



# A GRASP metaheuristic for the last-mile Vehicle Routing Problem with Delivery Options

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# Presentation Outline

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## Introduction

- The last-mile challenge
- Literature

## The Vehicle Routing Problem with Delivery Options

- Visual example
- Definition

## Solution methodology

- Initial solution construction
- Local search

## Computational Results

## Conclusions

# Introduction

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THE LAST-MILE CHALLENGE



# Last-mile delivery

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The **final step** of the delivery process where goods are transported from a **transportation hub** to their **final destination** (e.g., customer's home).

There has never been a time of **greater demand** for last-mile transport

- Last mile **market size** ([Global Market Insights, GMI 2024](#))
  - 2023: **175.3\$ billions**
  - 2032: **305.4\$ billions**
  - North America contributes the 37% of this (2023).
  - Increasing trend in many markets around the globe

Last mile is the **costliest link** in the **supply chain**

- **41%** of overall supply chain costs (almost double of all other processes, i.e., parcelling, warehousing) ([Cemex Ventures, 2023](#)).

# Challenges

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## Urban Congestion

Freight traffic contributes to **20%** of urban **traffic congestion**



## Environmental Impact

Freight transport accounts for approximately **25%** of **greenhouse gas** emissions



## High Delivery Costs

The net **profit margins** for many transport companies are minimal, often **negligible**



## Customer Expectations

There is an increasing **demand** for faster, more flexible, and reliable delivery options, including **shorter delivery windows**, accurate time predictions, and **same-day** deliveries.

# Optimization

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**Operations Research:** numerous **models** to optimize cases of delivery processes:

- enhancing **resources efficiency**
- minimizing **costs**
- increasing customer **satisfaction**

The **foundational VRP model** lies below several specialized case specific realistic variants:

- **CVRP** (Cumulative VRP): Manages accumulated cost (e.g., for satisfying latest arrival)
- **VRPTW** (VRP with Time Windows): Incorporates specific delivery time frames
- **VRPPD** (VRP with Pickup and Delivery): Handles both delivery and pickup tasks

# Vehicle Routing Problem with Delivery Options

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## Seminal papers:

- Tilk, C., Olkis, K. and Irnich, S. (2021), “The last-mile vehicle routing problem with delivery options”, *OR Spectrum*, Springer Berlin Heidelberg, Vol. 43 No. 4, pp. 877–904.
- Dumez, D., Lehuédé, F. and Péton, O. (2021), “A large neighborhood search approach to the vehicle routing problem with delivery options”, *Transportation Research Part B: Methodological*, Elsevier Ltd, Vol. 144, pp. 103–132.

Motivated by **last mile delivery challenges**: “the **bottleneck** of e-commerce” ([Wang et al. 2014](#)) & “the logistic service providers’ **nightmare(s)**” ([Savelsbergh and Van Woensel 2016](#)).

## Extends

- Vehicle Routing Problem with Time Windows (**VRPTW**)
- Generalized Vehicle Routing Problem (**GVRP**)

# Vehicle Routing Problem with Delivery Options

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## Innovation

- Alternative customer delivery **options** with **ranking**
- Capacitated shared facilities

## Challenges

- **Time windows**
- **Synchronized** resources (shared locations capacity, priorities)
- New **structure** of the search space, due to the presence of alternative delivery locations for each customer



# Literature & Motivation

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## Cardeneo (2005)

- Introduced the initial basic version of the Vehicle Routing Problem (VRP) with **alternative delivery locations**

## Los et al. (2018)

- Considered **service levels** and **customer preferences**, along with location selection, in the generalized pickup and delivery problem with time windows and preferences

## Ozbaygin et al. (2017); Reyes et al. (2017)

- Addressed the **vehicle routing problem with home and roaming delivery locations** (VRP(H)RDL), a special case of the VRPDO

## Lombard et al. (2018)

- Explored the VRP(H)RDL with **stochastic** travel times

## Tilk et al. (2021): Introduced the VRPDO

- **branch-and-price algorithm** featuring two different network structures, cutting planes, and branching rules
- **state-of-the-art algorithm** on benchmark instances for **VRPHRDL** and **VRPRDL**

## Dumez et al. (2021): Introduced the VRPDO

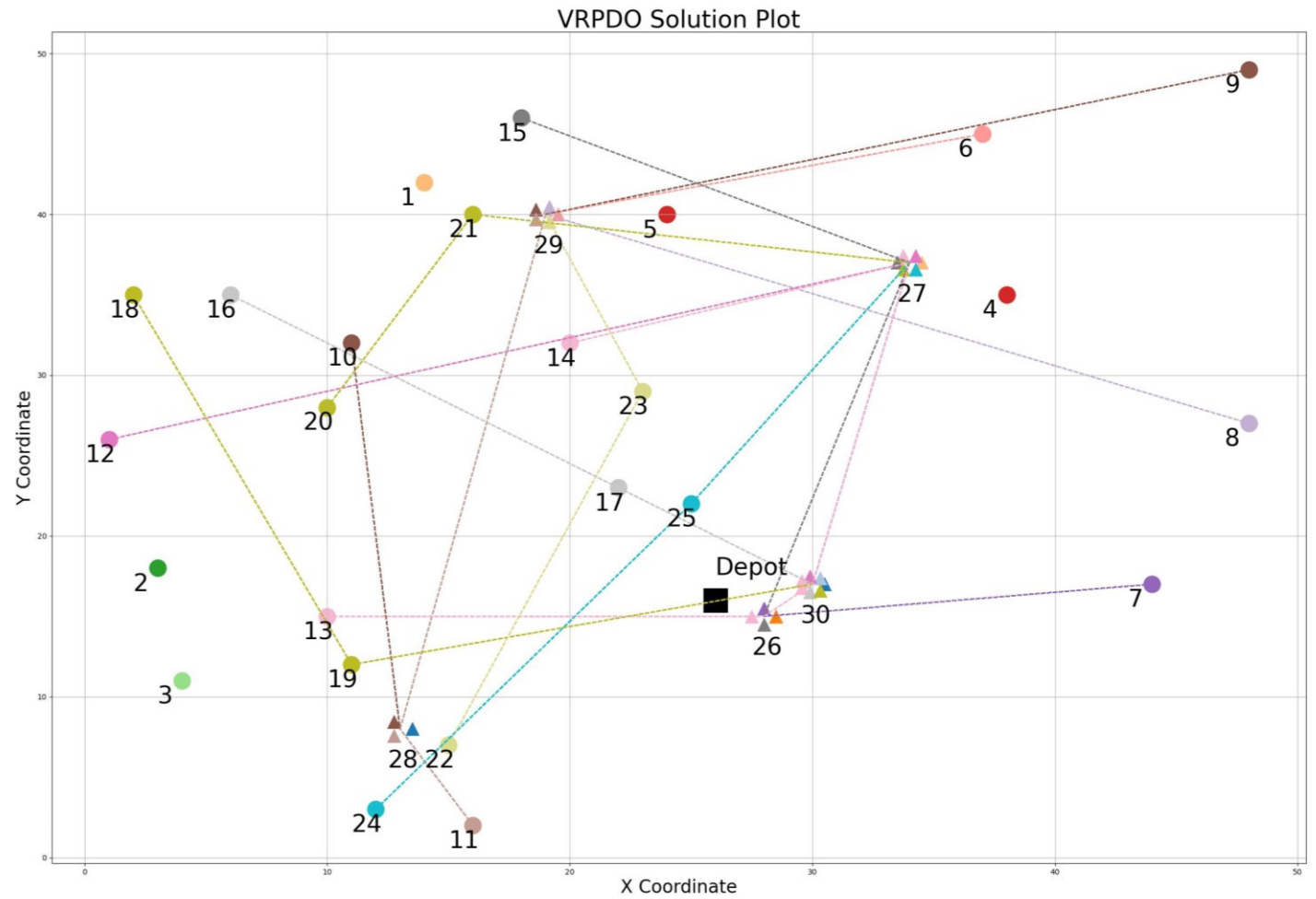
- **Large neighborhood search algorithm** with ruin and recreate operators
- A set partitioning problem is periodically used to reassemble routes

# Problem definition

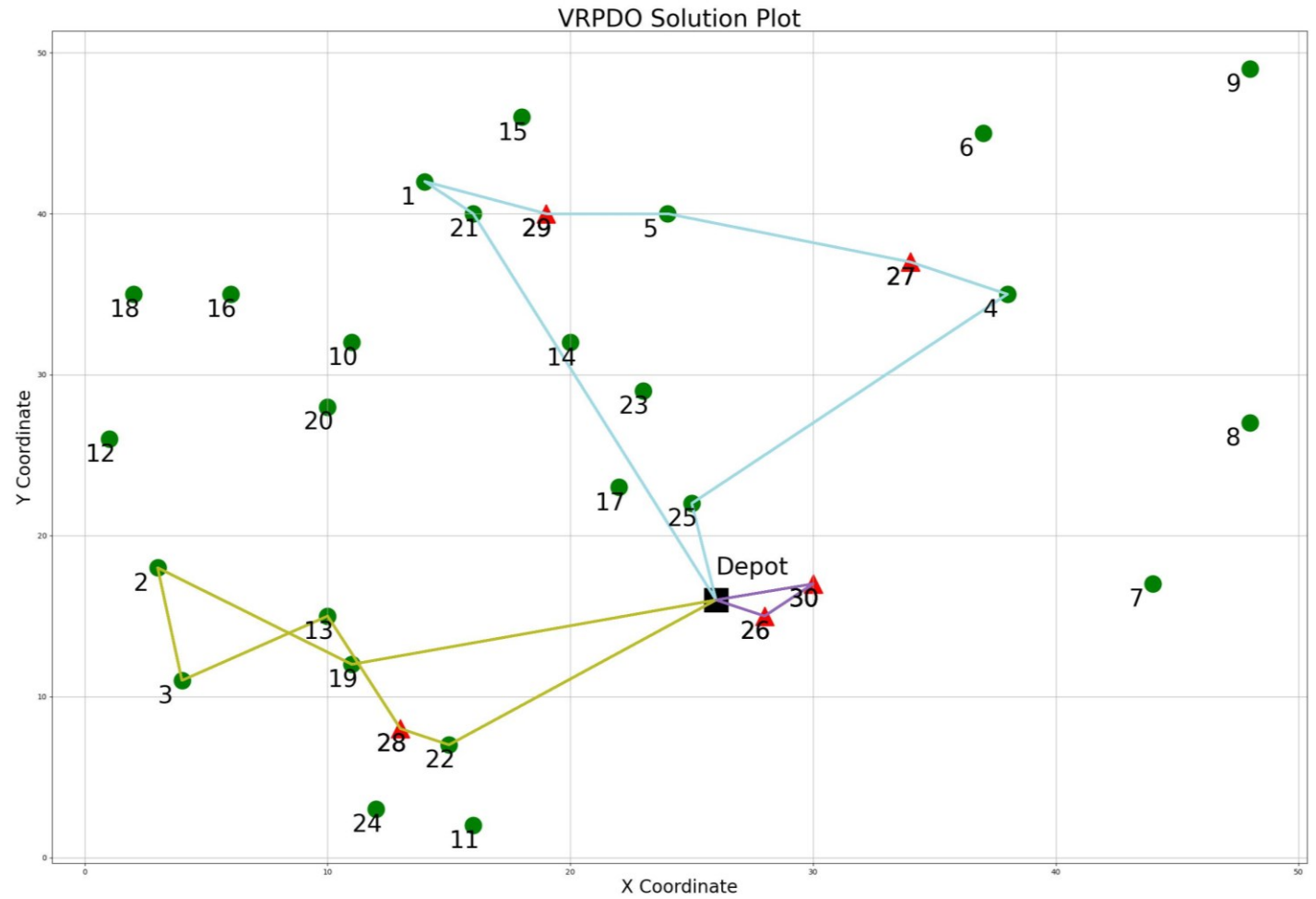
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VISUAL EXAMPLE AND MODEL

# Plotting an instance



# Plotting the solution



# Vehicle Routing Problem with Delivery Options

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## Objective:

**Minimizing** the number of **vehicles** and the total travel **cost** for serving all customers.



## Constraints:

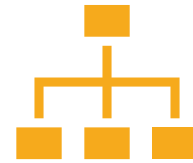
**Limited** number of capacitated **vehicles**

**Time windows**

**All customer** must be served

Shared **locations capacity**

**Service levels** to satisfy priorities



## Decisions:

**Which option** to chose for each customer?

Which are the **best routes** to serve the selected options?

The model is explained in detail in [Dumez et al. \(2021\)](#).

# Methodology

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GRASP METAHEURISTIC

# Grasp metaheuristic algorithm

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Iteratively follow **construct** initial solutions and **improve** via local search

## **Step 1:** Construct **initial solutions**

- **Minimum insertion** algorithm
- **Solution pool** for diversified option combinations

## **Step 2:** Iteratively **improve** the solutions via **Local search**

- **Scheme** design
- **Promises** to avoid cycling
- Routing and option **operators/neighborhoods**

# Approach 1 – Minimum insertion

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- **Minimum insertion:** Use minimum insertion to find the **best three** insertions in each loop until all customer are served.
- **Vehicle number minimization:** **Penalize** insertions in **empty or near empty vehicles** to ensure least number of vehicles while maintaining feasibility.
- **RCL (Restricted Candidate List):** Randomly select one of the top three solutions for **diversified**, high-quality restarts.

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**Algorithm 1** Overall Scheme - Minimum insertion

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```
1:  $S \leftarrow \text{MinimumInsertionAlgorithm}(), S^* \leftarrow \emptyset$ 
2: for  $i \leftarrow 1$  to restarts do
3:    $S_i^* \leftarrow \text{LocalSearch}(S_i)$ 
4:   if  $Z(S_i^*) > Z(S^*)$  then
5:      $S^* \leftarrow S_i^*$ 
6:   end if
7: end for
8: return  $S^*$ 
```

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**Problem:** **Low diversity in options** and each replacement alters the network.

# Approach 2 – Solution pool

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## **Predetermined Options:**

- Use a **weighted metric** to select options for each customer:
  - **Distance** from closest nodes
  - **Compatibility** of **time windows** with neighbors
  - Time windows **span**

## **Route Construction:**

- Apply the **minimum insertion algorithm** to route all preselected options.
- Repeat until a **satisfactory number of solutions** is available.

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- Apply the **minimum insertion algorithm** to route all preselected options.
- Repeat until a **satisfactory number of solutions** is available.

## **Solution Pool:**

- **Advantage:** Focus on the **problem network** affects the **search space**.
- **Disadvantage:** **Manual weighting**; a learning procedure is worth investigating.

# Approach 2 – Solution pool

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## Algorithm 2 Overall Scheme - Solution pool

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```
1:  $S^* \leftarrow \emptyset$ 
2:  $O^{\text{selected}} \leftarrow \text{FindBestOptions}()$ 
3:  $S \leftarrow \text{MinimumInsertionAlgorithm}(O^{\text{selected}})$ 
4: for  $i \leftarrow 1$  to  $\text{restarts}$  do
5:    $S_i^* \leftarrow \text{LocalSearch}(S_i)$ 
6:   if  $Z(S_i^*) > Z(S^*)$  then
7:      $S^* \leftarrow S_i^*$ 
8:   end if
9: end for
10: return  $S^*$ 
```

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# Local search scheme

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## Multiple Restarts:

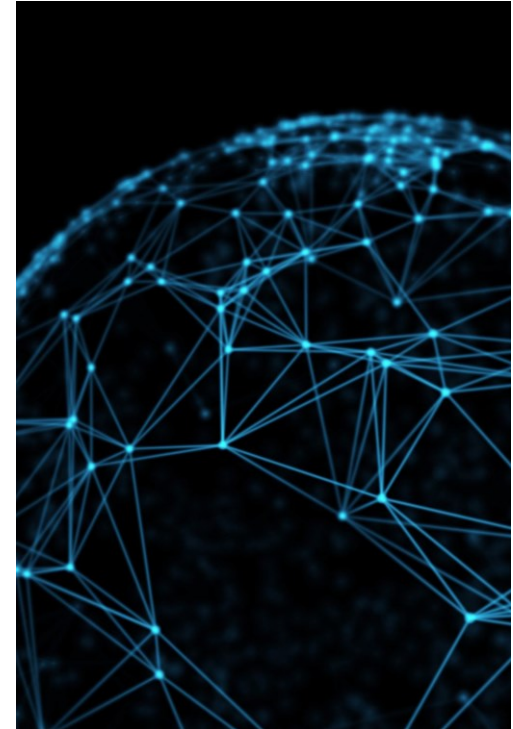
- Set **maximum iterations** and limit **non-improving iterations**.

## Move Filtering/Tabu Policy:

- Utilize the **promise** mechanism by [Zachariadis et al. \(2015\)](#).

## Neighborhood Exploration:

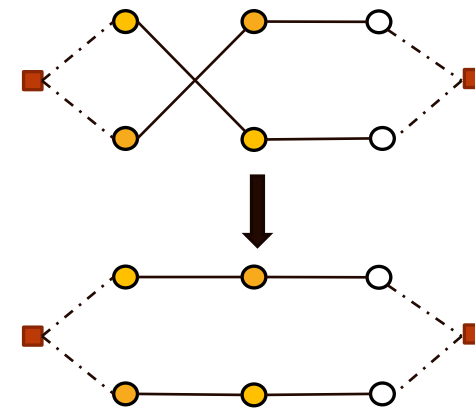
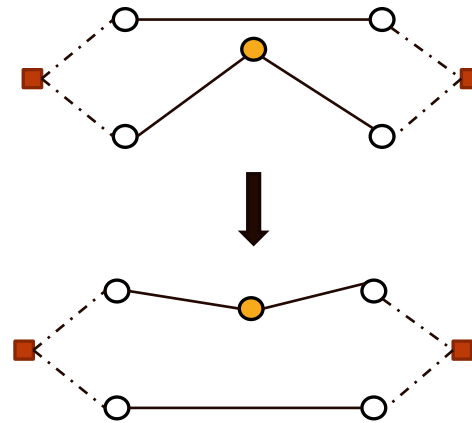
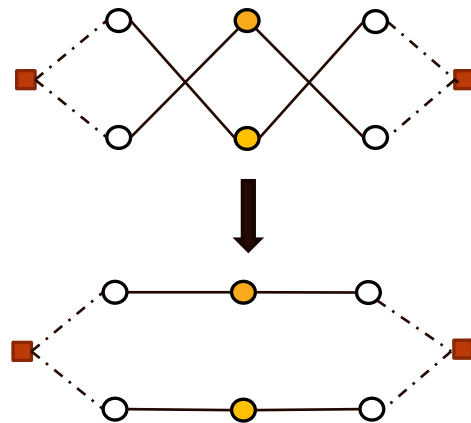
- Explore all neighborhoods in each iteration using **five operators**:
  - **three** classic **routing** operators
  - **two option-related** operators, controlled due to network alteration and combinatorial impact.



# Classic routing operators

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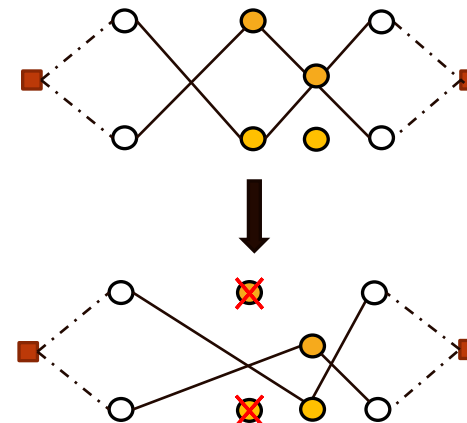
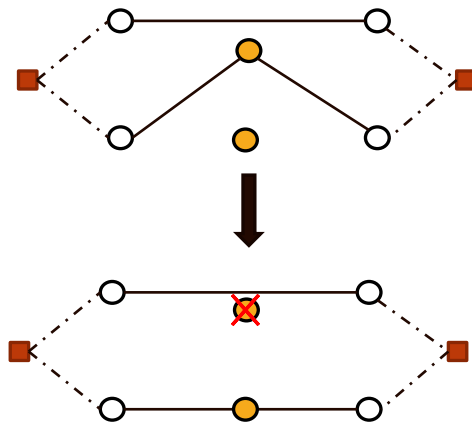
1. **Swap**: exchanges the positions of two selected options in the same or different routes.
2. **Relocation**: moves a single option from its current position to another position within the same route or to a different route.
3. **2-Opt**: remove two edges from the same or different routes and reconnect the two resulting paths in a different way to form a new route



# Option-related operators

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1. **Flip**: replaces one option with another option of the same customer (different location)
2. **Priority swap**: exchanges the positions of two selected options in the same or different routes replacing both option with other options of the same customer





# Computational Results

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BENCHMARKING AND EXPERIMENTS



# Computational Experiments

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- **Benchmarking** against **120 instances** of [Tilk et al. 2021](#):
  - **Requests**: 25 or 50
  - **Classes**: V (~1.5 options per request), U (~2 options per request). Priorities between 1 and 3 are uniformly distributed over the options of a request
  - **Time windows**: small (60-240 min), medium (120-480 min), large (240-600).
  - **Location preparation time** (e.g. parking): 6 min individual location, 4 min for shared
- Performance of BPC ([Tilk et al. 2021](#))
  - **VRPDO**: 78 of the 120 instances solved to **optimality**.
  - **VRPRDL**: 17 new **optimal** solutions (**x20 times faster** than the former state of the art)

# Computational Experiments

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## Implementation:

- **Language:** C# (.Net 6.0) with Visual Studio
- **Machine:** AMD Threadripper PRO 5955WX (16 cores, 4001 MHz), 128 GB RAM, x64 Windows 11

## Settings & Parameters:

- **Minimum insertions** construction algorithm
- 10 **restarts**
- Each restart ends after **5,000 non-improving iterations** or **15,000 total iterations**
- **Promises** restart after **1.5 times** the option set size



# Benchmarking

			Tilk et al. 2021				Our algorithm			Comparison	
Class	Customers	Time windows	Optimal	Avg routes	Avg cost	Avg time	Avg routes	Avg cost	Avg time	# New best (Opt)	Gap (%)
U	25	S	10	3.00	2455.80	23.56	3.00	2651.80	165.18	0 (0)	8.13
		M	9	3.10	2192.30	1089.33	3.00	2207.00	345.76	3 (2)	0.77
		L	9	3.00	2440.60	1595.00	3.00	2480.90	418.29	2 (1)	1.73
	50	S	6	5.88	3821.75	4020.63	5.30	4149.55	870.96	3 (0)	6.07
		M	1	5.67	4286.89	7014.24	5.60	4202.10	1598.74	6 (0)	0.93
		L	1	5.40	3807.20	6559.36	5.50	4142.10	1690.21	5 (0)	6.05
V	25	S	10	3.00	2616.90	33.94	3.00	2694.20	244.34	1 (1)	2.92
		M	10	3.00	2443.60	489.54	3.00	2492.70	333.98	3 (3)	2.09
		L	10	3.00	2114.70	938.31	3.00	2120.80	531.77	8 (8)	0.25
	50	S	6	5.70	4392.20	4090.85	5.90	4465.40	514.29	2 (0)	1.73
		M	4	5.38	3722.63	4861.73	5.50	3939.70	1931.95	2 (0)	2.89
		L	2	5.71	3816.86	6227.61	5.50	3854.20	2186.02	4 (0)	-0.35
			<b>78</b>	<b>4.32</b>	<b>3175.95</b>	<b>3078.68</b>	<b>4.28</b>	<b>3283.37</b>	<b>902.62</b>	<b>39 (15)</b>	<b>2.77</b>

# Benchmarking

## Observations

- Marginally less vehicles
- 24 new best & 15 optimal
- Faster on average

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# Conclusion

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KEY FINDINGS & FUTURE WORK

# Conclusion

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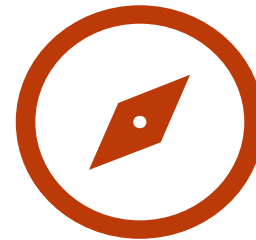
## Highlights

GRASP metaheuristic for VRPDO

Alternative construction heuristics (pool)

**24/120** new best solutions

**15/120** proven optimal



## Future Work

Solve [Dumez et al. \(2021\)](#) large instances (50-400)

Inject mathematical programming/constraint programming components into the scheme

Machine learning model for option selecting

# Bibliography

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Tilk, C., Olkis, K. and Irnich, S. (2021), "The last-mile vehicle routing problem with delivery options", OR Spectrum, Springer Berlin Heidelberg, Vol. 43 No. 4, pp. 877–904.

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Cardeneo A, (2005). Modellierung und Optimierung des B2C-Tourenplanungsproblems mit alternativen Lieferorten und -zeiten: Zugl.: Karlsruhe, Univ., Diss., (2005) volume 66 of Wissenschaftliche Berichte des Institutes für Fördertechnik und Logistiksysteme der Universität Karlsruhe (TH). Universitätsverlag, Karlsruhe, Germany (in German)

---

Los J, Spaan MTJ, Negenborn RR (2018) Fleet management for pickup and delivery problems with multiple locations and preferences. In: Freitag M, Kotzab H, Pannek J (eds) Dynamics in logistics, vol 61. Lecture notes in logistics. Springer, Cham, pp 86–94

---

Ozbaygin G, Ekin Karasan O, Savelsbergh M, Yaman H (2017) A branch-and-price algorithm for the vehicle routing problem with roaming delivery locations. Transp Res Part B: Methodol 100:115–137

---

Lombard A, Tamayo-Giraldo S, Fontane F (2018) Vehicle routing problem with roaming delivery locations and stochastic travel times (VRPRDL-S). Transp Res Procedia 30:167–177

---

Wang X, Zhan L, Ruan J, Zhang J (2014) How to choose last mile delivery modes for e-fulfillment. Math Probl Eng 2014:1–11

---

Savelsbergh M, Van Woensel T (2016) 50th anniversary invited article—city logistics: challenges and opportunities. Transp Sci 50(2):579–590

---

Global Market Insights, GMI (2024) Last Mile Delivery Market, <https://www.gminsights.com/industry-analysis/last-mile-delivery-market>

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CemexVentures (2023) Last mile delivery: The logistic challenge, <https://www.cemexventures.com/what-is-last-mile-delivery-2/>

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Dumez, D., Lehuédé, F. and Péton, O. (2021), "A large neighborhood search approach to the vehicle routing problem with delivery options", Transportation Research Part B: Methodological, Elsevier Ltd, Vol. 144, pp. 103–132.

# Acknowledgements

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The research project was supported by the Hellenic Foundation for Research and Innovation (H.F.R.I.) under the “2nd Call for H.F.R.I. Research Projects to support Faculty Members & Researchers” (Project Number: 02562).

**Thank You!**