# Literature review of modern and rich vehicle routing problems

## Contents



## <span id="page-1-0"></span>1 Introduction

Logistics and transportation form the backbone of global trade and commerce, facilitating the movement of goods and services across vast networks. Given the increasing customer demands, the cost increases and disruptions (e.g., COVID-19), the efficiency and effectiveness of these systems are crucial for economic growth, supply chain reliability, and customer satisfaction. Within this context, the role of operations research and optimization has become increasingly critical. These disciplines provide the analytical tools and methodologies needed to tackle complex logistical challenges, enabling organizations to enhance their operational performance and strategic decision-making.

Operations research (OR) is a field at the intersection of mathematics, engineering, and computing, dedicated to optimizing decision-making processes in complex systems. At its core, OR employs mathematical models, algorithms, and analytical techniques to tackle real-world problems efficiently. OR finds applications in various fields including logistics and transportation. From routing and scheduling to inventory management and production planning, optimization helps in minimizing costs, reducing delivery times, and improving overall service quality.

One of the most well-known and always timely challenges in OR is the Vehicle Routing Problem (VRP) [\(Toth and Vigo,](#page-20-0) [2014\)](#page-20-0). The classic VRP represents a fundamental class of problems in transportation logistics and operations research. It concerns the distribution of goods or services from a central depot to various delivery points, aiming to identify the minimum-cost routes subject to side constraints such as vehicle capacity. As a complex NP-hard combinatorial optimization problem, the VRP seeks to optimally assign delivery orders to a fleet of vehicles while determining the best sequence of delivery stops for each route, with the objective of minimizing total transportation costs, typically measured by the distance traveled. VRPs have attracted the interest of many researchers in the last 50 years due to their complexity (NP-Hard) but also their wide applicability in real cases. As tackling VRPs remain a fundamental problem in OR, VRPs continue to inspire research and innovation, driving advancements in optimization methodologies and practical applications across various industries.

Rich VRPs extend traditional models by incorporating a wider spectrum of real-world constraints and objectives, such as time windows, multiple depots, heterogeneous fleets, stochastic demands and many others. These enhancements allow for more realistic and applicable solutions, addressing the diverse needs of contemporary transportation systems. Additionally, integrated VRPs combine routing problems with other logistical functions like inventory management and production scheduling. This holistic approach ensures that decisions in one area support and enhance outcomes in others, leading to overall system optimization. The evolution of VRPs into rich and integrated problems reflects the growing complexity and dynamism of modern logistics environments.

In recent years, the landscape of vehicle routing problems (VRPs) has evolved to address a variety of rich and integrated challenges, reflecting the multifaceted nature of contemporary logistics management. This literature review explores promising directions in VRP research, highlighting specific aspects of these challenging problems. It focuses on addressing gaps in the literature, particularly the need for models that capture realistic logistics scenarios, the development of powerful optimization algorithms to handle large-scale instances, and the creation of experimental algorithmic designs that combine metaheuristics with exact methods.

Following the introduction, Section [2](#page-2-0) briefly presents rich VRPs and integrated problems, examining the complexities and discussing approached for tackling those problems. Section [3](#page-3-0) focuses on an important category of rich VRPs involving customer selection and resource budgeting, known as orienteering problems. Section [4](#page-5-0) explores the supply chain management aspect, discussing inventory management and the integration of production decisions into a holistic model. Next, Section [5](#page-9-0) examines logistics with humanitarian objectives and crowdsourcing that significantly differ from traditional objectives. Following this, Section [6](#page-11-1) explores VRPs with time windows, emphasizing hierarchical objectives and alternative approaches. Finally, the concluding Section [7](#page-14-1) summarizes the key findings and suggests potential avenues for future research, aiming to contribute to the ongoing development of robust and versatile solutions in transportation logistics management.

## <span id="page-2-0"></span>2 Rich VRPs and integrated problems

Naturally, VRPs have received a lot of research focus that has resulted in the development of state-of-the-art algorithms capable of handling thousands of customers [\(Vidal et al.,](#page-20-1) [2013,](#page-20-1) [2014;](#page-20-2) [Queiroga et al.,](#page-19-0) [2021\)](#page-19-0). As solution methods push the boundaries of the VRP instances that can be solved, Rich Vehicle Routing Problems (RVRPs) and other integrated logistics problems arise to model and capture multifaceted logistics challenges. These problems extend beyond basic formulations by incorporating additional features, constraints, or objectives to better reflect real-world scenarios. Rich VRPs encompass a spectrum of variants, including multi-objective VRPs, stochastic VRPs, and VRPs with dynamic or uncertain parameters. Integrated logistics problems combine VRP with other decision-making aspects, resulting in complex models that are more realistic but harder to tackle. Examples include VRPs integrated with inventory management, facility location, scheduling, or environmental considerations. By merging VRP with these additional features, integrated problems provide holistic solutions that optimize not only vehicle routing but also other crucial aspects of logistics operations. This approach enables decision-makers to consider various factors simultaneously, leading to more efficient and sustainable logistics solutions tailored to specific business requirements. The interested reader is referred to [Lahyani et al.](#page-18-0) [\(2015\)](#page-18-0) for a comprehensive review.

One example of complex realistic model enabled by the powerful optimization methods that have been proposed is the Inventory Routing Problem (IRP) formalizes the cost-saving Vendor Managed Inventory (VMI) policy, where the vendor is responsible for replenishing customer inventories over a specified time horizon, ensuring no stock-outs occur. This model can be extended to include production decisions, resulting in the even more complex Production Routing Problem (PRP), which accounts for production setup and lot-sizing decisions. Contemporary research in PRP addresses practical considerations such as product perishability, emissions, and other real-world factors, leading to significant cost savings for adopting organizations like Frito-Lay (Cetinkaya et al., [2009\)](#page-16-0). In addition to production, modern logistics problems also consider the time-scheduling of operations and jobs, and recent advances in warehousing include 3D packing and loading constraints. For example, the transportation of fuel products requires specific loading arrangements due to safety regulations. Moreover logistics networks can vary greatly depending on the business environment, with potential extensions including cross-docking cent,ers, satellite facilities, and multiple echelons with different fleets.

In summary, modern logistics problems involve a wide selection of optimization tasks that must address uncertainty, dynamism, and numerous real-life constraints related to time, distance, heterogeneous fleets, and integration with inventory and scheduling, as well as environmental considerations. Solving rich VRPs is particularly challenging due to their complexity and the need to balance multiple conflicting objectives. The corresponding formulations often involve largescale instances with numerous variables and constraints, making them computationally intensive. The presence of stochastic elements, such as varying customer demands and unpredictable travel times, adds further complexity, requiring robust and adaptive solutions. Integrating VRPs with other logistical functions necessitates sophisticated hybrid optimization algorithms that combine

metaheuristics with exact methods. These advanced approaches are essential for navigating the vast solution space efficiently. Real-time decision-making in dynamic environments demands algorithms that are both accurate and efficient, driving research into parallel and distributed computing techniques. Incorporating environmental and energy considerations into VRPs reflects the importance of sustainable logistics. Overall, the challenge of solving rich VRPs in modern logistics lies in developing robust, scalable, and efficient optimization algorithms that handle diverse constraints and objectives, adapting to the dynamic nature of real-world transportation networks. This literature review explores these challenges, highlighting the latest advancements and promising research directions in transportation logistics management.

## <span id="page-3-0"></span>3 Orienteering

Orienteering Problems (OPs) stand out in the realm of routing optimization by prioritizing the maximization of collected rewards along routes. On the contrary, to basic VRPs, which primarily seek to minimize operational costs, OPs aim to efficiently allocate resources to visit a subset of locations while maximizing the total collected rewards respecting specified constraints. These constraints typically consider factors such as time or capacity limitations. In OPs, each location or customer is associated with a reward value, and the challenge lies in designing routes that yield the highest cumulative reward ensuring that the constraints are satisfied. This feature introduces additional challenges as there is the need to select which locations to visit among many possible locations on top of deciding the optimal routes. The balancing of the contradicting exploration of high-reward locations and the efficient route planning under resource budget render OPs difficult to solve.

Indeed, the Orienteering Problem (OP) model has diverse and significant real-life applications across various domains. In tourism planning, OPs are utilized to design optimized tourist itineraries that maximize the experience for travelers given restricted time and transportation resources. Similarly, in mobile resource allocation, OPs help companies allocate resources such as delivery vehicles or service technicians to locations in a manner that maximizes overall efficiency and customer satisfaction. In recycling and waste collection, OPs aid in optimizing collection routes to minimize costs and environmental impact while maximizing the amount of material collected. Additionally, in facility servicing, OPs are used to efficiently schedule maintenance or repair visits to different locations, minimizing downtime and maximizing service quality. Furthermore, in critical scenarios like search and rescue missions, OPs assist in planning routes for rescue teams to quickly reach and assist individuals in distress. Overall, the versatility and effectiveness of the OP model make it an invaluable tool for addressing a wide range of resource management and optimization challenges in various real-world contexts.

The OP model is not new to the Operations Research literature and naturally, has attracted significant research interest. It was introduced by [Tsiligirides](#page-20-3) [\(1984\)](#page-20-3) and it can be seen as an extension of the Traveling Salesman Problem (TSP), for which the goal is to maximize the total profit collected from visiting a subset of customers without exceeding a specified time limit  $(T_{max})$ . Early methodological approaches for the OP involve the heuristic algorithms presented by [Tsiligirides](#page-20-3) [\(1984\)](#page-20-3) and [Chao et al.](#page-16-1) [\(1996\)](#page-16-1), as well as the branch-and-cut algorithm of [Fischetti](#page-17-0) [et al.](#page-17-0) [\(1998\)](#page-17-0). More recent and effective metaheuristic developments designed for the OP are the Pareto mimic algorithm of [Ke et al.](#page-17-1) [\(2016\)](#page-17-1) and the GRASP algorithm presented by [Keshtkaran](#page-17-2) [and Ziarati](#page-17-2) [\(2016\)](#page-17-2). [\(Vansteenwegen et al.,](#page-20-4) [2011\)](#page-20-4).

#### <span id="page-4-0"></span>3.1 Clustering in orienteering models

To accommodate different needs numerous variants of the basic OP model have been proposed. The focus of the research was on cluster related problems. One interesting variant is the Clustered Orienteering Problem (COP) which was introduced by [Angelelli et al.](#page-15-0) [\(2014\)](#page-15-0). COP considers customers to be grouped in clusters. To collect the profit associated with each cluster, all customers of the corresponding cluster must be served. [Angelelli et al.](#page-15-0) [\(2014\)](#page-15-0) propose a branch-and-cut method, as well as a tabu search metaheuristic to solve the problem. Another interesting variant which also considers group of customers is the newly introduced Set Orienteering Problem (SOP), a variant similar to the COP regarding the fact that customers belong to clusters. However, contrary to the COP, the Set Orienteering Problem assumes that the profit of a cluster (set) is collected, if any customer of the cluster is served. SOP was introduced by [Archetti](#page-15-1) [et al.](#page-15-1) [\(2018\)](#page-15-1) together with a mathematical formulation and a matheuristic algorithm based on tabu search and a mixed integer programming (MIP) based move for performing broad solution modifications. Later, Pěnička et al. [\(2019\)](#page-19-1) proposed a novel ILP formulation of the problem and developed a Variable Neighborhood Search method applying path and set relocations and exchanges. The most recent research work on SOP is published by [Carrabs](#page-15-2) [\(2020\)](#page-15-2). It presents a Biased Random-Key Genetic Algorithm (BRKGA) which makes use of three local search procedures to improve the fitness of the solution chromosomes. The solution chromosome is an array of random keys, with each key referring to a customer set. To decode a solution from a given array, the keys (sets) are firstly sorted. The sets whose values are lower than 0.5 are discarded. Then, from the resulting sequence of sets, the node sequence is determined by solving a suitably defined shortest path.

SOP is a new model in the OR literature that was developed to capture realistic needs especially in the logistics sector. Applications of the SOP can be found in the distribution of mass products, in which the carrier delivers the orders for all customers of a cluster by visiting a single customer, and then the customers within the cluster are independently served. Yet, as Pěnička et al. [\(2019\)](#page-19-1) state, the range of the SOP applicability can be extended to the GTSP and other variants of the OP, if certain assumptions for each case are made. For instance, the SOP model is valid for the travel guide problem, according to which attractions are grouped in clusters and a guide seeks to maximize the profit from visiting attractions in a limited time, given that having visited at least one attraction of a single cluster, the total cluster profit is collected.

#### <span id="page-4-1"></span>3.2 Research direction

Due to its significance and the scarcity of research addressing it, we have chosen to focus our research on the SOP. This decision stems from the problem's practical relevance and its relatively unexplored nature within academic literature. By concentrating our efforts on the SOP, we aim to explore multiple perspectives:

- 1. Model enhancement and extension: the model considers only one vehicle which is very restrictive. Therefore, we believe that the multi-vehicle version needs to be introduced and examined to promote the solution method applicability to real life cases.
- 2. Alternative mathematical formulations: exact algorithms may guarantee optimality but are restricted to small sized instances. Therefore, exploration of alternative formulations such as the two-commodity flow is a promising research direction [\(Manousakis et al.,](#page-18-1) [2021\)](#page-18-1).
- 3. Matheuristic optimization algorithms: last but not least, in order to be able to design and propose efficient metaheuristic approaches capable of solving large-scale instances of the problem, we aim to develop state-of-the-art matheuristic algorithms that combine the

flexibility and speed of heuristics as well as the high quality of exact methods aiming to find new best solutions for the literature benchmarks.

## <span id="page-5-0"></span>4 Supply chain management

As VRP lies in the heart of logistics and transportation, one of the most natural extensions is the consideration of inventory management, as well as production aspects. The scientific advances of the past years have enabled researchers and practitioners to formulate and solve such realistic and complicated models that better reflect the business environment's requirements.

## <span id="page-5-1"></span>4.1 Inventory management, periodicity, and the Vendor Managed Inventory policy

On this basis, one vast research area is formed around the combination of inventory management with routing optimization at the heart of which lies the well-known Inventory Routing Problem (IRP). The family of IRPs is a broad class of typically multi-period problems with numerous applications in several sectors and industries. The IRP calls for jointly determining the timing and quantity of customer deliveries, as well as the minimum cost vehicle routing in pursuit of the optimal coordination of inventory holding and transportation activities.

IRP implements the Vendor Managed Inventory (VMI) inventory replenishment policy according to which the supplier is responsible for delivering quantities to the customers, so that no stock-outs occur. The VMI policy commonly substitutes the short-sighted Retailer Managed Inventory policy under which each customer is responsible for placing orders. The VMI policy is one of the most important supply chain strategies followed by 84% of the companies with over a billion revenue [\(van den Bogaert and van Jaarsveld,](#page-20-5) [2021\)](#page-20-5). VMI allows the vendor to effectively coordinate delivery activities, in pursuit of distribution cost savings. On the other hand, customers receive cost incentives and save time and effort on inventory management. According to [Archetti and Speranza](#page-15-3) [\(2016\)](#page-15-3), the inventory and routing cost savings achieved by the VMI policy may range up to approximately 10% for well-known benchmark data sets. VMI systems were introduced in the literature in the context of liquified air products distribution. Since then, several VMI systems have been implemented for several sectors: automotive industries, electronics assembly, chemicals industries, vending machines for juice or foods, chain stores, maritime logistics and many other [\(Andersson et al.,](#page-15-4) [2010\)](#page-15-4). The interested reader is referred to the survey of [Coelho et al.](#page-16-2) [\(2014\)](#page-16-2) for an insightful look on the IRP literature.

As it becomes obvious, one challenging and realistic feature integrated problem is the periodicity of routing and inventory decisions. As long as routing decisions are concerned, VRPs consider different planning horizons for transportation activities: periodic deliveries, i.e., deliveries that are repeated in cycles of equal length are found in numerous fields and applications [\(Francis et al.,](#page-17-3) [2008\)](#page-17-3). On this basis, the Periodic Vehicle Routing Problem (PVRP) [Christofides](#page-16-3) [and Beasley](#page-16-3) [\(1984\)](#page-16-3) extends the classic VRP by considering a multi-period horizon during which the customer visits take place. [Campbell and Wilson](#page-15-5) [\(2014\)](#page-15-5) define the basic PVRP as the problem of selecting one of the given feasible visiting schedules of each customer (since each customer is not visited in every period), such that the total transportation cost is minimised. According to the same work, the delivery options for each customer found in the PVRP literature may be classified to three categories: i) predetermined set of candidate visit schedules, ii) requirement for equally distanced customer visits and iii) imposing minimum and maximum numbers of periods allowed between two consecutive customer visits. For all cases, given a customer visit schedule, the delivery quantity of every customer visit is fixed.

When periodicity is examined in the context of inventory decisions, it is common to define a time horizon as a period or cycle that is repeated (e.g., weekly distribution schedule) and therefore it is easier to predict and manage inventory. In comparison to the classic IRP model [\(Archetti et al.,](#page-15-6) [2014\)](#page-15-6), the Periodic Inventory Routing Problem (PIRP) incorporates additional constraints that enforce equality of the initial inventory levels and the inventory levels at the end of the planning horizon for both the depot and the customers. Therefore, the generated distribution schedule may be repeated if the input parameters remain unchanged (i.e., demand, etc.). This approach enables solving problems without optimizing over an infinite horizon [\(van](#page-20-6) [Anholt et al.,](#page-20-6) [2016\)](#page-20-6). The periodicity feature strengthens the practical applicability of PIRP: For example, [Gaur and Fisher](#page-17-4) [\(2004\)](#page-17-4) solve the problem of scheduling and routing replenishment operations for a supermarket network. The optimized schedule is repeated weekly. Over the first year of the schedule implementation, the distribution cost savings were equal to 4%. The Selective and Periodic IRP (SPIRP) is solved by [Aksen et al.](#page-14-2) [\(2014\)](#page-14-2) via an adaptive large neighborhood search algorithm. It calls for the generation of a distribution plan which is repeated on a weekly basis. The examined model is faced by a company which collects used vegetable oil from sources, such as restaurants and hotels and reuses it to produce biodiesel. In contrast to the basic IRP version, under which all customers must be visited to prevent stock-outs, this variant allows selecting which customers to visit based on profitability. Similarly, Montagné et al. [\(2019\)](#page-18-2) develop a constructive algorithm based on shortest path and split procedures for a real case of reusable waste oil collection in Canada. Interestingly, the tests performed on real-world problem instances of up to 3,000 customers served on a 30-day time horizon, show that the algorithm manages to outmatch the cost-effectiveness of the actual company solution by up to 20%.

In the PIRP model's case, the requirement for repeatable replenishment schedules is met by enforcing same inventory levels at the start and end of the studied time horizon. Therefore, the whole delivery schedule may be repeated in a cyclic manner, with the cycle time being equal to the time horizon considered (e.g., a week). Other IRP approaches incorporate the periodicity feature by allowing different cycle times for each customer or for each route. For instance, a customer with high demand and high proximity to the depot may be served every second day, whereas a customer with low demand rate and far from the depot may be served once per week. Similarly, one route may be repeated every day and another route every third day. In these cases, different delivery patterns are used for each customer and the whole schedule may be repeated with a cycle time equal to the lowest common multiple of the customer, or the route cycle times. On this basis, another IRP variant which is referred to as the Cyclic IRP (CIRP) has been developed [Raa and Aghezzaf](#page-19-2) [\(2009\)](#page-19-2). The objective of CIRP is to find a cyclic distribution pattern that minimizes the long-term transportation and inventory costs. The cycle time is an important decision variable, and it is often dictated by economic order quantity (EOQ) models.

[Aghezzaf et al.](#page-14-3) [\(2006\)](#page-14-3) propose a long-term IRP model with constant demands and consider economic order quantity (EOQ) policies for inventory management. A column generation based approximation method for solving the non-linear formulation is proposed. [Raa and Aghezzaf](#page-19-2) [\(2009\)](#page-19-2) develop a practical solution approach that considers cargo handling times, customer inventory capacities, and maximum driving limits for vehicles. [Chitsaz et al.](#page-16-4) [\(2016\)](#page-16-4) propose a two-phase iterative procedure which consists of two heuristic methods: the first one produces routes, whereas the second one combines and schedules routes to generate cyclic distribution plans. A two phase approach is also proposed by [Raa and Dullaert](#page-19-3) [\(2017\)](#page-19-3): in the first phase, cycle times are chosen and the routes are designed, whereas in the second phase, the routes are assigned to vehicles and the cycle times are adjusted, to minimise the number of vehicles used. The CIRP model is applicable to various industries. For example, [Zenker et al.](#page-20-7) [\(2016\)](#page-20-7) study the problem of producing cyclic tours when customers are located along a line. This operational scenario occurs in liner shipping (feeder ships that service inland ports along a stream) and fa-

cility logistics (tow trains which deliver bins to the stations of an assembly line). More recently, [Bertazzi et al.](#page-15-7) [\(2020\)](#page-15-7) study a problem where components are collected from a set of supplier locations and delivered to a manufacturing plant in cycles. The authors present polyhedral studies of the convex hull of the problem and propose a branch-and-cut algorithm to solve the provided problem formulation. Interestingly, computational experiments show that the cyclic formulation is significantly harder to solve compared to the standard non-cyclic IRP formulation. More specifically, optimal solutions are found for problem instances of up to 25 customers with the cyclic model, whereas for the non-cyclic version optimal solutions are found for problems of up to 50 customers when a single vehicle is considered.

#### <span id="page-7-0"></span>4.2 Taking it a step further: Integrating production management

Taking the IRP one step further is the challenging Production Routing Problem (PRP). The PRP is a hard-to-solve NP-hard combinatorial optimization problem that calls for the joint optimization of production, inventory, and routing decisions over a specific time horizon. A supplier is responsible for replenishing the inventories of geographically dispersed customers ensuring that no stock-outs occur at any period of the considered time horizon. Both customers and the depot/production facility face a per period inventory unit holding cost and have a maximum storage capacity. The problem incorporates the naturally cost-saving Vendor Managed Inventory policy according to which the supplier is responsible for determining the quantities and the timings of customer replenishment visits, as well as the routes serving the customers. In that sense, PRP generalizes the Inventory Routing Problem (IRP) and incorporates the well-known lot-sizing problem (LSP) and the vehicle routing problem (VRP).

The combined nature of the PRP is suitable for various industry fields and different appli-cations. In practice, companies such as Kellogg [\(Brown et al.,](#page-15-8) [2001\)](#page-15-8) and Frito-Lay (Cetinkaya [et al.,](#page-16-0) [2009\)](#page-16-0) have recorded significant savings after jointly optimizing production and distribution operations. Inspired by the requirements and the particularities of different industries and sectors, researchers have introduced and studied several PRP variants. Motivated by a food company that distributes fresh meat to a network of stores in China, [Qiu et al.](#page-19-4) [\(2019\)](#page-19-4) consider the PRP for perishable products and experiment with different selling policies to minimize the value losses. Also for the case of perishable products, [Chan et al.](#page-16-5) [\(2020\)](#page-16-5) extend the PRP in the context of sustainable food supply chain. They consider four objectives: the classic minimization of the total system costs, as well as, the maximization of the average food quality, the minimization of the  $CO<sub>2</sub>$  emissions in transportation and production and finally, the minimization of the total weighted delivery lead time. The perishability feature is also considered by [Ghasemkhani](#page-17-5) [et al.](#page-17-5) [\(2021\)](#page-17-5). The authors formulate a multi-perishable product and multiperiod PRP with heterogeneous fleet, time windows and fuzzy parameters. The objective of the proposed model is to maximise the total profit (selling revenue reduced by the aggregation of the holding, production, transportation, and utility preference costs).

Motivated by a real case, [Dayarian and Desaulniers](#page-16-6) [\(2019\)](#page-16-6) model and solve the PRP of a catering company in Montreal that delivers meals with short life-span. The authors take into account business requirements such as multi-trips and time-windows. With respect to the petrochemical industry, [Schenekemberg et al.](#page-20-8) [\(2021\)](#page-20-8) study the Two-Echelon PRP with pickups and inventory control of ethanol from the suppliers, production, and inventory control of pure and commercial gasoline, as well as deliveries of commercial gasoline to the final customers in South American countries. Most recently, [Farghadani-Chaharsooghi et al.](#page-16-7) [\(2021\)](#page-16-7) integrate the PRP model with workforce planning for the case of a company that processes and delivers organic fruits and vegetables. They consider stochastic times, perishable products, and quantify the effects of workforce planning on costs and productivity offering interesting managerial insights.

The interested reader is referred to the surveys of Díaz-Madroñero et al. [\(2015\)](#page-16-8) and [Adulyasak](#page-14-4) [et al.](#page-14-4) [\(2015\)](#page-14-4) on optimization models which integrate production and transportation decisions.

The basic PRP version [\(Adulyasak et al.,](#page-14-4) [2015\)](#page-14-4) has served for benchmarking several highquality solution approaches, most of which are matheuristic algorithms: [Russell](#page-20-9) [\(2017\)](#page-20-9) uses mathematical programming for a relaxed PRP version, to determine an initial solution. This solution is completed by a tabu search scheme based on the concept of seed routes. [Solyalı and](#page-20-10) Süral [\(2017\)](#page-20-10) propose a five-phase heuristic, with overlapping subproblems which are formulated as MIPs and solved via exact algorithms. In a similar manner, other multi-phase approaches decompose the problem into the setup, distribution and routing decision levels which may be tackled either by exact algorithms [\(Chitsaz et al.,](#page-16-9) [2019\)](#page-16-9), or with fix-and-optimize strategies [\(Li](#page-18-3) [et al.,](#page-18-3) [2019\)](#page-18-3). On the same basis, [Avci and Yildiz](#page-15-9) [\(2019\)](#page-15-9) propose a decomposition of the main problem into subproblems to reduce its complexity. The distribution and routing subproblems are solved heuristically (iterated local search), whereas the lot-sizing, inventory and delivery quantity decisions are handled by solving the corresponding MIP models. Hybrids of metaheuristic algorithms have been also introduced. [Qiu et al.](#page-19-5) [\(2018\)](#page-19-5) adopt a skewed general variable neighborhood search algorithm for the delivery schedule and a guided variable neighborhood descent algorithm for the routing subproblem, whereas the production and inventory quantities are obtained by solving a production-inventory MIP subproblem. Most recently, [Schenekemberg et al.](#page-20-8) [\(2021\)](#page-20-8) introduced a parallelized hybrid algorithm that combines local search procedures within a traditional BnC scheme.

#### <span id="page-8-0"></span>4.3 Research direction

The IRP model has been studied extensively, accumulating many papers proposing multiple solution approaches for many different variants. However, perhaps due to its complexity the PRP which is the natural extension of IRP has not been so studied. Moreover, an important aspect of real-world operational problems is the ability of an optimized set of decisions to be periodically applied in the long run, ensuring reduced variability and satisfactory performance [\(Grzegorz et al.,](#page-17-6) [2021\)](#page-17-6). It has been shown that for specific cases, the cyclic production planning can achieve a specified service target at lower total costs than a more cost-efficient but not non-cyclical approach [\(Nyen et al.,](#page-19-6) [2009\)](#page-19-6).

Despite the applicability of several PRP model variants on real-world cases, to the best of our knowledge, the feature of cyclicity has not been previously studied. In this context, the term cyclicity is used to describe the capability of repeating the production, distribution, and routing plans for several cycles. An optimal solution of the basic PRP model [\(Adulyasak et al.,](#page-14-5) [2014\)](#page-14-5) assumes zero final inventory for both the production facility and customers (according to the basic PRP, a non-zero inventory would indicate sub-optimal solution due to the incurred inventory holding costs). Therefore, at the end of the considered time horizon, the whole supply chain network (production site and customers) is out of products. Thus, the generated optimised schedules cannot be repeated in a cyclic manner: in fact, stock-outs are inevitable right after the end of the examined time horizon. To overcome this limitation, the classic PRP model can be revisited through the prism of a cyclic time horizon to ensure that the final inventory levels of the depot and the customers are equal to the starting ones. In that sense, the production and distribution schedules along with the routing plans can be repeated for several cycles of length equal to the considered time horizon provided that the various problem parameters remain unchanged.

## <span id="page-9-0"></span>5 Humanitarian logistics and crowdsourced VRPs

Natural and man-made disasters such as droughts, hurricanes, floods, famines, earthquakes, refugee crises, terrorist attacks, wars, etc. are becoming more and more frequent around the world. In 2019 alone, 361 natural disasters were reported. The significant surge in the frequency and magnitude of disasters, has affected a lot of people. According to [Ritchie and Rosado](#page-19-7) [\(2022\)](#page-19-7), the average annual toll of disaster-affected individuals stood at 176,543,870 for the decade spanning 2010-2019, with an estimated average of 114,844,770 individuals affected annually from 2020 onwards.

In this context, logistics models with humanitarian aspects have been developed. Humanitarian logistics can be understood as a specialized category of rich vehicle routing problems (VRPs), characterized by complex constraints and objectives that go beyond those found in conventional logistics. In this context, the primary goal is not merely to minimize costs or travel times, but to optimize the delivery of critical supplies and services under emergency conditions, often with incomplete information (e.g., blocked roads) and rapidly changing environments. Humanitarian VRPs must account for factors such as the urgency and priority of deliveries, accessibility of disaster-stricken areas, and the need for equitable distribution of aid. This complexity requires sophisticated algorithms and often real-time data integration to ensure efficient and effective relief operations, ultimately aiming to save lives and alleviate suffering in the wake of natural disasters and crises.

Relatively recently, a significant development has unfolded within the field of humanitarian logistics: the increasing acknowledgment of crowdsourcing as a powerful asset for disaster relief endeavors. Technologies such as GIS, mobile apps, and social media platforms play a crucial role in this context, enabling real-time data collection and analysis. For example, as highlighted by [Parappathodi and Archetti](#page-19-8) [\(2022\)](#page-19-8), during the Haiti earthquake and the 2018 floods in India, crowdsourced information was vital in coordinating relief efforts and directing resources where they were most needed. By leveraging collective intelligence and decentralized networks to tackle humanitarian crises, the response to such events is quick and massive.

#### <span id="page-9-1"></span>5.1 Humanitarian logistics

During disasters and crises, logistics are vital for determining the success or failure of relief efforts. Such disasters do not only affect human lives, but they also result in significant economic losses. Consequently, disaster management and relief efforts necessitate intricate logistical operations, as the required resources are typically not found at the disaster site. These logistical operations are commonly known as humanitarian logistics [Chiappetta Jabbour et al.](#page-16-10) [\(2019\)](#page-16-10). According to [Behl and Dutta](#page-15-10) [\(2019\)](#page-15-10), the work of [Kovacs and Spens](#page-18-4) [\(2007\)](#page-18-4) is considered the seminal paper and an important milestone for humanitarian logistics aiming to understand logistics operations during humanitarian crises.

Tackling crises involves a great number of different organization, individuals, structures called actors (e.g., government, local population, NGOs, media, donors, etc.). The critical need for coordination among humanitarian actors became evident after the response to the Rwandan humanitarian crisis that began in 1994. In 1996, the challenges and inefficiencies encountered during this crisis led to the creation of the Sphere Project. By January 2000, the Sphere Project had released its first handbook, outlining minimum standards in key lifesaving sectors to enhance the quality and accountability of NGOs during humanitarian responses. These standards cover four main response areas: water supply, sanitation and hygiene promotion, food security and nutrition, and shelter, settlement, and health [Paciarotti et al.](#page-19-9) [\(2021\)](#page-19-9).

One particular reason that makes humanitarian logistics challenging and discriminate them

from traditional logistics is the objective function. In the context of optimizing humanitarian logistics, fairness is a crucial consideration that goes beyond traditional efficiency metrics. While the primary objective often involves minimizing costs or delivery times, incorporating fairness ensures that aid distribution is equitable and prioritizes the most vulnerable populations. For example, instead of solely focusing on the quickest routes, algorithms can be designed to ensure that remote or marginalized communities receive necessary supplies, even if it means longer travel times or higher costs. Alternative objective functions may include minimizing unmet demand across all affected areas or maximizing the coverage of aid within a specified time horizon. For instance, during the response to the 2010 Haiti earthquake, fairness considerations could involve prioritizing aid delivery to severely affected areas like Léogâne, which might otherwise be overlooked due to logistical challenges. Another example is the minimization of the average rescue time over all people or even more sophisticated the minimization of the difference between rescue times of the last person and the first person to be rescued. By integrating fairness into objective functions, humanitarian logistics can better address the diverse needs of disasterstricken populations and ensure a more fair distribution of resources.

The interested reader is referred to the works of  $Ozdamar$  and Ertem  $(2015)$  and [Behl and](#page-15-10) [Dutta](#page-15-10) [\(2019\)](#page-15-10) for surveys on humanitarian logistics and humanitarian supply chain management in general.

#### <span id="page-10-0"></span>5.2 Crowdsourcing deliveries

Crowdsourcing has become widely utilized across different aspects of logistics, spanning transportation, warehousing, inventory management, and route optimization. Particularly in disaster relief contexts, crowdsourcing practices can play crucial role in quickly mobilizing resources, ensuring fast reach to impacted regions. Apart from their role in disaster relief, crowdsourcing finds applications in urban logistics. Crowdsourced delivery networks provide economical alternatives to alleviate the high last-mile delivery, capitalizing on local insights and resources to streamline delivery routes and minimize delivery duration [\(Alnaggar et al.,](#page-15-11) [2021\)](#page-15-11).

Crowdsourcing in logistics offers numerous benefits and challenges that businesses must balance to leverage its potential. On the positive side, crowdsourcing can significantly reduce operational costs by utilizing an on-demand workforce, ensuring scalability and flexibility to meet fluctuating demands. It enhances last-mile delivery by using local resources, resulting in faster and more efficient deliveries while extending reach to hard-to-service areas. On the downside, crowdsourcing can be challenging as consistent quality, accountability and availability cannot be guaranteed. Therefore, it has been observed that companies hesitate to adopt crowdsourcing solution for shipping products, possibly because of a lower on-time performance observed in such a setting when compared to a dedicated fleet. Interestingly, [Castillo et al.](#page-16-11) [\(2018\)](#page-16-11) underlines that the total deliveries are higher for a crowdsourced logistics network when compared to dedicated fleet.

Naturally, crowdsourcing has drawn researchers attention and has been studied from different perspectives and within different application areas. In the area of urban freight logistics, crowdsourcing is considered a more sustainable solution [\(Buldeo Rai et al.,](#page-15-12) [2017\)](#page-15-12). In the context of the classic VRP, [Archetti et al.](#page-15-13) [\(2016\)](#page-15-13) formulates a crowdshipping problem with occasional drivers. A multi-start heuristic is proposed to tackle the problem including a series of small integer programming problems to assign subsets of customers to occasional drivers. Two compensation schemes are studied, one depending solely on the delivery location and one taking into account the deviation required with respect to the occasional driver's route. [Li et al.](#page-18-5) [\(2014\)](#page-18-5) explores two multi commodity sharing models in an urban setting. As for travelling in increasingly crowded cities, dynamic ridesharing is a recent alternative in which people with similar travel plans are matched and travel together. [Lee and Savelsbergh](#page-18-6) [\(2015\)](#page-18-6) investigate the benefits, complexities, and costs of employing a small number of dedicated drivers to serve riders who would otherwise remain unmatched. An extensive computational study demonstrates the potential benefits of dedicated drivers (e.g., taxi drivers) and identifies environments in which dedicated drivers are most useful.

In a crowdsourced disaster relief system, vehicles and supplies are provided by individuals spread across the affected area, making routing complex but offering advantages over centralized systems. Centralized systems require vehicles to start from a central depot, leading to delays and inefficiencies. Crowdsourcing reduces mobilization time, cuts down on travel time, and improves access to isolated areas, especially when road networks are disrupted, by utilizing geographically dispersed resources. In this context, [Parappathodi and Archetti](#page-19-8) [\(2022\)](#page-19-8) introduce the crowdsourced humanitarian relief vehicle routing problem (CHR-VRP) in a post-disaster context, where drivers with homogeneous vehicles volunteer to deliver emergency kits to people in need. The objective is to minimize the time until the last person receives aid. The problem assumes that each demand point requires one kit, vehicles have finite capacity, and supply points have sufficient kits. Simplifying assumptions are made to ensure problem tractability, with the ultimate goal of developing routes that serve all demand points, respect vehicle capacities, and minimize the length of the longest route.

#### <span id="page-11-0"></span>5.3 Research direction

In summary, crowdsourcing has emerged as a means of transportation of increasing interest. It has persuaded even major stakeholders of its efficacy in balancing the challenging last mile delivery costs in many industries. Serving as a way to delegate a part of the delivery operations, it presents a notably cost-effective alternative. Furthermore, it can increase the maximum transportation capabilities and help the organization adapt to demand fluctuations, in contrast to rigid dedicated fleets. As for the humanitarian logistics, crowdsourcing seems of great importance. Its ability to swiftly mobilize a substantial delivery force proves indispensable, particularly in scenarios where the road infrastructure is compromised, as seen in instances such as earthquakes.

The CHR-VRP is a realistic and relevant model, recently introduced by [Parappathodi and](#page-19-8) [Archetti](#page-19-8) [\(2022\)](#page-19-8) inspired by recent distastes such as the 2018 India floods. Although it was only recently introduced and has not been extensively studied, it offers significant potential for improving disaster relief efforts. So far only a iterated Local Search implementation is given for solving it. We aim to formulate powerful matheuristic solution methodologies for tackling the problem. The purpose is also to be design a rigid solution framework that will be able to balance the challenging different KPIs and objectives as discussed above.

## <span id="page-11-1"></span>6 Exploring VRPs and time windows

The concept of time in logistics literature has been examined in different ways. For instance, as seen before, the periodicity is found in the context of inventory and production planning. Another aspect of time that integrates into the domain of classic VRP models is the concept of time windows. Time windows, in the context of vehicle routing, are predefined time intervals during which customers/locations are serviced. These intervals serve as constraints dictating the allowed arrival and departure times for vehicles. Time windows play a pivotal role in modeling real-world logistics scenarios and depict different operational constraints according to which customers have time requirements for receiving deliveries or services. Time windows can be of different types, including hard time windows, where vehicles must arrive or depart within the specified interval, and soft time windows, where deviations from the specified window may incur penalties or

costs. The consideration of time windows adds complexity to vehicle routing problems but also enhances the realism and practical applicability of the models. Effective management of time windows allows for improved scheduling of deliveries, reduced waiting times, and better overall service levels, leading to increased customer satisfaction and operational efficiency. For this reason, the literature corpus is rich in terms of time windows models and methodologies.

One of the most well-known models is the Capacitated Vehicle Routing Problem with Time Windows (CVRPTW or VRPTW). In this problem customers have specific demand while they also need to be served within a predefined time window. The VRPTW accumulated more than 35 years of research. Due to its complexity the VRPTW has been the subject of numerous research efforts that focus on solving the problem using different optimization methodologies. On this basis, the benchmark set of [Solomon](#page-20-12) [\(1987\)](#page-20-12) is a widely accepted test bed that has been used for the evaluation of most optimization methods developed for the VRPTW. The interested reader is referred to the literature review of [Zhang et al.](#page-20-13) [\(2022\)](#page-20-13) for more information.

Early efforts focus mainly on exact methods and heuristics, while nowadays dedicated efficient meta- and math- heuristics are being developed to tackle larger and more realistic instances of the problem. Regarding exact methods, [Kolen et al.](#page-18-7) [\(1987\)](#page-18-7) developed a Branch and Bound (BnB) method for solving the problem, whereas [Fisher](#page-17-7) [\(1994\)](#page-17-7) proposed an optimization model based on k-trees in order to solve both the CVRP and its extension the VRPTW. On the other hand, [Desrochers et al.](#page-16-12) [\(1992\)](#page-16-12) propose a branch and price (BnP) algorithm for solving the problem, which was improved by [Kohl et al.](#page-18-8) [\(1999\)](#page-18-8) with the addition of 2-path inequalities to the LPrelaxation of the Set Partitioning (SP) formulation proposed initially by [Balinski and Quandt](#page-15-14) [\(1964\)](#page-15-14). The BnP algorithm of [Kohl et al.](#page-18-8) [\(1999\)](#page-18-8) calculated the optimal solutions for more than 15 unsolved Solomon instances, however it failed to solve several small-size problem instances. Later on, [Jepsen et al.](#page-17-8) [\(2008\)](#page-17-8) proposed a branch-and-cut-and-price (BCP) framework and introduced a family of inequalities that provide better lower bounds. However, for larger instances the new inequalities significantly increased the complexity of the pricing problem. For this purpose, the authors propose several exact and heuristic techniques to accelerate its solving process. Their results demonstrate an improvement compared to the works of [Irnich and Villeneuve](#page-17-9) [\(2006\)](#page-17-9) and [Chabrier](#page-16-13) [\(2006\)](#page-16-13). The framework of [Jepsen et al.](#page-17-8) [\(2008\)](#page-17-8) was further improved by [Desaulniers](#page-16-14) [et al.](#page-16-14) [\(2008\)](#page-16-14) where the authors incorporate k-path along with the aforementioned inequalities into the pricing problem. Moreover, a tabu search algorithm is employed to generate reduced cost routes more efficiently. This method was able to outperform all existing algorithms by decreasing the computational time required to solve large-scale Solomon instances, whereas it also solved 5 out of 10 open problem instances. Lastly, [Baldacci et al.](#page-15-15) [\(2012\)](#page-15-15) further expanded this framework by introducing new route relaxations. The proposed exact algorithm is considered highly efficient in terms of computational effort.

Regarding heuristic optimization methods, various approaches have been proposed throughout the years. [Gambardella et al.](#page-17-10) [\(1999\)](#page-17-10) developed an ant colony optimization method for the VRPTW. The algorithm utilizes two ant colonies. The first is responsible for the minimization of the number of vehicles, while the second is responsible for the minimization of the total traveled distance. A specialized pheromone update mechanism is used in an effort to exchange information between the two ant colonies. On a similar basis, G $\acute{o}$ mez S. et al. [\(2014\)](#page-17-11) incorporated several heuristics in an ant colony optimization framework. Learning level heuristics are used for characterizing time windows and distribution of customers to increase the effectiveness of the proposed heuristics. With regards to population-based methods, [Hu et al.](#page-17-12) [\(2013\)](#page-17-12) adopted a particle swarm optimization framework that employs a chaos algorithm to re-initialize the particle swarm, whereas [Repoussis et al.](#page-19-10) [\(2009\)](#page-19-10) presented an arc-guided evolutionary algorithm that evolves a population of solutions by generating new solutions that combine arcs from parent solutions. The required arcs are selected by considering both the appearance frequency of the arcs in high quality solutions, but also the diversity of the population. [Nagata et al.](#page-19-11) [\(2010\)](#page-19-11) developed a memetic algorithm that combines dedicated crossover operators along with a novel penalty function to eliminate violations of time window constraints during the generation of offspring solutions.

Towards the state-of-the-art, [Vidal et al.](#page-20-1) [\(2013\)](#page-20-1) proposed a hybrid genetic search algorithm with adaptive diversity control that was designed to solve a handful of vehicle routing problems with time window constraints such as the VRPTW, the PVRPTW, the MDVRPTW and the SDVRPTW. Extending this work [Vidal et al.](#page-20-2) [\(2014\)](#page-20-2) proposed a unified solution framework for multi-attribute vehicle routing problems. This framework adopts a component-based design which can be easily extended to support various types of problem attributes such as soft/hard time windows, multiple depots, simultaneous deliveries, and pickups. Despite its generic nature, the authors highlight its performance and effectiveness compared to the state-of-the-art. Lastly, [Nalepa and Blocho](#page-19-12) [\(2016\)](#page-19-12) presented a self-adapting memetic algorithm that is capable of automatically tuning the algorithm's parameters during execution. The algorithm incorporates dedicated crossover operators inspired by [Nagata et al.](#page-19-11) [\(2010\)](#page-19-11) and extensive computational experiments showcase the effectiveness of the algorithm.

#### <span id="page-13-0"></span>6.1 Objectives hierarchy and alternatives

As it becomes clear, the classic version of VRPTW is a challenging problem with numerous applications that have attracted research interest. It is worth highlighting that in terms of optimization objectives, the vast majority of research efforts in the VRPTW context focus mainly on minimizing the number of vehicles and minimizing the total travel distance. In general, multiobjective problems can be treated in different ways (e.g., sequential optimization, weighted global objective function, etc.). In the VRPTW context, most research works focus on minimizing both objectives hierarchically. Firstly, the number of vehicles is minimized and after this the total travel distance is minimized. Only a few papers attempt to optimize the objectives simultaneously. The interested reader is referred to the works of [Gehring](#page-17-13) [\(1999\)](#page-17-13), [Bent and Van Hentenryck](#page-15-16) [\(2004\)](#page-15-16) and [Ghoseiri and Ghannadpour](#page-17-14) [\(2010\)](#page-17-14).

In the context of VRP model an interesting variant with an alternative objective is the Cumulative Capacitated VRP (CCVRP). This problem was first proposed and solved by [Ngueveu](#page-19-13) [et al.](#page-19-13) [\(2010\)](#page-19-13). The problem definition is similar too the CVRP with the main difference being the optimization objective. According to the CCVRP model, the goal is to minimize the cumulative sum of arrival times to every customer. The traditional objective of minimizing the total travel distance is not considered at all. The authors in [Ngueveu et al.](#page-19-13) [\(2010\)](#page-19-13) proposed two memetic algorithms which employ local search to intensify the search of promising solution regions. [Lysgaard and Wøhlk](#page-18-9) [\(2014\)](#page-18-9) proposed a BCP algorithm for solving several instances of the CCVRP, whereas [Mattos Ribeiro and Laporte](#page-18-10) [\(2012\)](#page-18-10) proposed an adaptive large neighborhood search algorithm to solve the problem. The latter incorporated an adaptive probabilistic model in an effort to choose the most effective destruction/repair operators. After this, [Ozsoydan and](#page-19-14) [Sipahioglu](#page-19-14) [\(2013\)](#page-19-14) compare the performance of different optimization algorithms on the CCVRP including a genetic algorithm, a particle swarm optimization algorithm, as well as a Tabu Search algorithm. More recently, [Ke](#page-17-15) [\(2018\)](#page-17-15) presented a brainstorm optimization algorithm which is capable of solving large-scale instances of up to 1200 customers. Lastly, [Kyriakakis et al.](#page-18-11) [\(2021\)](#page-18-11) implemented an ant colony optimization algorithm, as well as a variable neighborhood descent hybrid algorithm to tackle the problem.

#### <span id="page-14-0"></span>6.2 Research direction

Despite the rich literature on the CCVRP and its extensions [\(Rivera et al.,](#page-20-14) [2015,](#page-20-14) [2014;](#page-20-15) [Lalla-Ruiz](#page-18-12) [and Voß,](#page-18-12) [2020\)](#page-18-12), very few research efforts have studied the CCVRP with time window constraints (CCVRPTW). To our knowledge there are only two works that study the CCVRPTW. [Liu and](#page-18-13) [Jiang](#page-18-13) [\(2019\)](#page-18-13) were the first to present and study the problem. The authors developed a hybrid large-neighborhood search algorithm adopts a constraint relaxation scheme in an effort to extend the search space and allow for the exploration of both feasible and infeasible neighboring solutions. Most recently, [Kyriakakis et al.](#page-18-14) [\(2022\)](#page-18-14) present a hybrid tabu search- variable neighborhood descent algorithm for solving the problem. Therefore, the CCVRPTW seems a unexplored hard to tackle problem that combines realistic modeling features such as time windows. Together with the alternative objective it becomes a strong tool for modelling complex operation in the logistics sector.

## <span id="page-14-1"></span>7 Conclusion

In conclusion, the extensive literature review has not only offered valuable insights into the diverse landscape of VRP models but has also underscored the multifaceted nature of this field. Throughout the analysis, various features of VRPs have been highlighted, each playing a crucial role in shaping the complexities and nuances of routing optimization. These features include profit collection, inventory management, production scheduling considerations, the unique challenges posed by humanitarian logistics, and notably, the pivotal role of time windows.

Looking ahead, future research in VRPs is poised to explore innovative approaches to tackle these challenges, including the development of advanced algorithms, the integration of real-time data analytics, and the adoption of artificial intelligence and machine learning techniques. Additionally, there is a growing emphasis on sustainability concerns, with researchers exploring ways to minimize environmental impacts and promote eco-friendly routing solutions. Ultimately, the significance of VRP research lies in its practical application in addressing real-world logistics challenges. By optimizing vehicle routes, VRP models contribute to enhanced operational efficiency, reduced transportation costs, and improved customer satisfaction. As logistics operations evolve and become more complex, ongoing research in VRPs remains essential for driving innovation and facilitating sustainable solutions in logistics and transportation.

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