 3, defines our basic model that examines the interactions among the involved agents, in the benchmark case where there is neither def nor $pref$. Section 4 introduces $pref$, which means that the expert will produce his forecasts on ΔY_{t+1} by means of a misspecified model which integrates sociental preferences instead of DM's actual preferences. In this section we assume that the expert never learns about his specification error (no learning mechanism exists), which in turn implies that the expert ends up having a subjective probability of ΔY_{t+1} different than the objective one (the expert is not the bearer of objective chance). Section 4.1 relaxes the no-learning assumption and derives the asymptotic distribution of ΔY_{t+1} under recursive Ordinary Least Squares (OLS) learning. An interesting feature of this case is that the forecast error committed by the expert never goes to zero. However, even under the aforementioned asymptotic bias, the stochastic process $\{\Delta Y_{t+1}\}$ converges-in-law. Section 5 gives a brief description of how def can be introduced in the model, together with its potential interactions with $pref$ and their combined effects on the probabilistic properties of $\{\Delta Y_{t+1}\}$. Section 6 summarizes the main findings, concludes the paper and draws lines for future research. The technical details are in the Appendix.

3 The Basic Model

The basic model (benchmark case) is defined by the following assumptions: (i) DM is interested in experts' point forecasts (conditional expectations) rather than their views about the complete distribution of ΔY_{t+1}. (ii) DM is assumed to be "projectivist", who always (i.e. for each t) acts in such a way as to bring the actual Y_{t+1} in line with the level $Y^*_{t,t+1}$ designated by society at t as optimal for $t+1$. (iii) DM acts upon Z_t, which means that DM's perception about the optimal level Y_{t+1} coincides with that of the society. In such a case, her projectivist attitude can be fulfilled. (iv) There is only one expert who knows the structural form of the statistical model describing the probabilistic properties of $\{\Delta Y_{t+1}\}$, as well as the true values of the model's structural parameters. As a byproduct assumptions (iii) and (iv), the expert knows that DM acts on the basis of Z.

Obviously, in our benchmark case, there is neither def nor $pref$. As such, this case is equivalent to the basic case of Baillon, Cabantous and Wakker (2012) (referred to as the "source risk") according to which "both agents are Bayesian and agree with each other (and everyone else). This is the common case of generally accepted objective probabilities, with no ambiguity involved" (pp.116). However, the aforementioned authors do not allow for any interactions between DM and the expert: "We also assume that there is no interaction between the agents themselves, or between the agents and the decision maker, so that no group process is involved." (pp. 116). On the contrary, our model not only allows for such interactions, but makes them our central topic of research.

Let us begin with introducing some basic concepts and notation. Assume that $\mathcal{E}_t(Y_{t+1})$ denotes the DM's forecast today for the actual level of Y at $t+1$, whereas Y^*_t (a simplified notation for $Y^*_{t,t+1}$) stands for the level of Y that the society at t thinks of as optimal (or desired) at $t+1$. As already mentioned, DM is supposed to act in line with society's preferences. This implies that whenever $\mathcal{E}_t(Y_{t+1}) > Y^*_t$ ($\mathcal{E}_t(Y_{t+1}) < Y^*_t$) DM acts in such a way as to produce a negative (positive) actual change ΔY_{t+1}. As far as Y^*_t is concerned, we assume that it is the sum of the actual Y_t and another variable Z_t, with the latter representing society's preferences at t for the next period's value of Y, i.e.

$$Y^*_t = Y_t + Z_t \tag{1}$$

If $Z_t > 0$ ($Z_t < 0$), the society prefers (at t) a higher (lower) value of Y_{t+1} than the one that currently prevails (namely Y_t). Let $\mu_Z = E(Z_t)$ and $\sigma^2_Z = V(Z_t)$. The more frequently the society's targets change over time, the larger the value of σ^2_Z. What are the reasons that cause the society to change the desired level of Y over time? One important such reason may be a change in the society's perception/understanding about the effects of GHG emissions on social welfare. For example, if the society becomes more environmentally aware, translating in the belief that a marginal change on GHG emissions will produce higher social welfare losses, then the society will prefer a lower future level of Y. As McKitrick (2014) puts it: "In a low-sensitivity model, GHG (greenhouse gases) emissions lead only to minor changes in temperature, so the socioeconomic costs associated with the emissions are minimal. In a high-sensitivity model, large temperature changes would occur, so marginal economic damages of CO2 emissions are larger." (pp. 1).

DM adopts society's target Z_t and since she is supposed to act in the best interests of the society, $(\mathcal{E}_t(Y_{t+1}) - Y_t^*)$ enters as a causal factor in the determination of ΔY_{t+1} with a negative coefficient. We may refer to this factor as the "human" factor. In addition, there is a physical variable (assumed to be exogenous in the standard sense) X_{t+1}, affecting ΔY_{t+1}, which may be referred to as the "physical" factor. Bringing these two factors together results in the following equation,

$$\Delta Y_{t+1} = Y_{t+1} - Y_t = \alpha(\mathcal{E}_t(Y_{t+1}) - Y_t^*) + \beta X_{t+1}, \tag{2}$$

The structural parameters α and β are assumed to be time invariant. With respect to α (key parameter in the ensuing analysis) we assume $-1 < \alpha < 0$ in order to capture DM's socially-sensitive behavior. Concerning the exogenous variable, we assume for simplicity that X_t is a Gaussian IID process with zero mean,

$$X_t \sim NIID(0, \sigma_X^2). \tag{3}$$

Under the assumptions made thus far, (2) becomes

$$\Delta Y_{t+1} = \alpha(\mathcal{E}_t(\Delta Y_{t+1}) - Z_t) + \beta X_{t+1} \tag{4}$$

As far as the experts are concerned, we assume that there is only one expert who knows the structural model given by equations, (1), (2) and (3) together with the value of the parameter vector $\theta = [\alpha, \beta, \mu_Z, \sigma_Z^2, \sigma_X^2]$. This means that the expert (at t) specifies in his model the same variable Z_t that DM adopts (no $pref$). Furthermore, we assume that DM always defers to expert's point forecast, $E_t(\Delta Y_{t+1})$, and the expert is aware of this fact (no def). Below we summarize the policy implications under the benchmark case. Section 1 in Appendix provides the technical details.

*Implication i***:** If the society desires, on average, a positive (negative) change in next period's level of emissions, this desire will be translated (via DM's actions) into an actual average positive (negative) change in emissions.

*Implication ii***:** The human involvement in the generation process of Y_{t+1} always results in an increase in the variability of ΔY_{t+1} (compared to the case where the DM's and society's involvment is absent). Put differently, even if the society (almost) always desires a lower level of next period's emissions (that is $Z_t < 0$), the DM's actions to achieve this target will produce an increase in the volatility of the actual changes in emissions, compared to the cases that (i) DM is inactive ($\alpha = 0$) or (ii) DM is active ($\alpha \neq 0$) but the society does not change its preferences over time ($Z_t = 0$).

*Implication iii***:** Given that the society prefers a negative ($\mu_Z < 0$) (positive $\mu_Z > 0$) change in GHG emissions, the more radical the DM (a decreases) is in terms of her policy actions towards the satisfaction of social preferences (e.g. rather than regulating GHG emissions, DM decides to adopt a fully renewable energy production model), the bigger the negative (positive) change in the actual GHG emissions ΔY_{t+1}. As far as the variance of GHG emissions is concerned, there is no ambiguity: the more radical the DM, the higher the increase in the variance of ΔY_{t+1}, irrespective of the preferences of the society. The aforementioned discussion bears some interesting policy implications regarding the degree of DM's radicality on "tail risks" (that is, the probability of realization of a very large change in Y_{t+1}). Consider the case in which $\mu_Z < 0$, that is the case in which the society exhibits aversion to emissions. The probability of observing larger values, given on the one hand, that the mean of the distribution of ΔY_{t+1} is shifted to the left and on the other, that the variance increases, could either increase or decrease, depending on the distribution of ΔY_{t+1}. Under the normality assumption (3), however, the aforementioned ambiguity is eliminated: the probability of observing positive changes in GHG emissions decreases as the DM becomes more radical (a decreases). The graph below, depicts the probability of the event $E = \{\Delta Y_{t+1} > 0\}$ as a function of α assuming, without loss of generality, that $\mu_Z = -0.5, \sigma_Z^2 = \beta = \sigma_X^2 = 1$:

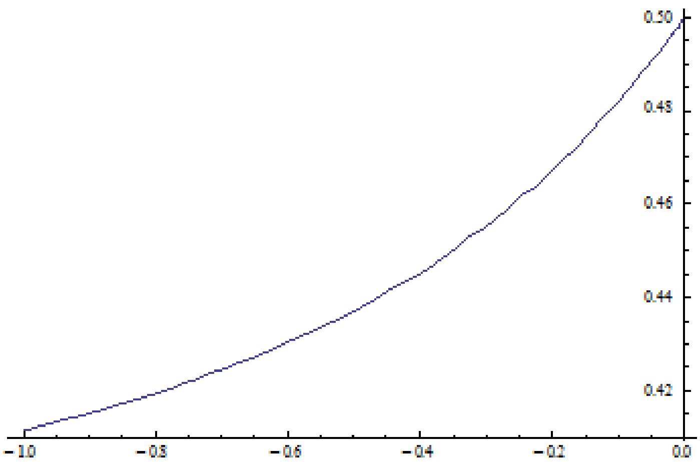

Figure 1: $P(E)$ as a function of α

4 Introducing Preferential Ambiguity

In this Section we introduce $pref$ and investigate its effects on the interaction among the agents involved and the resulting change in GHG emissions. In doing so, we retain the first two assumptions of the foregoing benchmark case and replace (iii) and (iv), with (iiia) and (iva), respectively: (iiia) DM acts upon W_t rather than Z_t. She may or may not know Z_t, while her decision to act on W_t is not necessarily a strategic act towards the fullfilment of her own self-interest/agenta. As Tunney and Ziegler (2015) remark, "surrogate decision makers may not have as their goal to match the wishes of the recipient, but instead to make what they perceive to be an optimal or benevolent decision." (2015, pp 884). (iva) The unique expert knows all the features of the structural form of the model of $\{\Delta Y_{t+1}\}$, except fo the fact that DM employs W_t instead of Z_t. This means that the expert will produce his forecasts of ΔY_{t+1} by means of a misspecified model, which includes the wrong variable Z_t instead of the true one, W_t. We also assume that the expert never learns about his specification error, which in turn implies that the expert ends up having a subjective probability of ΔY_{t+1} different than the objective one (the expert is not the bearer of objective chance). Below we summarize the policy implications under the case of $pref$, while Section 2 in Appendix provides the technical details.

Implication i: There are cases in which DM's focusing on W instead of Z, that is her "deviant behavior", results in significant shifts in the unconditional distribution of Y_{t+1}, which in turn may prove beneficial for the society in the future. Under a certain technical condition (see Appendix (10)), DM's actions shift the unconditional distribution to the left, thus reducing the probability of an extremely large value of ΔY_{t+1} (tail event) in the future. This may be interpreted as the result of DM's benevolent behavior who acts on the basis of what the society should prefer at t (normative stance) rather than what the society does prefer at t (descriptive stance).

Implication ii: A similar observation can be made with respect to the unconditional variance. Under certain technical conditions (see Appendix (11)), the variance in the case with $pref$, is smaller than that in the benchmark case. In other words, under (11), $pref$ decreases the unconditional variance, in comparison to the case of no $pref$. The human involvement in the regulation of Y_t always results in an increase in the variability of ΔY_{t+1}, compared to the case of no such involvement.

Implication iii: The case in which DM focuses on W instead of Z with a time-invariant policy reaction (i.e. constant parameter α) is equivalent to the case of a DM focusing on Z with a time-varying policy reaction (i.e. time-varying parameter γ_t). This means that DM does not have to exhibit "deviant behavior" in order to achieve her goals. The latter may be equivalently achieved if

DM exhibits "politically correct" behavior combined with a specific time-varying degree of policy reaction.

4.1 Introducing Learning

Let us now assume that the expert, utilizes the information that is being accumulated over time, to update his model by repeatedly comparing the realized values of ΔY_{t+1} to those implied by his model. Below we mention the policy implication under the $pref$ case with learning, while Section 3 in Appendix provides the technical details.[3]

Implication: For each t, the objective distribution of ΔY_{t+1} does not coincide to the corresponding subjective distribution of the expert. Specifically, for each t, the difference between the objective conditional mean and the subjective conditional mean of ΔY_{t+1} is, in general, different from 0. As a result, there exists a non-zero bias at each point t. The important question is whether this error asymptotically vanishes. The answer to this question is, in general, negative. This means that in spite of the learning process, the expert never achieves a full understanding of the situation, thus committing a forecast error even asymptotically.

5 Introducing Deferential Ambiguity

Let us now introduce a second expert who (initially or permanently) disagrees with the first regarding the values of the structural parameters, θ, of the true model. More specifically, we assume that both experts know the true structural form of the model (including the DM's preference variable Z) but take different views on the values of its parameters, with none of these experts knowing the true value of θ. This case corresponds to the so-called "conflict ambiguity" in Baillon, Cabantous and Wakker (2012): "For the second source of uncertainty, each agent alone fully satisfies Bayesianism, with a precise probability judgment. However, the two agents give different judgments, generating ambiguity for the decision maker aggregating their beliefs. This source of uncertainty, which is characterized by between-agent ambiguity (heterogeneous beliefs), is called conflict (C-)ambiguity in this paper." (pp. 117). As already mentioned, this type of ambiguity does not affect exclusively DM; instead because of the endogeneity of DM's forecasts, conflict ambiguity produces a "boomerang effect" by injecting this ambiguity back into the process of forecast formation by the experts. Each expert does not know at each point in time whether DM will defer to him or his competitor. Hence each expert should account for this ambiguity by introducing it

[3] In Section 4 in Appendix we derive a formula for the unconditional variance that ecompases the benchmark, the $pref$ with no learning and the $pref$ with learning cases.

explicitly into his model for the generation of Y_{t+1}. Below we summarize the policy implications under the def case, while Section 5 in Appendix provides the technical details.

Implication i: If the society desires, on average, a positive (negative) change in next period's level of emissions, this desire will be translated (via DM's actions) into an actual average positive (negative) change in emissions, as in the benchmark case. This implication suggests that the existence of def does not affect the result that the society will end up with a change in GHG emissions that it prefers.

Implication ii: If both experts assume that DM is more radical that she actually is (a more negative a), the unconditional variance of the change in GHG emissions will always be smaller than in the benchmark case. In particular, in such a case, the experts' forecasts will be suggesting smaller changes in the GHG emissions and therefore, the DM will take less radical/fewer actions, which reduces the unconditional variance of ΔY_{t+1}. If the two experts disagree on the DM's degree of radicalness and in particular the first (second) expert assumes that the DM's actions will be more (less) radical, while the other believes that she is less radical, the unconditional variance will be larger when the probability of deference to the first expert is small enough (see Appendix (14)). If both experts assume that DM will be less radical than she actually is, then the unconditional variance of the change in GHG emissions in the def case of two experts, will always be larger than in the benchmark case.

6 Conclusions

This is the first paper, to the best of our knowledge, that introduces ambiguity that derives from the interaction among the different agents relevant in climate change regulation. The salient features of our approach are the following: (i) Ambiguity is an epistemic state which characterizes not only DM but the scientific experts as well. We distinguish between preferential ambiguity, which is defined as the expert's uncertainty about DM's preference variables and deferential ambiguity, which arises in the case of multiple experts. Deferential ambiguity may be born by both DM and the experts and stems from the potential difficulty of DM to decide which of the experts should refer to. (ii) DM's ambiguity does not affect the formation of her prior probability function (which is the standard assumption in the ambiguity aversion literature). Instead, it affects the formation of DM's posterior distribution, in the sense that DM is uncertain about the piece of information that she should condition upon. As a result, DM's ambiguity is compatible with probabilistic sophistication. (iii) Both types of ambiguity have significant effects on the probabilistic properties of environmental policy variables. With respect to the policy relevant question of whether these types of ambiguity increase the probability of a "tail event" (i.e. extreme changes in GHG emissions), we show that the answer to this question depends on the probabilsitic properties of DM's adherence to the social preferences, on the extent to which the expert(s) learns from experience, on how DM combines experts' information and on the pattern of interaction between preferential and deferential ambiguity.

7 Appendix

7.1 Section 3 - Technical Details

Taking expectations on both sides of (4), we get $E_t(\Delta Y_{t+1}) = -\frac{\alpha}{1-\alpha} Z_t$ and therefore $\Delta Y_{t+1} = -\frac{\alpha}{1-\alpha} Z_t + \beta X_{t+1}$. The conditional distribution of ΔY_{t+1} is

$$\Delta Y_{t+1} \mid \mathcal{F}_t \sim N\left(-\frac{\alpha}{1-\alpha} Z_t, \beta^2 \sigma_X^2\right),$$

where $\mathcal{F}_t$ represents the information until t. The corresponding unconditional distribution is

$$\Delta Y_{t+1} \sim N\left(-\frac{\alpha}{1-\alpha}\mu_Z, \left(\frac{\alpha}{1-\alpha}\right)^2 \sigma_Z^2 + \beta^2 \sigma_X^2\right). \tag{5}$$

Comments on the Implications

(i) Since the coefficient $-\frac{\alpha}{1-\alpha}$ is always positive, the unconditional mean of ΔY_{t+1} is positive (negative) whenever the mean, μ_Z, of Z is positive (negative), i.e., on average, a positive (negative) change in next period's level of emissions, this desire will be translated (via DM's actions) into an actual average positive (negative) change in emissions.

(ii) The increase in the variability of ΔY_{t+1} in the case where $a \neq 0$ and $Z_t \neq 0$ (i.e. the case with human involvement), is equal to $\left(\frac{\alpha}{1-\alpha}\right)^2 \sigma_Z^2$.

7.2 Section 4 - Technical Details

DM acts upon her own preference variable W rather than that of the society, i.e. DM's and society's preferences are not aligned. Concerning W, let $\mu_W = E(W_t)$, $\sigma_W^2 = V(W_t)$, $corr(W_t, Z_t) = \rho_{WZ}$ and $corr(W_t, X_{t+1}) = 0$. As a result, (2) becomes

$$\Delta Y_{t+1} = \alpha(\mathcal{E}_t(\Delta Y_{t+1}) - W_t) + \beta X_{t+1} \tag{6}$$

The expert fails to recognize the discrepancy between DM's and society's preferences. Hence, he believes that the law of motion of Y_{t+1} is given by $\Delta Y_{t+1} = \alpha(\mathcal{E}_t(\Delta Y_{t+1}) - Z_t) + \beta X_{t+1}$. He also believes (correctly) that $\mathcal{E}_t(Y_{t+1}) = E_t(Y_{t+1})$ i.e. there is no def. In this case, the expert's subjective conditional distribution (the one perceived by the expert as true) is given by

$$\Delta Y_{t+1} \mid \mathcal{F}_t \sim N\left(-\frac{\alpha}{1-\alpha} Z_t, \beta^2 \sigma_X^2\right) \tag{7}$$

Since DM always defers to the expert, it follows that $\Delta Y_{t+1} = -\frac{\alpha^2}{1-\alpha} Z_t - \alpha W_t + \beta X_{t+1}$. The objective conditional distribution of ΔY_{t+1} is

$$\Delta Y_{t+1} \mid \mathcal{F}_{t-1} \sim N\left(-a\left(\frac{\alpha}{1-\alpha}Z_t + W_t\right), \beta^2\sigma_X^2\right), \tag{8}$$

whereas, the corresponding objective unconditional distribution is

$$\Delta Y_{t+1} \sim N\left(-\alpha\left(\frac{\alpha}{1-\alpha}\mu_Z + \mu_W\right), \left(\frac{\alpha^2}{1-\alpha}\right)^2 \sigma_Z^2 + \alpha^2\sigma_W^2 + \beta^2\sigma_X^2 + 2\frac{\alpha^3}{1-\alpha}\rho_{WZ}\sigma_W\sigma_Z\right). \tag{9}$$

Comments on the Implications

(i) If

$$\mu_W < -\frac{\alpha}{1-\alpha}\mu_Z, \tag{10}$$

then DM's actions shift the unconditional distribution to the left, thus reducing the probability of an extremely large value of ΔY_{t+1} (tail event) in the future.

(ii) If

$$\sigma_Z > \sigma_W \text{ and } \rho_{WZ} > \frac{(1+\alpha)\,\sigma_Z^2 - (1-\alpha)\,\sigma_W^2}{2\alpha\sigma_W\sigma_Z}, \tag{11}$$

the variance in (9) is smaller than that in (5).

(iii) The case in which DM focuses on W instead of Z with a time-invariant reaction parameter α is equivalent to the case of a DM focusing on Z with a time-varying reaction parameter γ_t.

Proof of Comment (iii): We want to find a process $\{\gamma_t\}$ such that $\alpha\,(E_t(\Delta Y_{t+1}) - W_t) + \beta X_{t+1} = \gamma_t\,(E_t(\Delta Y_{t+1}) - Z_t) + \beta X_{t+1}$. Solving for γ_t, we get

$$\gamma_t = \frac{\alpha\,(E_t(\Delta Y_{t+1}) - W_t)}{E_t(\Delta Y_{t+1}) - Z_t}$$

Define $\rho_t = \frac{W_t}{Z_t}$. Since $E_t(\Delta Y_{t+1}) = -\frac{\alpha}{1-\alpha}Z_t$ the above equation becomes

$$\gamma_t = \frac{\alpha\left(-\frac{\alpha}{1-\alpha}Z_t - \rho_t Z_t\right)}{-\frac{\alpha}{1-\alpha}Z_t - Z_t}$$

and as a result, $\gamma_t = a^2 + \rho_t\,(1-\alpha)\,.$

7.3 Section 4.1 - Technical Details

Retaining the assumption that the expert fails to observe the discrepancy between DM's and society's preferences, the only possible form of learning, is "parameter updating". Let the perceived (by the expert) law of motion (PLM) be

$$\Delta Y_{t+1} = AZ_t + u_{t+1} \tag{12}$$

where A is the parameter that the expert tries to estimate and u_{t+1} is a Gaussian IID process with zero mean. PLM is the reduced form model that the expert has in his mind when communicating his forecasts of ΔY_{t+1}, i.e. $E_t(\Delta Y_{t+1})$, to the expert. To update the parameter A, he applies the recursive least squares (RLS) to (12). This methodology produces an estimate $\hat{A}_t$ for each time t, which minimizes the mean squared error, namely $E\left(\Delta Y_t - E_{t-1}(\Delta Y_t)\right)^2$.

Recursive Least Squares: The least squares estimate is

$$A_t = \left(\sum_{s=1}^{t} Z_{s-1}^2\right)^{-1}\left(\sum_{s=1}^{t} Z_{s-1}\Delta Y_s\right)$$

More conveniently, the least squares estimates may be written in a recursive manner as

$$\begin{aligned} A_t &= A_{t-1} + t^{-1}R_t^{-1}Z_{t-1}(\Delta Y_t - A_{t-1}Z_{t-1}) \\ R_t &= R_{t-1} + t^{-1}(Z_{t-1}^2 - R_{t-1}) \end{aligned}$$

where $R_t = t^{-1}\left(\sum_{s=1}^{t} Z_{s-1}^2\right)$. The objective is to find the asymptotic value of A_t, denoted by A^*, and the conditions that lead to $A_t \to A^*$?

The expert's forecast of ΔY_t at time $t-1$, is given by $E_{t-1}(\Delta Y_t) = A_{t-1}Z_{t-1}$, which under (6) yields

$$\Delta Y_t = \alpha\left(A_{t-1}Z_{t-1} - W_{t-1}\right) + \beta X_t.$$

Hence, the RLS system can be written as

$$\begin{aligned} \phi_t &= \phi_{t-1} + t^{-1}R_t^{-1}Z_{t-1}((\alpha-1)\,A_{t-1}Z_{t-1} - \alpha W_{t-1} + \beta X_t) \\ R_t &= R_{t-1} + t^{-1}(z_{t-1}z_{t-1}' - R_{t-1}) \end{aligned}$$

In order to apply the standard convergence results of stochastic recursive algorithms, we need to set $S_{t-1} = R_t$, in order for the term R_t^{-1} in the lhs of the first equation to be a lagged variable:

$$\begin{aligned} \phi_t &= \phi_{t-1} + t^{-1}S_{t-1}^{-1}Z_{t-1}((\alpha-1)\,A_{t-1}Z_{t-1} - \alpha W_{t-1} + \beta X_t) \\ S_t &= S_{t-1} + t^{-1}\left(\frac{t}{t+1}\right)(Z_t^2 - S_{t-1}). \end{aligned}$$

The associated ordinary differential equation (ODE) that governs stability of the system above is

$$\frac{d\Phi}{d\tau} = h(\Phi) = \lim_{t\to\infty} E\left(Q\left(t,\Phi,z_t\right)\right)$$

where $\Phi = (A,S)^{'}$, $z_t = (Z_{t-1}, W_{t-1}, X_t)$ and E denotes the expectation of $Q\left(t,\Phi,z_t\right)$ taken over the invariant distribution of z_t, for fixed $\Phi.Q\left(t,\Phi,z_t\right)$ is derived by the RLS system and is defined as

$$Q\left(t,\Phi,z_t\right) = \begin{pmatrix} S^{-1}Z_{t-1}((\alpha-1)\,AZ_{t-1} - \alpha W_{t-1} + \beta X_t) \\ \left(\frac{t}{t+1}\right)(Z_t^2 - S) \end{pmatrix}$$

It follows that

$$\begin{aligned} h_\theta(\Phi) &= \lim_{t\to\infty} E\left(S^{-1}Z_{t-1}((\alpha-1)\,AZ_{t-1} - \alpha W_{t-1} + \beta X_t)\right) \\ h_S(\Phi) &= \lim_{t\to\infty}\left(\frac{t}{t+1}\right) E(Z_t^2 - S) = \left(\sigma_Z^2 + \mu_Z^2\right) - S \end{aligned}$$

The second relationship gives $S \to \left(\sigma_Z^2 + \mu_Z^2\right)$, and therefore,

$$\begin{aligned} h_\theta(\Phi) &= \lim_{t\to\infty} E\left(\left(\sigma_Z^2+\mu_Z^2\right)^{-1} Z_{t-1}((\alpha-1)\,AZ_{t-1} - \alpha W_{t-1} + \beta X_t)\right) = \\ &= (\alpha-1)\,A - \alpha\left(\sigma_Z^2+\mu_Z^2\right)^{-1}\left(\rho_{WZ}\sigma_W\sigma_Z + \mu_Z\mu_W\right) \end{aligned} \tag{13}$$

The ODE (13) gives the system

$$\dot{A} = (\alpha-1)\,A - \alpha\left(\sigma_Z^2+\mu_Z^2\right)^{-1}\left(\rho_{WZ}\sigma_W\sigma_Z + \mu_Z\mu_W\right)$$

whose solution is given by

$$A^* = -\frac{\alpha}{1-\alpha}\frac{\rho_{WZ}\sigma_W\sigma_Z + \mu_Z\mu_W}{\sigma_Z^2+\mu_Z^2}$$

The convergence to A^* is convergence in probability, i.e. $\forall\varepsilon > 0$, $\lim_{t\to\infty} P\left(|A_t - A^*| \geq \varepsilon\right) = 0$. The E-stability amounts to the condition $\alpha < 1$, which holds by assumption. Therefore, with probability 1, the system will converge to the equilibrium, irrespective of the initial estimations A_0. Hence, asymptotically, the expert's view on A will settle down on A^* (which is different than $-\frac{\alpha}{1-\alpha}$). The asymptotic objective conditional distribution of ΔY_{t+1} is

$$\Delta Y_{t+1} \mid \mathcal{F}_{t-1} \sim N\left(a\left(A^* Z_t - W_t\right), \beta^2\sigma_X^2\right),$$

whereas, the corresponding objective unconditional distribution is

$$\Delta Y_{t+1} \sim N\left(a\left(A^{*}\mu_{Z}-\mu_{W}\right), \left(\alpha A^{*}\right)^{2}\sigma_{Z}^{2}+\alpha^{2}\sigma_{W}^{2}+\beta^{2}\sigma_{X}^{2}-2\alpha^{2}A^{*}\rho_{WZ}\sigma_{W}\sigma_{Z}\right).$$

Comments on the Implications

(i) For each t, the difference between the objective conditional mean and the subjective conditional mean of ΔY_{t+1} is given by $(a-1)\,A_{t}Z_{t}-\alpha W_{t}$, which, in general, is different from 0. The corresponding difference in the asymptotic means is $\alpha\left(\frac{\rho_{WZ}\sigma_{W}\sigma_{Z}+\mu_{Z}\mu_{W}}{\sigma_{Z}^{2}+\mu_{Z}^{2}}\mu_{Z}-\mu_{W}\right)$, which is zero iff $\rho_{WZ}=\frac{\mu_{W}\sigma_{Z}}{\mu_{Z}\sigma_{W}}$. Therefore, the expert commits a forecast error even asymptotically.

(ii) The asymptotic parameter A^{*} can be decomposed in two terms: $-\frac{\alpha}{1-\alpha}$ and $\frac{\rho_{WZ}\sigma_{W}\sigma_{Z}+\mu_{Z}\mu_{W}}{\sigma_{Z}^{2}+\mu_{Z}^{2}}$. The second term may be thought of as an "adjustment factor" that captures the effects of learning. Specifically, in the no-learning case the expert is always under the impression that the coefficient of Z_{t} is $-\frac{\alpha}{1-\alpha}$ (7) whereas under learning he ends up believing that this coefficient is A^{*}. As expected, when $W_{t}\equiv Z_{t}$, the second term is equal to 1 and, A^{*} collapses to $-\frac{\alpha}{1-\alpha}$, that is the coefficient of the benchmark case.

7.4 Comparison of Variances in Sections 3 & 4

The unconditional variance for all the above cases can be written in a eneral form as

$$\alpha^{2}\left[\left(\frac{\alpha}{1-\alpha}R\sigma_{Z}+\sigma_{W}\right)^{2}-2\frac{\alpha}{1-\alpha}R\left(1-\rho_{WZ}\right)\sigma_{Z}\sigma_{W}\right]+\beta^{2}\sigma_{X}^{2}$$

To arrive at the first case, we have that $R=\rho_{WZ}=1,\sigma_{W}=\sigma_{Z}$. For the second case, we need $R=1$ and the third $R=\frac{\rho_{WZ}\sigma_{W}\sigma_{Z}+\mu_{Z}\mu_{W}}{\sigma_{Z}^{2}+\mu_{Z}^{2}}$. Note that the unconditional variance is increasing (decreasing) in R if $R>(<)-\frac{1-a}{\alpha}\frac{\sigma_{W}}{\sigma_{Z}}\rho_{WZ}$.

7.5 Section 5 - Technical Details

The assumptions that we make are the following: (i) DM adopts Z_{t} (rather than W_{t}) at t (there is no preferential ambiguity). (ii) There are two experts who believe that the structural model is given by equations, (1), (2) and (3). The two experts agree on $\mu_{Z},\sigma_{Z}^{2},\sigma_{X}^{2}$ but disagree on α and β. Hence, the agent i's epistemic state is represented by $\theta_{i}=[\alpha_{i},\beta_{i},\mu_{Z},\sigma_{Z}^{2},\sigma_{X}^{2}]$, $i=1,2$. Without loss of generality, assume that $\alpha_{2}>\alpha_{1}$. (iii) The objective probability that DM at t defers to expert's i point forecast, $E_{t}^{i}(\Delta Y_{t+1})$, is p_{i}. (iv) DM combines experts forecasts by means of a linear pool using p_{1} and p_{2} as weights. (v) Both experts know the objective deferential probabilities p_{1} and p_{2} as well as DM's aggregation rule.

Under the foregoing assumptions, we get $E_t^i(\Delta Y_{t+1}) = -\frac{\alpha_i}{1-\alpha_i} Z_t$. Now the actual law of motion becomes $\Delta Y_{t+1} = -\alpha(p\frac{\alpha_1}{1-\alpha_1} + (1-p)\frac{\alpha_2}{1-\alpha_2} + 1)Z_t + \beta X_{t+1}$. The conditional distribution is

$$\Delta Y_{t+1} \mid \mathcal{F}_{t-1} \sim N\left(-\alpha\left(p\frac{\alpha_1}{1-\alpha_1} + (1-p)\frac{\alpha_2}{1-\alpha_2} + 1\right)Z_t, \beta^2\sigma_X^2\right).$$

The unconditional distribution is

$$\Delta Y_{t+1} \sim N\left(-\alpha\left(p\frac{\alpha_1}{1-\alpha_1} + (1-p)\frac{\alpha_2}{1-\alpha_2} + 1\right)\mu_Z, \left(\alpha\left(p\frac{\alpha_1}{1-\alpha_1} + (1-p)\frac{\alpha_2}{1-\alpha_2} + 1\right)\right)^2\sigma_Z^2 + \beta^2\sigma_X^2\right).$$

Comments

(i) Since the coefficient $-\alpha$ is always positive, the unconditional mean of ΔY_{t+1} will always be positive (negative) whenever the mean, μ_Z, of Z is positive (negative).

(ii) If $\alpha_1, \alpha_2 < \alpha$, the unconditional variance is always smaller than in the benchmark case, since their opinions will be closer to the "no action" case. Specifically, the term $\mathcal{E}_{t-1}(Y_t) - Y_{t-1}^* = \mathcal{E}_{t-1}(\Delta Y_t) - Z_{t-1}$ is closer to 0, i.e. the "no action" case. As a result, the DM will take fewer actions, which reduces the unconditional variance. If $\alpha_1 < \alpha < \alpha_2$, i.e. the first (second) expert assumes that the DM's actions will be more (less) significant, the unconditional variance will be larger when

$$p < \frac{(1-\alpha_1)(\alpha_2-\alpha)}{(\alpha_2-\alpha_1)(1-\alpha)}. \tag{14}$$

If $\alpha < \alpha_1 < \alpha_2$, then the unconditional variance in the case of two experts will always be larger.

References

Al-Najjar, N. and J. L. Weinstein, 2009, The Ambiguity Aversion Literature: A Critical Assessment, *Economics and Philosophy*, 25: 249-284.

Baillon A, Cabantous L, Wakker P.P., 2012, Aggregating imprecise or conflicting beliefs: An experimental investigation using modern ambiguity theories, *Journal of Risk and Uncertainty,* 44:115–147.

Carnap, R., 1950, Logical Foundations of Probability, Chicago, IL: University of Chicago Press.

Clemen, R.T. and R. L. Winkler, 1999, Combining probability distributions from experts in risk analysis, *Risk Analysis,* 19(2): 187–203.

Coenen, F., D. Huitema, and L. O'Toole, Jr., eds. 1998, Participation and the quality of environmental decision making, Dordrecht, Netherlands: Kluwer.

French, S., 1981, Updating of belief in the light of someone else's opinion, *Journal of the Royal Statistical Society,* Ser. A 143 43-48.

Gilboa, I. and D. Schmeidler, 1989, Maxmin Expected Utility with a Non-Unique Prior, *Journal of Mathematical Economics*, 18: 141-153.

Heal G. and A. Millner, 2014, Uncertainty and Decision Making in Climate Change Economics, *Review of Environmental Economics and Policy* 8.1, 120–137.

IPCC, 1997, The Regional Impacts of Climate Change: An Assessment of Vulnerability, Geneva: *Cambridge University Press.*

IPCC, 2013, Climate Change 2013: The Physical Science Basis. Contribution of Working Group I to Fifth Assessment Report of the Intergovernmental Panel on Climate Change", New York: *Cambridge University Press.*

Jacobs R. A., 1995, Methods for combining experts' probability assessments, *Neural Computation*, vol. 7, pp. 867–888.

Klibanoff, P., M. Marinacci, and S. Mukerji, 2005, A Smooth Model of Decision Making under Ambiguity, *Econometrica*, 73: 1849-1892.

Lempert R., S. Popper, and S. Bankes, 2003, Shaping the Next One Hundred Years: New Methods for Quantitative Long-Term Policy Analysis, RAND Corporation (MR-1626).

Lempert R., D. Groves, S. Popper, and S. Bankes, 2006, A General, Analytic Method for Generating Robust Strategies and Narrative Scenarios, *Management Science*, 52(4).

Lewis, D., 1980, A Subjectivist Guide to Objective Chance, In Richard C. Jeffrey, ed., *Studies in Inductive Logic and Probability*, vol. II. Berkeley: University of California Press. Reprinted with postscripts in Lewis 1986, pp. 83–132.

Lindley, D. V., 1985, Reconciliation of discrete probability distributions, *Bayesian Statistics* 2, North-Holland (Amsterdam).

McKitrick, Ross R., 2014, Climate Policy Implications of the Hiatus in Global Warming, Vancouver: Fraser Institute, October 2, 2014.

Meacham, C., 2007, Chance and the Dynamics of De Se Beliefs, Doctoral Dissertation.

Press, D., 1994, Democratic dilemmas in the age of ecology. Trees and toxics in the American West, Durham.

Schmeidler, D., 1989, Subjective Probability and Expected Utility without Additivity, *Econometrica*, 57: 571-587.

Tunney, R.J., and F.V. Ziegler, 2015, Surrogate utility estimation by long-term partners and unfamiliar dyads, *Frontiers in Psychology*, 6. doi: 10.3389/fpsyg.2015.00315 .