Department of Economics

Athens University of Economics and Business

WORKING PAPER no. 17-2024

Gaussian Stochastic Volatility, Misspecified Volatility Filters and Indirect Inference Estimation

Stelios Arvanitis, Antonis Demos

November 2024

Πατησίων 76, 104 34 Αθήνα. Τηλ.: 210 8203303-5. E-mail: econ@aueb.gr / www.aueb.gr
76, Patission Street, Athens 104 34 Greece. Tel.: (+30) 210 8203303-5

The Working Papers in this series circulate mainly for early presentation and discussion, as well as for the information of the Academic Community and all interested in our current research activity.

The authors assume full responsibility for the accuracy of their paper as well as for the opinions expressed therein.

Πατησίων 76, 104 34 Αθήνα. Τηλ.: 210 8203303-5. E-mail: econ@aueb.gr / www.aueb.gr
76, Patission Street, Athens 104 34 Greece. Tel.: (+30) 210 8203303-5

Gaussian Stochastic Volatility, Misspecified Volatility Filters and Indirect Inference Estimation

Stelios Arvanitis,[*] *Antonis Demos*[†]

Abstract

The theory of the indirect inference estimation of a conditionally Gaussian asymmetric SV in mean model via the auxiliary use of Gaussian QML estimators based on misspecified volatility filters is investigated. A general assumption framework is provided and it is proven that the binding function between the associated models-the DGP and the one that the filter is based upon-is a well defined injection. The derivation of this new result is based on arguments related to ergodic optimization. The framework allows then for the establishment of a strong consistency property for the indirect inference estimator based on the uniform pseudo-consistency of the auxiliary one, and the derivation of a Gaussian limit theory with standard rates. A consistent estimator of the limiting variance is also discussed that allows for inference. A Monte Carlo simulation and an application on financial data, employing competing EGARCH and GQARCH type filters provided with some initial indication that favors the model defined by a recursion that "bears stronger resemblance" to the SV volatility recursion.
JEL Codes: C01, C10, C13, C58.
Keywords: Gaussian stochastic volatility, EGARCH, GQARCH, volatility filter, misspecification, indirect inference estimation, ergodic optimization, financial returns.

[*]Arvanitis is Professor of Theoretical and Financial Econometrics at the Department of Economics of Athens University of Economics and Business; 10434, Athens; Greece; email: stelios@aueb.gr

[†] Demos is Professor of Finance at the Department of International and European Economic Studies of Athens University of Economics and Business, 10434, Athens, Greece; e-mail: demos@aueb.gr

1 Introduction

The paper investigates the theory of indirect inference estimation of a conditionally Gaussian asymmetric stochastic volatility in mean model via the auxiliary use of Gaussian QML estimators based on misspecified volatility filters. Stochastic volatility models comprise a subclass of conditional heteroskedasticity that is characterized by the fact that volatility is exogenous; the log-conditional variance process is driven by innovations that are different processes from the innovations that drive the martingale difference noise that appears in the specification of the conditional mean. The two innovation processes need not however be independent. This volatility specification is in sharp contrast to the "endogenous" GARCH-type volatility models' specification; there the innovations in conditional variance recursions are measurable transformations of the conditional mean noise.

Both endogenous and exogenous volatility specifications, are in several cases tailored so that their probabilistic properties match several well documented statistical regularities, the so-called stylized facts, that appear in financial data. Examples of such empirical regularities include a. volatility clustering, i.e. that, on average, periods of high (low) volatility are followed period with high (low) volatility, b. dynamic asymmetry/leverage effects manifested in negative correlations between current returns and future squared returns, c. indirect evidence a positive correlation between the expected risk premium and ex ante volatility, d. small to insignificant autocorrelations, e. leptokurtosis, i.e. significant empirical excess kurtosis. For a relevant comprehensive introduction to the stylized facts and detailed literature see for example Straumann (2006) (43) or Tsay (2005) (47).

Volatility processes are unobservable. Hence statistical inference on time series models involving time varying conditional variance requires filtering of the latent volatility via the sample of returns and the properties of the model at hand. Some of the endogenous volatility models enjoy a property of invertibility, see Straumann (2006) (43) or Wintenberger (2013) (48); volatility is a measurable function of past observations. This property facilitates filtering which is then usually performed without smoothing, something that is in sharp contrast with the exogenous volatility models, making the endogenous invertible models more tractable w.r.t. inference.

The time series of asset returns was modeled using mainly endogenous volatility in mean specifications-see indicatively, among many others, Gonzales-Rivera (1996) (24), Choudhry (1996) (16), Dunne (1999) (21), Tai (2000) (44) and (2001) (45) , Ortiz and Arjona (2001) (38), Arvanitis and Demos (2004) (7) and (2004a) (8), and Bali and Peng (2006) (10).

Stochastic volatility models can be customized to replicate the aforementioned stylized facts, making them competitive with their endogenous volatility counterparts. Furthermore, they naturally appear as representations of information flows in financial markets (see, for example, Andersen (1996) (2)), and their continuous-time versions serve as natural diffusion limits of GARCH-type models (see, for instance, Nelson (1990) (37)). However, because they typically lack the property of invertibility, applying volatility filtering to these models often requires complex smoothing techniques. This complexity can make parameter estimation cumbersome (see, for example, Andersen and Benzoni 2009 (1), and Broto and Ruiz 2004 (14)), thereby reducing their appeal. Thus, endogenous volatility processes can also serve as useful statistical approximations for stochastic volatility models.

The direct likelihood-based estimation methods that have been proposed for stochastic volatility models can be divided into two main groups; those that try to reconstruct the full likelihood function and those that approximate it (see e.g. Taylor 1986 (46), and Harvey et al. (1994) (26)). The estimation method based on evaluating the full likelihood function can be found in, for example, Jacquier et al. (1994) (27) et al. (1998) (30), Sandmann and Koopman (1998) (39), Fridman and Harris (1998) (22), and Koopman and Uspensky (2002) (32). Several method of moment approaches have also been employed to estimate the SV model parameters such as the, so called, efficient method of moments (Gallant and Tauchen 1996 (23)), the Indirect Inference (Smith (1993) (42) and Gourieroux et al. (1993) (25)), the spectral method of moments (e.g. Knight et al. (2002) (31)), the simulated method of moments (Duffie and Singleton (1993) (20)) and the generalized method of moments (Andersen and Sorensen 1996 (3)).

This paper explores the estimation of a conditionally Gaussian asymmetric stochastic volatility in mean model using an indirect inference methodology (see e.g. Gourieroux et al. (1993) (25), Andersen and Sorensen (1996) (3)); the likelihood is approximated by a Gaussian likelihood involving a misspecified filter for the volatility, emerging from a general invertible endogenous volatility model. Hence in the spirit of the aforementioned convenient statistical approximation, the DGP likelihood is replaced by a misspecified one, the limiting properties of which establish a binding function between the DGP endogenous model and the one producing the volatility filter. The function essentially represents the pseudo-consistency behavior of the auxiliary Gaussian QMLE for the parameters of the misspecified filter. Invertibility of this so-called binding function, implies the possibility of estimating the DGP parameters, via the auxiliary estimator of the parameters of the filter and the inversion at it of the binding function, or some approximation of it. Invertibility of the endogenous auxiliary volatility model associated with the misspecified filter, implies that the Gaussian QMLE for the auxiliary parameters is not numerically cumbersome and is performable using standard smooth optimization numerical techniques; e.g. Newton-Ralphson-type methods. Then an approximate inversion of the latent binding function is provided by the Gallant and Tauchen (1996) (23) methodology, which is connected to minimal computational burden for the inversion, as it involves Monte Carlo integration of the auxiliary likelihood score, evaluated at the auxiliary estimator based on the original sample. Hence it does not require the evaluation of the auxiliary estimator at the Monte Carlo samples, while it enjoys first order asymptotic equivalence with more computationally involved indirect inference estimators like the ones introduced by Gourieroux et al. (1993) (25).

The fundamental idea of the indirect inference methodology is the binding function. This latent function typically maps the parameters of the accurately specified statistical model to those of the possibly misspecified auxiliary model. It is crucial that the binding function is well-defined, meaning it should not be multivalued. This ensures the pseudo-consistency of the auxiliary estimator, allowing it to have a meaningful and well-defined limit theory. The binding function should also be injective; this is essentially a restriction of indirect identification for the underlying well specified model from the auxiliary one. This implies that inverting any sufficiently close data dependent approximation of the binding function evaluated at the auxiliary estimator could result to a sufficiently close approximation of the DGP parameters, giving rise to several indirect inference estimators that usually employ some sort of Monte Carlo integration and potentially cumbersome optimization.

In this set up of the stochastic volatility in mean DGP, with a Gaussian likelihood emerging from an auxiliary misspecified endogenous volatility filter, the binding function is not only latent but also very difficult to approximate analytically. Hence the required properties of it being a well-defined injective function are difficult to establish analytically. The major contribution of the present paper, is that given an assumption framework that is reminiscent of the stationary and ergodic framework employed in the asymptotic analysis of the Gaussian QMLE in GARCH-type models (see for example Straumann (2006) (43)), it is proven that the binding function between the associated models-the DGP and the one that the filter is based upon-has the aforementioned properties. The derivation of this new result is based on arguments related to the concept of ergodic optimization Jenkinson (2019) (29)). This mathematical formulation investigates the properties of the set of optimizing measures of the integral of a given function w.r.t. to a collection of probability measures that are invariant w.r.t. the lag-operator. The convexity properties of this collection enable the establishment of the results about the binding function.

The assumption framework allows then for the establishment of a strong consistency property for the indirect inference estimator based on the uniform pseudo-consistency of the auxiliary one, and the derivation of a Gaussian limit theory with standard rates. A consistent estimator of the limiting variance is also discussed that allows for inference based on the estimator. A Monte Carlo simulation and an application on financial data, employing competing EGARCH(1,1) and GQARCH(1,1) type filters, with parameter restrictions that verify the assumptions, provides with some experimental indication that favors the filter emerging from the model defined by a recursion that "bears stronger resemblance" to the stochastic volatility recursion, in terms of Monte Carlo bias and Mean Squared Error. These motivate the theoretical consideration of the question of the optimal choice of a computationally convenient endogenous misspecified filter as further research.

The rest of the paper proceeds as follows. Section 2 introduces the model, the general form of the volatility filter and the subsequent auxiliary estimator. Section 3 provides with the limit theory of the auxiliary estimator under model misspecification and among others introduces and derives the properties of the binding function. Section 4 introduces the indirect inference estimator and derives its asymptotic properties. Section 5 performs Monte Carlo experiments utilizing the competing EGARCH and GQARCH volatility filters. Section 6 provides estimates on real financial data. In section 7 we conclude. A Supplementary Appendix provides with details concerning the construction of the indirect inference estimator for both the aforementioned filters, along with further information regarding the Monte Carlo experiments.

2 Framework

The following conditionally Gaussian SV(1,1)-M model is considered:

$$\begin{cases} y_t = m_\delta(\exp(v_t)) + \exp(\frac{v_t}{2})z_t, \\ v_t = \omega + \beta v_{t-1} + \sigma u_{t-1},\ t \in \mathbb{Z}, \\ (z_t, u_t)^{\mathrm{T}} \overset{\text{iid}}{\sim} N(\mathbf{0}, \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}). \end{cases} \tag{1}$$

There, for the s-dimensional real vector δ, $m_\delta : \mathbb{R} \to \mathbb{R}$ is a measurable invertible function, while z_k is independent of u_l, for all $k, l \in \mathbb{Z}$. The vector $\theta := (\delta^T, \omega, \beta, \sigma, \rho)^{\mathrm{T}}$, organizes the parameters that appear in the model and assumes its' values in some compact set $\Theta \subset \mathbb{R}^p$, $p = s + 4$.

For the filtration $\mathcal{F}^{(z,u)}$, where the algebra $\mathcal{F}_t^{(z,u)}$ represents the innovation history $\sigma(z_{t-j}, u_{t-j}, j \geq 1)$, the process $(m_\delta(\exp(v_t)))$ is the conditional mean process of (y_t). The process (v_t) is the corresponding logarithm of the conditional variance; the latter is hereafter denoted with (σ_t^2). The conditional mean thus depends on the variance, and the conditional variance is exogenous, as it is formulated without explicit dependence on the noise process (z_t). The error process $(\sigma_t z_t)$ is a martingale transform w.r.t. the aforementioned filtration.

The processes $(z_t), (u_t), (v_t)$ as well as the true values of the parameters are latent. The process (y_t) is observable, in the sense that a sample $(y_t)_{t=1,\ldots,T}$ is available to the analyst. The function m_δ is known up to the true value of the associated parameter. Thus there is no semi-parametric component in the DGP stochastic volatility model.

For reasons of inference, and via the sample above, the analyst uses the system

$$\begin{cases} y_t = m_\delta(h_t) + \sqrt{h_t} z_t \\ h_t = g^\star_{\phi^\star}\left(y_{t-1}, h_{t-1}\right) \end{cases}, \ t \in \mathbb{Z}, \tag{2}$$

as well as a -possibly stochastic- initial value $h^\star_0(\phi^\star)$ in order to construct a filter for the latent volatility $\exp(v_t)$. Here, for each value of the auxiliary parameter $\phi := (\delta^{\mathrm{T}}, \phi^{\star\mathrm{T}})^{\mathrm{T}}$ lying in some compact parameter space $\Phi \subset \mathbb{R}^q$, $q = \dim(\delta) + \dim(\phi^\star)$. For brevity, and in order to avoid issues involving inference on optimal weighting matrices for the construction of indirect estimators, we assume that $p = q$, a condition that is inline with our applications. The interested reader can use the relevant derivations of Arvanitis and Demos (2018) (5) in order to see how the results that follow about the indirect estimator can be generalized to the case $q > p$. Furthermore, $g^\star_{\phi^\star}$ is a non-negative measurable function on $\mathbb{R} \times \mathbb{R}_+$. Specifically, the filter $(h^\star_t(\phi^\star))_{t=0,\ldots,T}$ is constructed via the recursion:

$$h^\star_t(\phi^\star) = \begin{cases} h^\star_0(\phi^\star) & t = 0 \\ g^\star_{\phi^\star}\left(y_{t-1}, h^\star_{t-1}(\phi^\star)\right) & t > 0. \end{cases} \tag{3}$$

The filter thus corresponds to an invertible volatility process, see Straumann (2006) (43); this must be necessarily endogenous, meaning that the volatility dynamics directly depend on the noise and the conditional variance history. Hence the filter is misspecified, in the sense that it is a process with positive probability distinguishable from the DGP volatility process in (1). It is used for inference on ϕ; specifically, under the marginal Gaussianity of z_0; for $\ell_t(\phi) := \ln(h^\star(\phi^\star)) + \frac{(y_t - m_\delta(h^\star(\phi^\star)))^2}{h^\star(\phi^\star)}$, an auxiliary estimator for ϕ is definable via:

$$\phi_T \in \arg\min_{\phi\in\Phi} {}^1\!/_T \sum_{t=1}^{T} \ell_t(\phi).$$

If $g^\star_{\phi^\star}$ and the initial value $h^\star_0(\phi^\star)$ are continuous w.r.t. ϕ, then standard arguments along with the Theorem of Measurable Projections (see Par. 1.7 of van der Vaart and

Wellner (1996) (49)), ensure the existence of ϕ_T. The estimator ϕ_T can be regarded as a Gaussian QMLE based on a misspecified filter. As mentioned above, the derivation of the estimator usually incorporates a mild computational burden since it is not associated with costly smoothing and can be performed via Newton-Ralphson type optimization procedures.

The above are extendable to models of higher orders. Furthermore, they are easily extendable to cases where the auxiliary estimator is defined as an approximate minimizer.

In our Monte Carlo experiments and the application the conditional mean function is $m_\delta(h) := \delta_1 + \delta_2\sqrt{h}$. Two alternative misspecified filters are considered: (i) $g^\star_{\phi^\star}(y,h) := \exp(\omega^\star + (\alpha^\star|y| + \gamma^\star y)h^{-1/2} + \beta^\star \log(h))$, that corresponds to the EGARCH(1,1) endogenous volatility specification of Nelson (1991) (37), and (ii) $g^\star_{\phi^\star}(y,h) := \omega^\star + \alpha^\star y^2 + \gamma^\star y + \beta^\star h$, that in turn corresponds to the GQARCH(1,1) endogenous volatility specification of Sentana (1991) (41). The auxiliary parameter vector is $\phi := (\delta, \phi^{\star\mathrm{T}})$, where $\phi^\star := (\omega^\star, \alpha^\star, \gamma^\star, \beta^\star)^{\mathrm{T}}$, which however satisfies different restrictions in each case.

Due to the specific structure of the auxiliary likelihood function, choosing an endogenous volatility filter is crucial for both the implementability of indirect inference estimation and the properties of the resulting estimator. In the following, a comprehensive assumption framework is outlined which ensures that the asymptotic properties of the auxiliary estimator ϕ_T enable indirect identification. This means that the estimator converges to a distinct value of ϕ for each given θ, and the mapping between their respective parameter spaces is injective. Moreover, the framework ensures a conventional Gaussian limit theory for the auxiliary estimator, which is subsequently converted into a corresponding limit theory for its indirect inference version, facilitating inference.

3 Limit theory for the QMLE under filter misspecification

We are thus interested in studying the asymptotic properties of ϕ_T as $T \to \infty$. Under conditions specified below, the QMLE converges to an invertible function of the SV parameter θ. It is called binding function and will be denoted by $\phi(\theta) : \Theta \to \Phi$. The limit theory sought can be used in order to construct an indirect estimator of θ by inverting an implicit approximation of $\phi(\theta)$, as well as to establish the limit theory of the subsequent estimator.

We operate within a framework where the original process exhibits stationarity and ergodicity, while both stationarity, ergodicity, and uniform invertibility are applicable to the filters used. Extending these results to non-stationary contexts presents an interesting path for future study.

3.1 Pseudo-consistency and the binding function

In order to establish convergence of ϕ_T, along with the existence of $\phi(\theta)$ as a well defined singleton-valued function and its invertibility, we work with the following set of assumptions:

A1: *$|\beta| < 1$ and m_δ is Borel measurable.*

This along with Theorem 3.1 of Bougerol and Picard (1992) (13) and Proposition 2.1.1 of Straumann (2006) (43) directly imply stationarity and ergodicity for the process $(y_t, \exp(u_t))_{t\in\mathbb{Z}}$, thus providing an ergodic framework for the stochastic recurrence equations that define the filter in 2-3. Borel measurability for $m_\delta(h) = \delta_1 + \delta_2\sqrt{h}$ is readily the case.

In what follows let $\psi := (\theta^{\mathrm{T}}, \phi^{\mathrm{T}})^{\mathrm{T}}$ and $\Psi := \Theta \times \Phi$.

A2: *Suppose that: (i). $\mathbb{E}(\log^+ \sup_{\psi\in\Psi} |g^\star_{\phi^\star}(y_0, h)|) < +\infty$. (ii). $g^\star_{\phi^\star}(y, h)$ is almost surely Lipschitz continuous in h, with Lipschitz coefficient $\Lambda_i(\psi, y)$, and such that, (a). the map $\Psi \ni \theta \to \delta_2(\psi, y_0)$ is almost surely continuous, (b). $\mathbb{E}(\sup_{\psi\in\Psi} \log^+ \delta_2(\psi, y_0)) < 0$, where $\log^+$ is the positive part of the logarithmic function. (iii) $g^\star_{\phi^\star}(y_0, h)$ is almost surely continuous in ψ.*

Given **A1**, and the compactness of Ψ, the (i)-(ii) parts of the assumption imply the continuous invertibility (see for example Par. 3 and Prop. 3.1 of Blasques et al. (2018) (11)) of the filter $(h^\star_t(\phi^\star))$. In the GQARCH specification it suffices $\beta^\star < 1$. For the EGARCH specification the results of Wintenberger (2013) (48) readily imply that it is ensured whenever $\alpha^\star > |\gamma^\star|$, and $\mathbb{E}(\log(\sup_{\theta\in\Theta} \max(\beta^\star, 2^{-1}(\alpha^\star|y_0| + \gamma^\star y_0)\exp(\frac{\omega^\star}{2(1-\beta^\star)} - \beta^\star)))) < 0$ along with **A3** below. The latter is not trivial to verify, however a uniform integrability argument enabled by the compactness of Φ and the fact that y_0 follows a mixture of log-normal distributions imply that the expectation of the rhs of the previous inequality is continuous w.r.t. (θ, ϕ), and thereby for any $\theta_0 \in \Theta$ there exists a Φ such that for any θ in a neighborhood of θ_0, the condition holds for any $\phi \in \Phi$. Prop. 3.1 of Blasques et al. (2018) (11) imply the existence of a unique stationary and ergodic solution $(h_t(\phi))_{t\in\mathbb{Z}}$ of 2, the existence of which is ensured by the basic framework and Assumptions **A.1**-**A.2**; this is the exponentially fast almost sure approximator of the filter defined in 3 uniformly in θ. **A2**.(iii) implies then that $h_0(\phi^\star)$ is almost surely continuous in ϕ. The assumption obviously holds for both the specifications considered given the SV specification.

A3: *There exists some* $C > 0$ *such that* $\inf_{\psi\in\Psi} h_0 > C$.

A3 along with the form of ℓ_t and **A1**-**A2** imply Assumption C4 of Blasques et al. (11) and thus enables the exponentially fast almost sure approximation of the likelihood contributions by stationary and ergodic analogues uniformly over Ψ. For the GQARCH it follows when $\gamma^\star \leq 0$ and $\beta^\star < 1$ due to Lemma 2.1 of Arvanitis and Louka (2015) (6). For the EGARCH specification, it follows from **A2** due to the results of Wintenberger (2013) (48).

A1-**A3** and the form of the likelihood function then imply that $\mathbb{E}(\sup_{\phi\in\Phi}|\ell_0(\phi))| < +\infty$. Given this, and using the arguments in the proof of Theorem 4.1 of Blasques et al. (2018) (11), and Theorem 3.4 (Ch. 5) of Molchanov (2005) (34) we have that almost surely $\limsup_{T\to\infty}\phi_T \subset \phi(\theta) := \arg\min_{\phi\in\Phi}\mathbb{E}(\frac{(y_0(\theta)-m_\delta(h_0(\phi^\star)))^2}{h_0(\phi^\star)} - \ln(\frac{\exp(v_0(\theta))}{h_0(\phi^\star)}))$-actually this holds uniformly in Θ. Even though $\phi(\theta)$ is a well defined upper semi-continuous correspondence due to Th. 3.4 of Molchanov (2005) (34), as pointed out in a more general context by Blasques et al. (2018) (11) it may not be single-valued for some values of θ. The following assumption, is a usual identification condition in the relevant literature-see for example condition C.5 in Chapter 5 of Straumann (2006) (43)-that it is surprisingly sufficient for $\phi(\theta)$ to be single-valued and 1-1.

A4: $\Phi \ni \psi_1 \neq \psi_2 \Rightarrow h_0(\psi_1) \neq h_0(\psi_2)$ almost surely, and $\delta \to m_\delta(h)$ is 1-1 in for all h.

The first part of **A4** holds for both the GQARCH and EGARCH specifications since h_0', the gradient of the ergodic form of the filter w.r.t ψ, exists and it is comprised of linearly independent random variables due to the Gaussian part of the specification of the SV process. The second part readily holds for $m_\delta(h) = \delta_1 + \delta_2\sqrt{h}$.

A4 then implies that the auxiliary estimator is pseudo-consistent in an injective manner: ϕ_θ is not only a well defined function but also 1-1. This is described in the following result, which to the best of our knowledge is the first for models of such complexity, and it utilizes the notion of ergodic optimization (see for example Jenkinson (2019) (29)); the general structure of this mathematical formulation is the study of the set of optimizing measures of the integral of a given function w.r.t. to a collection of probability measures that are invariant w.r.t. the lag-operator. A crucial general result is that the set of optimizers is convex with extreme points the set of the ergodic measures in the collection. This can be utilized in order to create a contradiction that leads to:

Proposition 1. *Under* ***A1***-***A4*** *the binding correspondence*

$$\phi(\theta) := \arg\min_{\phi\in\Phi} \mathbb{E}(\frac{(y_0(\theta) - m_\delta(h_0(\phi^\star)))^2}{h_0(\phi^\star)} - \ln(\frac{\exp(v_0(\theta))}{h_0(\phi^\star)}),$$

is single valued and 1-1.

Proof. Suppose that the result does not hold. Then the ergodic optimization problem $\max_\mu \int(\frac{(y_0(\theta)-m_\delta(h_0(\phi^\star)))^2}{h_0(\phi^\star)} - \ln(\frac{\exp(v_0(\theta))}{h_0(\phi^\star)}))d\mu$ where μ denotes the joint distribution of $(y_0(\theta), h_0(\phi^\star), m_\delta(h_0(\phi^\star)))$ (see Relation (1) in Jenkinson (2019) (29)), would have at least two ergodic solutions, say $\mu, \mu^\star$ due to Proposition 2.3 of Jenkinson (2019) (29)) which is valid since Θ and Φ are compact and the parameterizations are continuous in the topology of weak convergence. This-due to the linearity of the integral in μ- would in turn imply that any mixture $\lambda\mu + (1 - \lambda\mu^\star)$ would also solve the problem above. Let A denote an invariant set of the underlying dynamical system w.r.t. the lag operator. We have that due to ergodicity $\mu(A) = 0, 1$, $\mu^\star(A) = 0, 1$. It must be that $\mu(A) = \mu^\star(A)$, for if it were not, then the support of one of those measure, would be a negligible set for the other. This cannot be the case due to the conditional Gaussianity assumption in the definition of the SV process, **A4** and the non degeneracy of the limiting filters. Hence $(\lambda\mu + (1 - \lambda\mu^\star))(A) = 0, 1$ which means that the mixture measure is also ergodic. But this is impossible due to Proposition 2.3 of Jenkinson (2019) (29). □

In both the GQARCH and the EGARCH specifications the binding functions seem analytically intractable. To our knowledge, Proposition 1 is the initial finding that identifies it as a well-defined function. Typically, similar outcomes are presented as assumptions in the field of misspecified models' estimation, as illustrated by Blasques et al. (2018) (11).

The proposition then directly implies uniform in θ pseudo-consistency for ϕ_T; in what follows $\|\cdot\|$ denotes any norm on Ψ:

Theorem 1. *Under* ***A1***-***A4****, for any* $\varepsilon > 0$, $\sup_{\theta\in\Theta} \mathbb{P}(\|\phi_T - \phi(\theta)\| > \varepsilon) \to 0$.

Proof. Follows from Theorem 4.3 of Blasques et al. (2018) (11), Fatou's Lemma, Lemma AL.1 of Arvanitis and Demos (2018) (5), and Proposition 1. □

The pseudo-consistency of the QMLE does not rely on bijectivity. The one-to-one property will later serve as an indirect identification attribute for ensuring the consistency of indirect estimators.

3.2 Rate and limiting Gaussianity

The following additional framework facilitates the derivation of the rate and limiting distribution of the QMLE based on the misspecified volatility filter. This is important because, combined with the injectivity of the binding function and arguments related to uniform integrability and the inverse function theorem, it supports a comparable limit theory for the indirect inference estimator, mainly through the Delta method. Now, $\|\cdot\|$ denotes the Euclidean or the Frobenius norm depending on the context.

B1: *For any $\theta \in \Theta$, there exists an open neighborhood, say $B_{\phi^\star}$, of $\phi^\star(\theta) := (\phi(\theta))_{i=s,\dots q}$ such that: (i) $g_{\phi^\star}$ is twice continuously differentiable w.r.t. $(\psi^\star, h)$ on $B_{\phi^\star(\theta)} \times \mathbb{R}_+$, for almost every value of its remaining arguments. (ii). For $g_{\partial\phi^\star}$ denoting the SRE obtained by recursive differentiation of $g_{\phi^\star}$ w.r.t. $\phi^\star$, we have that $\mathbb{E}(\log^+ \sup_{\phi^\star \in B_{\phi^\star}} \|g_{\partial\phi^\star 0}(\cdot)\|) < +\infty$. (A). $g_{\partial\phi^\star,t}$ is almost surely Lipschitz continuous w.r.t. $g_{\partial\phi^\star,t-1}$, with Lipschitz coefficient $\Lambda_t^{(\partial\phi^\star)}(\psi)$, and such that, (B). the map $B_{\phi^\star(\theta)} \ni \phi \to \Lambda_0^{(\partial\phi^\star)}(\psi)$ is almost surely continuous and, (C). $\mathbb{E}(\sup_{\phi^\star \in B_{\phi^\star(\theta)}} \log^+ \Lambda_0^{(\partial\phi^\star)}(\psi)) < 0$. (iii). Furthermore, let $g_{\partial\phi^\star\partial\phi^{\star\mathrm{T}}}$ denote the SRE obtained by recursive differentiation of $g_{\partial\phi^\star}$ w.r.t. $\phi^\star$. Then, suppose analogously that $\mathbb{E}(\log^+ \sup_{\phi \in B_{\phi^\star(\theta)}} \|g_{\partial\phi^\star\partial\phi^{\star\mathrm{T}}}(\cdot)\|) < +\infty$. (A). $g_{\partial\phi^\star\partial\phi^{\star\mathrm{T}},t}$ is almost surely Lipschitz continuous in $g_{\partial\phi^\star\partial\phi^{\star\mathrm{T}},t-1}$, with Lipschitz coefficient $\Lambda_t^{(\partial\phi^\star\partial\phi^{\star\mathrm{T}})}(\psi)$, and such that, (B). the map $B_{\phi^\star(\theta)} \ni \phi^\star \to \Lambda^{(\partial\phi^\star\partial\phi^{\star\mathrm{T}})}$ is almost surely continuous and, (C). $\mathbb{E}(\sup_{\phi^\star \in B_{\phi^\star(\theta)}} \log^+ \Lambda_0^{(\partial\phi^\star\partial\phi^{\star\mathrm{T}})}(\psi)) < 0$. Finally, (iv), m_δ is three times differentiable in $(\delta^{\mathrm{T}}, h)^{\mathrm{T}}$, and there exists some neighborhood of $\phi(\theta)$ for which $\mathbb{E}\log^+(\sup_{\delta,h}(\|m'_\delta(h)\| + \|m''_\delta(h)\|) < +\infty$, where m'_δ and m''_δ denote the gradient and the Hessian respectively of $m_\delta(h)$ w.r.t. $(\delta^{\mathrm{T}}, h)^{\mathrm{T}}$.*

The first three parts of the assumption facilitate among others the continuous invertibility of the filter derivatives. In the GQARCH specification, those parts follow directly from the parameter restriction conditions that ensure **A2**-**A3** due to the relevant derivations in the Appendix of Arvanitis and Louka (2015) (6) and the Gaussian part of the SV specification. The derivations in Lemma 1 of Wintenberger (2013) (48) imply that for the EGARCH specification they hold if $\mathbb{E}(\log^+ \sup_{\phi^\star \in B_{\phi^\star(\theta)}}(\beta^\star - \frac{1}{2}(a^\star|z_0| + \gamma^\star z_0)\frac{\exp(\frac{v_0(\theta)}{2})}{\sqrt{h_0(\phi^\star)}})) < 0$. For the aforementioned specifications, the final part holds for the case $m_\delta(h) = \delta_1 + \delta_2\sqrt{h}$ due the previous, and Lemma 2.5.3 of Straumann (2006) (43).

In what follows $h^{\star\prime}$ and $h^{\star\prime\prime}$ (respectively h' and h'') denote the gradient and the

Hessian of the filter w.r.t. $\phi^\star$ (respectively of their ergodic approximators).

B3: *For* $B_{\phi^\star(\theta)}$ *as above and for some* $\epsilon > 0$, $\mathbb{E}(\sup_{\phi \in B_{\phi^\star(\theta)}} \|\frac{h_0'}{h_0}\|^{2+\epsilon}) + \mathbb{E}(\sup_{\phi \in B_{\phi^\star(\theta)}} \frac{m_\delta^{4+\epsilon}(h_0)}{h_0^{2+\epsilon}})$ $+\mathbb{E}(\sup_{\phi \in B_{\phi^\star(\theta)}} m_\delta'^{4+\epsilon}(h_0)\|h_0'\|^{2+\epsilon}) + \mathbb{E}(\sup_{\phi \in B_{\phi^\star(\theta)}} (m_\delta(h_0)\|m_\delta'(h_0)\|)^{2+\epsilon}) + \mathbb{E}(\sup_{\phi \in B_{\phi^\star(\theta)}} \|\frac{h_0''}{h_0}\|^{1+\epsilon})$ $+\mathbb{E}(\sup_{\phi \in B_{\phi^\star(\theta)}} \|\frac{m_\delta''(h_0)}{h_0^2}\|^{1+\epsilon}) + \mathbb{E}(\sup_{\phi \in B_{\phi^\star(\theta)}} \|\frac{m_\delta'(h_0)}{h_0^2}\|^{1+\epsilon}) < +\infty$. *Furthermore,* $\mathbb{E}(\|\frac{h_0'}{h_0}\|^4) +$ $\mathbb{E}(\frac{m_\delta^8(h_0)}{h_0^4}) + \mathbb{E}((m_\delta(h_0)\|m_\delta'(h_0)\|)^4) < +\infty$.

Given the derivations in the Appendix of Arvanitis and Louka (2015) (6) and the Gaussianity involved in the definition of the SV process, the required moments' existence for the GQARCH specification follows when β is bounded below one. For the EGARCH specification, the derivations in Lemma 1 of Wintenberger (2013) (48) as well as the derivations in the Appendix of Demos and Kyriakopoulou (2009) (17), and the aforementioned Gaussianity imply that the required conditions hold as long as $\mathbb{E}(\sup_{\phi^\star \in B_{\phi^\star(\theta)}} (\beta^\star - \frac{1}{2}(a^\star|z_0| + \gamma^\star z_0)\frac{\exp(\frac{v_0(\theta)}{2})}{\sqrt{h_0(\phi^\star)}})^{4+\delta}) < 1$, a condition that is stronger compared to the analogous logarithmic moment existence that appeared above. The conditions involving m_δ follow easily for the specification $\delta_1 + \delta_2\sqrt{h}$ due to the above and **A3**.

B4: *For any* $\theta \in \Theta$, *and* $\phi_1 \neq \phi_2$, $\frac{\partial \ell_0(h_0(\phi_1))}{\partial \phi} \neq \frac{\partial \ell_0(h_0(\phi_2))}{\partial \phi}$ *almost surely.*

Again, the derivations in the Appendix of Arvanitis and Louka (2015) (6), the derivations in Lemma 1 of Wintenberger (2013) (48) as well as the derivations in the Appendix of Demos and Kyriakopoulou (2009) (17), and the Gaussianity involved in the definition of the SV process imply that **B3** follows for the GQARCH and EGARCH filter specifications.

Utilizing the totality of our assumption framework, we obtain the following limit theory for the QMLE-there $\rightsquigarrow$ denotes convergence in distribution:

Theorem 2. *Under the premises of Theorem 1, if moreover **B1-B5** hold and* $\theta \in \mathrm{Int}(\Theta)$, *then*

$$\sqrt{T}(\phi_T - \phi(\theta)) \rightsquigarrow N(\mathbf{0}, V_{\phi(\theta)}),$$

where $V_{\phi(\theta)} := (\partial_\phi \partial_{\phi^{\mathrm{T}}} \mathbb{E}(\ell_0(\phi(\theta))))^{-1} \mathbb{E}(\partial_\phi \ell_0(\phi(\theta)) \partial_\phi \ell_0^{\mathrm{T}}(\phi(\theta)))(\partial_\phi \partial_{\phi^T} \mathbb{E}(\ell_0(\phi(\theta))))^{-1}$, *for the stationary and ergodic versions of the associated derivatives.*

Proof. First notice that the first part of **B3**, along with differentiability of the SV model w.r.t. θ, the interior condition and uniform integrability imply commutativity between the integral and all the employed derivative operators. This along with Proposition 1 and the indirect information equality (see the preparation before Proposition 5 of Gourieroux et al. (1993) (25)) implies that $\phi(\theta)$ is continuously differentiable on the interior of Θ with a full rank gradient, hence a homeomorphism $\mathrm{Int}(\Theta) \to \mathrm{Int}(\Phi)$. **B4** and mean value expansion of the first order derivative imply full rank for $\partial_{\phi}\partial_{\phi^{\mathrm{T}}}\mathbb{E}(\ell_0(\phi(\theta)))$ and via the aforementioned equality it implies algebraic linear independence for the elements of $\partial_{\phi}\ell_0(\phi(\theta))$, and thereby full rank for $\mathbb{E}(\partial_{\phi}\ell_0(\phi(\theta))\partial_{\phi}\ell_0^{\mathrm{T}}(\phi(\theta)))$. **B1** implies exponentially fast almost sure convergence of the filter derivatives to their ergodic analogues locally uniformly in $\phi(\theta)$, and combined with **B3** it implies almost sure approximation of the first and second order derivatives of the likelihood by their ergodic analogues locally uniformly in $\phi(\theta)$. The first part of **B3** implies convergence of the ergodic version of the empirical Hessian to its population analogue due to Birkhoff's LLN locally uniformly in $\phi(\theta)$. This and Theorem 1 imply almost sure convergence of the empirical Hessian evaluated anywhere on the line connecting ϕ_T and $\phi(\theta)$ to the population Hessian at $\phi(\theta)$. Furthermore, the first derivative of the likelihood contributions evaluated at $\phi(\theta))$ conditionally on the information at $t-1$, lies in the normal domain of attraction of a zero mean Gaussian distribution (see Theorem 2.6.5 in Ibragimov and Linnik (1975) (33)). Then stationarity and ergodicity and second order integrability of the limiting filter of the first derivatives as well as the almost sure boundedness of the derivatives of the remaining processes along with the principle of conditioning (see Jakubowski (1986) (28)), implies $O_p(\sqrt{T})$ asymptotic tightness and limiting zero mean Gaussianity for the empirical average of the score. The result then follows from a Mean Value expansion of the f.o.c.s. of the optimization problem that defines the estimator, which holds w.h.p. due to that $\phi(\theta)$ is open. $\square$

A consistent estimator for the asymptotic variance that appears in the limit theory is $V_T := (\partial_{\phi}\partial_{\phi^{\mathrm{T}}}\mathbb{E}_T(\ell_t(\phi_T)))^{-1}\mathbb{E}_T(\partial_{\phi}\ell_t(\phi_T)\partial_{\phi}\ell_t^{\mathrm{T}}(\phi_T))(\partial_{\phi}\partial_{\phi^T}\mathbb{E}_T(\ell_t(\phi_T)))^{-1}$, where $\mathbb{E}_T$ denotes integration via the empirical measure. This directly follows from the assumption framework above, Theorem 1 and the Continuous Mapping Theorem (CMT). Notice that the derivatives involved in the construction of the Hessian in V_T can be evaluated via numerical differentiation. It is easy to see that the above imply that when this is the case and the evaluation of the forward differences occurs at a parameter value, say $\phi^{\star}$, such that $\|\phi^{\star} - \phi_T\|$ almost surely converges to zero, while $T\|\phi^{\star} - \phi_T\|$ almost surely diverges to infinity, then this version of V_T also converges to $V_{\phi(\theta)}$ almost surely.

4 The indirect estimator based on the score

The assumption framework above produces interior openess for the binding function and locally uniform convergence for the score. Those along with the properties of the limiting version of the likelihood imply that the inverse of the binding function, when restricted to the interior of Θ, can be recovered via the variational problem $\min_{\theta\in\mathrm{Int}(\Theta)} \|\mathbb{E}(\partial_\phi \ell_0(\phi(\theta)))\|^2$. An empirical approximation of the latent expectation of the score, can be obtained via Monte Carlo integration. Thus, an indirect estimator for θ, termed the Gallant and Tauchen indirect inference estimator (see Gallant and Tauchen(1996) (23) and Gourieroux et al. (1996) (25)), θ_T, can be defined by

$$\theta_T \in \arg\min_{\theta\in\Theta} \|\mathbb{E}_{S,\theta}\mathbb{E}_T(\partial_\phi \ell_0(\phi_T))\|^2,$$

where $\mathbb{E}_{S,\theta}$ denotes integration w.r.t. the empirical Monte Carlo distribution of S independent samples of $(y_{t,s}(\theta))_{t=1,\ldots,T,s=1,\ldots,S}$ from model (1). The particular estimator is connected to minimal computational burden among competing indirect inference estimators (see again Gourieroux et al. (1996) (25)), as it does not require the derivation of the auxiliary estimate at each Monte Carlo sample, while being asymptotically equivalent to them. It may possess, however, poorer properties in terms of higher-order asymptotics, see Arvanitis and Demos (2018) (5).

The above results on the asymptotic behavior of the auxiliary likelihood and estimator, and the properties of the binding function, directly provide the limit theory of θ_T. The derivation essentially works via the Delta method and the inverse function theorem:

Theorem 3. *Under the premises of Theorem 2, for any S, and as $T \to \infty$, a. θ_T is strongly consistent for any $\theta \in \mathrm{Int}(\Theta)$, and b. for any such θ,*

$$\sqrt{T}(\theta_T - \theta) \rightsquigarrow N(\mathbf{0}, (1 + {}^1\!/\!{}_S)(\partial_\theta\phi(\theta))^{-1} V_{\phi(\theta)})(\partial_\theta\phi(\theta))^{-1\mathrm{T}}),$$

where $\partial_\theta\phi := \frac{\partial\phi}{\partial\theta^{\mathrm{T}}}$.

Proof. a. follows from that due to $B1$-$B3$ and Birkhoff's LLN $\|\mathbb{E}_{S,\theta}\mathbb{E}_T(\partial_\phi \ell_0(\phi(\theta)))\|$ converges locally uniformly almost surely to $\|\mathbb{E}(\partial_\phi \ell_0(\phi(\theta)))\|$ and Proposition 1. b. then follows from the Theorem 2 and its proof and Appendix 1 of Gourieroux et al. (1996) (25). □

If S is allowed to diverge to infinity, the result predicted for the general first order theory for the GT estimators established in Arvanitis and Demos (2018) (5) is recovered. If the boundary of Φ is representable by restrictions amenable to Kuhn-Tucker optimization conditions, the above definition and results can be extended to the case where $\theta \in \mathrm{Bd}(\Theta)$ via the Lagrange multiplier technology developed in Calzolari et al. (2004) (15).

The uniform consistency result for the auxiliary estimator in Theorem 1 and the first result of the previous theorem, imply that a consistent estimator of the Jacobian of the binding function, say $\partial_\theta \phi_T$ is obtainable via numerical differentiation of $\phi_T(\theta_T)$, as long as the evaluation of the forward differences occurs at a parameter value, say $\theta^\star$, such that $\|\theta^\star - \theta_T\|$ converges almost surely to zero, while $T\|\theta^\star - \theta_T\|$ almost surely diverges to infinity. The CMT and the discussion in the previous section then imply that $(1+{}^1\!/\!{}_S)(\partial_\theta \phi_T)^{-1} V_T (\partial_\theta \phi_T)^{-1\mathrm{T}}$ is a strongly consistent estimator of the asymptotic variance of θ_T. This allows inferential tasks such as constructing confidence regions or performing Wald-type tests using θ_T to be practically implementable.

5 Monte Carlo Simulations

We employ Monte Carlo simulations to assess the small sample bias and Mean Squared Error properties of indirect inference estimators that utilize EGARCH(1,1) and GQARCH(1,1) models. Both auxiliary volatility models are endogenous and align with the theory mentioned above given the specified parameter constraints. They also agree with the volatility stylized facts highlighted in the Introduction, which informed the customization of the stochastic volatility model. The first auxiliary filter-corresponding to the EGARCH model-is defined by a recursive equation that bears a stronger resemblance to the stochastic volatility recursion than the second filter, especially when the absolute value of ρ assumes high positive values.

The following Monte Carlo simulations might offer insights into whether a heuristic approach to selecting a volatility filter correlates with improved performance in small samples. This is evidently related to the broader issue of choosing a computationally efficient volatility filter that is optimal, at least in an asymptotic sense. The theoretical aspects of this issue appear to be an intriguing subject for future research. The Monte Carlo simulations also offer some insight into the computational effort required to obtain the estimates, as well as issues related to occasional process failures.

The results in Jacquier et al. (1994) (27) and in Calzolari et al. (2004) (15) imply that an important determinant of the performance of the different estimators is the unconditional coefficient of variation of the unobserved volatility level $\sigma_t^2 := \exp(v_t)$, say CV, where

$$\mathrm{CV}^2 = \frac{\mathrm{Var}\left(\sigma_0^2\right)}{\mathbb{E}^2\left(\sigma_0^2\right)} = \exp\left(\frac{\sigma^2}{1-\beta^2}\right) - 1.$$

When CV is low, the observed process is close to Gaussian white noise, and consequently estimating the stochastic volatility parameters is difficult. Furthermore, CV is independent of ρ.

The simulated paths were generated employing 3 sets of parameter values. For the the first one we set $\omega = -0.1, \beta = 0.9, \rho = -0.8, and \sigma = 0.3629$ getting $CV^2 = 1.0$, for the second one we chose $\omega = 0.0, \beta = 0.9$, $\rho = -0.95$ and $\sigma = 0.31623$ with $\mathrm{CV}^2 = 0.693$ as in Monfardini (1998) (35), and for the third one we chose $\omega = -0.736$, $\beta = 0.9$, $\rho = -0.95$ and $\sigma = 0.363$ with $\mathrm{CV}^2 = 1.0$ as in Jacquier et al. (1994) (27). Notice that the third set of parameters values has been employed by among others Andersen and Sorensen (1996) (3), as well. However, the previous articles are dealing with symmetric SV models; $\rho = 0$. For the mean specifications, we set $(\delta_1, \delta_2) = (0.0, 0.111)$ for the first set, $(\delta_1, \delta_2) = (0, 0.111)$ for the second one, and $(\delta_1, \delta_2) = (0.07, 0.08)$ for the third one. In the Supplement we also present Monte Carlo results for the zero mean asymmetric SV models and compare our results with those of previous research.

In all simulations we choose $S = 200$ for $T = 1000$, 2000 and 3000, and $S = 150$ for $T = 5000$, 7500 and 10000, and perform 500 Monte Carlo simulations for each score generator. The choice of S is based mainly in computational time considerations, as higher value of S results in smaller asymptotic variance of the estimators and consequently increases the stability of the estimation (see below on this) but increases the time needed for the program to converge. These values of S are far smaller than the ones employed in the application with real data section.

In the first parameter scenario is obtained that $\mathbb{E}\left(y_t\right) = 0.074$ giving an Annualized Rate of Return of 3.89%, and $\mathrm{Var}\left(y_t\right) = 0.520$. In Figure 1 the Bias and Root MSE of the indirect inference estimators are presented for the first set of parameters values. Notice that only for T=1000 the GQARCH-M score generator has smaller Bias and Root MSE.

Place Figure 1 about here

For the second set of parameter values we have that $\mathbb{E}\left(y_t\right) = 0.043$, with Annualized Rate of Return 2.45%, and $\mathrm{Var}\left(y_t\right) = 1.301$. For this set it is obvious that the Biases of the EGARCH-M score generator are much smaller of the respective GQARCH-M ones (see Figure 2). In terms Root MSEs, the ones of the GQARCH-M generator are smaller for T=2000 and T=5000.

Place Figure 2 about here

Finally, for the third set of parameters we get $\mathbb{E}\left(y_t\right) = 0.123$, Annualized Rate of Return 6.59%, and $\mathrm{Var}\left(y_t\right) = 0.521$. Here, the Biases and Root MSEs of the E-GARCH-M score generator are smaller as compared to the GQARCH-M ones, for all the examined values of T.

Place Figure 3 about here

In the supplement all biases and root MSEs are presented for all three parameter values' sets. In terms of estimated biases, in almost all cases, the auxiliary EGARCH estimates are closer to the true ones. Further, the estimated root MSE of the auxiliary EGARCH estimation procedure is by far smaller than the equivalent of GQARCH estimation procedure.

In simulation exercises concerning stochastic volatility models, it has been observed that a portion of the draws does not lead to convergence of the optimization algorithms, regardless of whether the Generalized Method of Moments (3), the Efficient Method of Moments (4), or the Quasi-Maximum Likelihood (27) techniques are applied for estimation. This issue is evident in our study as well, particularly when using the GQARCH-M filter.

In particular, concerning the specifics of the numerical methods utilized, we observe the following: our approach consists of a two-stage optimization process. Initially, we maximize the quasi-likelihood function of the auxiliary model by minimizing the gradient norm concerning the auxiliary parameters. Subsequently, we minimize the norm of the estimated expected gradient of the auxiliary quasi-likelihood, calculated at the derived auxiliary parameters, focusing on the model parameters. Either of these numerical optimizations might not succeed. We utilized the E04JBF minimization routine from the NAG library for both stages of optimization. In the second step, the initial values were set to the true values across all replications and parameter sets. In contrast, the initial values for the auxiliary parameters were chosen to be realistic, approximately aligning with the theoretical values of various moments for both the auxiliary and the true model, using the formulas from Demos (2023) (18).

In case that in the first step optimization we got a failure, IFAIL=2 (the routine has not found the minimum but the maximum number of iterations, set to 320, has been reached) or IFAIL=3 (the conditions for termination have all been met, but a lower point could not be found) we leave the program to proceed to the second step and obtain the estimates of the parameters of the true model. On the other hand, if we get a failure on the second step we consider that the program failed at this replication. We could deal with the failures in a number of different ways. First, we could increase S, and/or increase the number of iterations, and/or change the initial values of the parameters. However, the first two solutions would increase the time needed for the program to converge, which is extremely time consuming for a simulation exercise with three different parameter sets and six different sample sizes. We set aside the third option because our aim is to handle each simulation path uniformly. Consequently, we opted to ignore the failed simulation paths and select alternative ones. This approach necessitates careful interpretation of comparisons among auxiliary models, particularly concerning root MSEs.

The following table presents the number of program failures for the considered two auxiliary models, 3 set of parameters and all the considered sample sizes.

For all the considered cases the number of failures with the EGARCH-M auxiliary is

Tab. 1: Optimization Routine Failures

Number of Failures						
	1st Parameter Set		2nd Parameter Set		3rd Parameter Set	
	EGARCH-M	GQARCH-M	EGARCH-M	GQARCH-M	EGARCH-M	GQARCH-M
T=1000	1	117	27	88	19	116
T=2000	3	59	5	58	0	55
T=3000	0	70	0	53	0	57
T=5000	0	82	0	14	0	59
T=7500	0	34	0	28	0	70
T=10000	8	38	0	18	0	69

by far less the failure cases with the GQARCH-M one. It seems that the EGARCH-M auxiliary is a better choice not only in terms of bias and root MSE but also in terms of facilitation of routine convergence and computational easiness. These are an indications of validity of the heuristic mentioned in the beginning of the present paragraph and could be related to geometrical/probabilistic aspects of the relations between the statistical models employed.

6 An application to International Markets.

The aforementioned methods of estimation are employed on samples of weekly excess returns of four indices of international markets, i.e. the S&P, the FTSE, the DAX and the Nikkey. The motivation is similar to the Monte Carlo exercises above; in the context of the question of the optimal selection of a volatility filters, we want to explore whether the indirect inference estimator based on the EGARCH(1,1) filter is able to better reproduce the empirical characteristics of the aforementioned data compared to the one based on the GQARCH(1,1) filter. In this context and in the the following Table we present some descriptive statistics for the 4 indices, along with the period of estimation and the number of observations. It is obvious that the standard deviation of returns is almost 22 times higher than the average return. Further, in all markets the skewness and kurtosis coefficients are far from the corresponding of the normal distribution ones. The asymptotic confidence interval for the autocorrelations is $(-0.041, 0.041)$ for the 3 markets and $(-0.049, 0.049)$ for FTSE, indicating that, apart from Nikkey, either the 1st or the second order autocorrelations are significant. However, it is known that in the presents of GARCH-type effects the asymptotic distribution of the correlation coefficients are affected (see e.g. Diebold (1986) (19)). $Q(4)$ is the fourth order Ljng-Box statistic, asymptotically distributed as χ_4^2 under the null of zero autocorrelation up to order 4.

Tab. 2: Statistics Weekly Excess Returns

Index	S&P	FTSE	DAX	Nikkey
Period	$1973-2017$	$1987-2017$	$1973-2017$	$1973-2017$
No. of Obs.	2299	1621	2300	2299
Average	0.103	0.104	0.134	0.037
St. Dev..	2.299	2.299	2.767	2.485
Skewness	-0.542	-0.541	-0.592	-0.614
Kurtosis	8.309	8.304	8.003	7.578
Jarque − Bera	2811.4	2807.5	2533.9	2151.9
$\widehat{\rho}\,(y_t, y_{t-1})$	-0.063	-0.064	-0.005	0.000
$\widehat{\rho}\,(y_t, y_{t-2})$	0.038	0.037	0.058	0.038
$Q(4)$	15.359	15.408	13.635	4.220
$\widehat{\rho}\left(y_t^2, y_{t-1}^2\right)$	0.267	0.267	0.203	0.217
$\widehat{\rho}\left(y_t^2, y_{t-2}^2\right)$	0.168	0.168	0.252	0.143
$Q^{(2}\,(4)$	363.53	363.78	425.78	201.19
Dyn. Asym.(1)	-0.198	-0.198	-0.176	-0.137

The first and second order autocorrelation of the squared returns is significant indicating strong volatility clustering effects. This is justified by the fourth Ljung-Box statistic for the squared returns $Q^{(2}\,(4)$. The estimated Dynamic Asymmetry, $\widehat{\rho}\,(y_t^2, y_{t-1})$, is negative and significant in all cases. Notice that the theoretical dynamic asymmetry depends on the leverage effect parameter ρ as well as the parameter λ (see Demos 2023 (18) and Bollerslev and Zhou (2006) (12)).

The estimates along with their z-type statistics for significance (in parentheses) appear in Table 3. The estimation in all cases is carried out under the restriction that $\delta_1 = 0$ due to that initial estimation results-available upon request-showed insignificance of the particular parameter in all cases. To avoid inflating the estimator variances we have chosen $S = 90000$.

Tab. 3: Constrained Estimation

	S&P		FTSE	
	EGARCH − M	GQARCH − M	EGARCH − M	GQARCH − M
Parameter				
δ_2	0.053	0.051	0.023	0.302
	(2.630)	(0.208)	(0.986)	(0.123)
ω	0.079	0.109	0.086	0.088
	(1.103)	(0.502)	(0.0891)	(0.481)
β	0.939	0.916	0.945	0.944
	(5.953)	(2.958)	(5.243)	(3.877)
ρ	−0.591	−0.538	−0.672	−0.559
	(−2.178)	(−0.341)	(−2.129)	(−0.341)
σ	0.254	0.308	0.243	0.260
	(2.654)	(0.489)	(1.998)	(0.498)
	DAX		Nikkey	
	EGARCH − M	GQARCH − M	EGARCH − M	GQARCH − M
Parameter				
δ_2	0.070	0.030	0.015	0.030
	(2.751)	(0.123)	(0.713)	(0.123)
ω	0.077	0.088	0.067	0.088
	(0.891)	(0.481)	(0.725)	(0.481)
β	0.952	0.944	0.957	0.944
	(5.308)	(3.878)	(4.886)	(3.877)
ρ	−0.435	−0.560	−0.358	−0.559
	(−1.369)	(−0.341)	(−1.008)	(−0.341)
σ	0.235	0.260	0.233	0.260
	(2.141)	(0.498)	(2.058)	(0.498)

It is worth noticing that, with the exemption of Nikkey, the price of risk, δ_2, is significant under the EGARCH-M auxiliary, whereas it is insignificant for all markets employing GQARCH-M auxiliary. In fact, in almost all cases, it seems that the EGQARCH-M auxiliary estimates the parameters with greater precision than the ones of the GQARCH-M auxiliary.

7 Conclusions

We investigated the theory of the estimation of a conditionally Gaussian asymmetric SV model with possibly time varying risk premia, via the auxiliary use of Gaussian QML estimators based on misspecified volatility filters and the subsequent employment of an indirect inference estimation procedure based on the simulated score.

We have developed a general assumption framework and demonstrated that it ensures the binding function between the associated models—the DGP and the model upon which the filter is based—is well defined and injective. This new result is derived using arguments related to ergodic optimization. Consequently, this framework facilitates establishing a strong consistency property for the indirect inference estimator, grounded in the uniform pseudo-consistency of the auxiliary estimator, leading to the derivation of a standard rate Gaussian limit theory. A consistent estimator of the limiting variance is also discussed, allowing for inference.

The critical issue of selecting a computationally affordable auxiliary volatility filter that has at least optimal asymptotic properties is an interesting path for further research. A Monte Carlo simulation and an application on data from international financial markets, employing competing EGARCH(1,1) and GQARCH(1,1) type filters provided with some initial indication that favors the model defined by a recursion that "bears stronger resemblance" to the SV volatility recursion. The theoretical underpinning of such like heuristics, as well as issues related with the computational burden associated with the derivation of the estimation procedures involved, could be benefited from geometrical/probabilistic aspects of the relations between the statistical models at hand as exemplified by fields like Information Geometry (see Ay et al. (2017) (9)). Similar considerations could be also helpful in determining the optimal selection of filters when the underlying stochastic volatility model is itself misspecified due to misspecification of the conditional distribution, and potentially facilitate parametric model selection in semi-parametric stochastic volatility frameworks. These considerations are delegated to further research.

References

[1] Andersen, T.G. and L. Benzoni (2009), Stochastic Volatility, in Meyers R. (eds) *Complex Systems in Finance and Econometrics*, Springer N. York

[2] Andersen, T.G., 1996. Return Volatility and Trading Volume: An Information Flow Interpretation of Stochastic Volatility, Journal of Finance 51, 169-204.

[3] Andersen, T.G. and B.E. Sorensen, 1996. GMM Estimation of a Stochastic

Volatility Model: a Monte Carlo Study, Journal of Business and Economic Statistics 14, 328–352.

[4] Andersen, T. G., Chung, H. J., & Sørensen, B. E., 1999. Efficient method of moments estimation of a stochastic volatility model: A Monte Carlo study. Journal of Econometrics, 91(1), 61-87.

[5] Arvanitis, S., & Demos, A., 2018. On the Validity of Edgeworth Expansions and Moment Approximations for Three Indirect Inference Estimators. Journal of Econometric Methods, 7(1), 20150009.

[6] Arvanitis, S. and Louka, A., 2015. Limit Theory for the QMLE of the GQARCH (1, 1) model. Communications in Statistics-Theory and Methods, 44(17), pp.3549-3575.

[7] Arvanitis, S. and A. Demos, 2004. Time Dependence and Moments of a family of Time-Varying parameter $GARCH$ in Mean Models, Journal of Time Series Analysis 25, 1-25.

[8] Arvanitis, S. and A. Demos, 2004a, Conditional Heteroskedasticity in Mean Models, in *Quantitative Methods in Finance in Honor of Prof. A. Kintis*, Editor A. Refenes, Typothito.

[9] Ay, N., Jost, J., Vân Lê, H., & Schwachhöfer, L., 2017. Information geometry (Vol. 64). Cham: Springer.

[10] Bali, T.G. and Peng, L., 2006. Is there a risk–return trade-off? Evidence from high-frequency data. Journal of Applied Econometrics, 21(8), pp.1169-1198.

[11] Blasques, F., Gorgi, P., Koopman, S.J. and Wintenberger, O., 2018. Feasible invertibility conditions and maximum likelihood estimation for observation-driven models. Electron. J. Statist. 12 (1) 1019 - 1052. https://doi.org/10.1214/18-EJS1416

[12] Bollerslev, T. and H. Zhou, 2006. Volatility puzzles: a simple framework for gauging return-volatility regressions, Journal of Econometrics 131, 123–150.

[13] Bougerol, P. and Picard, N., 1992. Strict stationarity of generalized autoregressive processes. The Annals of Probability, 20(4), pp.1714-1730.

[14] Broto, C. and E. Ruiz, 2004. Estimation Methods for Stochastic Volatility Models: A Survey, Journal of Economic Surveys 18, 613-649.

[15] Calzolari, G., Fiorentini, G. and Sentana, E., 2004. Constrained indirect estimation. The Review of Economic Studies, 71(4), pp.945-973.

[16] Choudhry, T., 1996. Stock Market Volatility and the Crash of 1997: Evidence Six Emerging Markets, Journal of International Money and Finance 15, 969-981.

[17] Demos, A. and Kyriakopoulou, D., 2009. Asymptotic expansions of the QMLEs in the EGARCH (1, 1) model. preprint.

[18] Demos, A., 2023. Statistical Properties of Two Asymmetric Stochastic Volatility in Mean Models, DP 23-03, Dpt. IEES, AUEB

[19] Diebold, F.X., 1986. Testing for Serial Correlation in the Presence of ARCH, in Proceedings of the American Statistical Association, Business and Economics Statistics Section, 323–328.

[20] Duffie, D. and Singleton, K.J., 1997. An econometric model of the term structure of interest-rate swap yields. The Journal of Finance, 52(4), pp.1287-1321.

[21] Dunne, P.G., 1999. Size and Book to Market Factors in a Multivariate GARCH in Mean Asset Pricing Application, International Review of Financial Analysis 8, 61, 35-52.

[22] Fridman, M. and L. Harris, 1998. A Maximum Likelihood Approach for Non-Gaussian Stochastic Volatility Models, Journal of Business and Economic Statistics 16, 284-291.

[23] Gallant, R. A. and G. Tauchen, 1996. Which Moments to Match, Econometric Theory 12, 657-81.

[24] Gonzales-Rivera G. ,1996. Time Varying Risk. The Case of The American Computer Industry, Journal of Empirical Finance 2, 333-342.

[25] Gourieroux, C., Monfort, A., & Renault, E., 1993. Indirect inference. Journal of applied econometrics, 8(S1), S85-S118.

[26] Harvey, A., E. Ruiz and N. Shephard, 1994. Multivariate Stochastic Variance Models, Review of Economic Studies 61, 247-264.

[27] Jacquier, E., N.G. Polson and P.E. Rossi, 1994. Bayesian Analysis of Stochastic Volatility Models, Journal of Business and Economic Statistics 12, 371-417.

[28] Jakubowski, A., 1986. Principle of conditioning in limit theorems for sums of random variables. The Annals of Probability, pp.902-915.

[29] Jenkinson, O., 2019. Ergodic optimization in dynamical systems. Ergodic Theory and Dynamical Systems, 39(10), pp.2593-2618.

[30] Kim, S., N. Shephard and S. Chib, 1998. Stochastic Volatility: Likelihood Inference and comparison with $ARCH$ Models, Review of Economic Studies 65, 361-393.

[31] Knight, J.L., Satchell, S.E. and Yu, J., 2002. Theory & methods: Estimation of the stochastic volatility model by the empirical characteristic function method. Australian & New Zealand Journal of Statistics, 44(3), pp.319-335.

[32] Koopman, S.J. and E. H. Uspensky, 2002. The Stochastic Volatility in Mean Model: Empirical Evidence from International Stock Markets, Journal of Applied Econometrics 17, 667-689.

[33] Ibragimov, I., 1975. Independent and stationary sequences of random variables. Wolters, Noordhoff Pub.

[34] Molchanov, I. and Molchanov, I.S., 2005. Theory of random sets (Vol. 19, No. 2). London: Springer.

[35] Monfardini, C., 1998. Estimating Stochastic Volatility Models through Indirect Inference, Econometrics Journal 1, 113-128.

[36] Nelson, D.B., 1990. ARCH models as diffusion approximations. Journal of econometrics, 45(1-2), pp.7-38.

[37] Nelson, D.B., 1991. Conditional heteroskedasticity in asset returns: A new approach. Econometrica: Journal of the econometric society, pp.347-370.

[38] Ortiz, E. and E . Arjona, 2001. Heteroskedastic Behavior of the Latin America Emerging Stock Markets, International Review of Financial Analysis 10, 287-305.

[39] Sandmann, G. and S. J. Koopman, 1998. Estimation of stochastic volatility models via Monte Carlo maximum likelihood, Journal of Econometrics 87, 271-301.

[40] Schwert, G.W., 1989. Why does Stock Market Volatility Change over Time?, Journal of Finance 44, 1115-1153.

[41] Sentana, E., 1995. Quadratic ARCH models. Review of Economic Studies 62, 639–661.

[42] Smith, A.A., 1993. Estimating Nonlinear Time-Series Models using Si1mulated Vector Autoregressions, Journal of Applied Econometrics 8, S63-S84.

[43] Straumann, D., 2006. Estimation in conditionally heteroscedastic time series models (Vol. 181). Springer Science & Business Media.

[44] Tai, C.S., 2000. Time Varying Market, Interest Rates and Exchange Rates Risk Premia in the US Commercial Bank Stock Returns, Journal of Multinational Financial Management 10 , 397-420.

[45] Tai, C.S. (2001), A Multivariate $GARCH$ in Mean Approach to Testing Uncovered Interest Parity: Evidence from Asia-Pacific Foreign Exchange Rates Markets, *The Quarterly Review of Economics and Finance* 41, 441-460.

[46] Taylor, S.J., 1986. Modelling Financial Time Series, J. Wiley, Chichester.

[47] Tsay, R.S., 2005. Analysis of financial time series. John Eiley and Sons.

[48] Wintenberger, O., 2013. Continuous invertibility and stable QML estimation of the EGARCH (1, 1) model. Scandinavian Journal of Statistics, 40(4), pp.846-867.

[49] van der Vaart, A. W., & Wellner, J. A., 1996. Weak convergence and empirical processes with applications to statistics. Springer-Verlag New York.

Figures

Fig. 1: Param. Set 1; $\delta_1 = 0$, $\delta_2 = 0.11$, $\omega = -0.1, \beta = 0.9, \rho = -0.8$, and $\sigma = 0.36$.

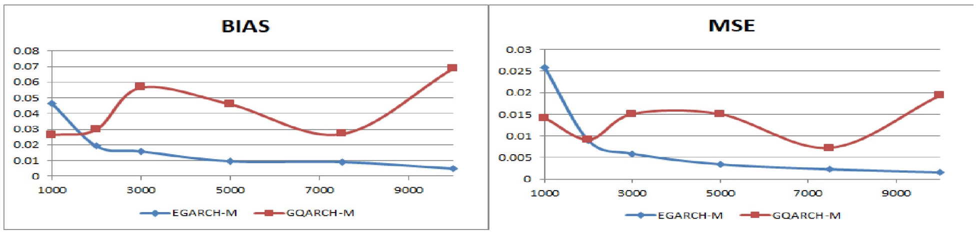


Fig. 2: Param. Set 2; $\delta_1 = 0$, $\delta_2 = 0.04$, $\omega = 0.0$, $\beta = 0.9$, $\rho = -0.95$ and $\sigma = 0.31$.

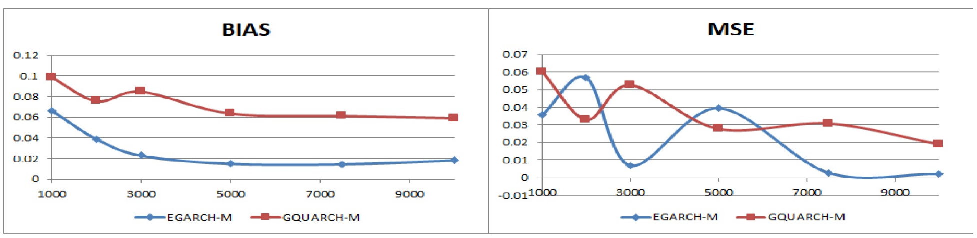
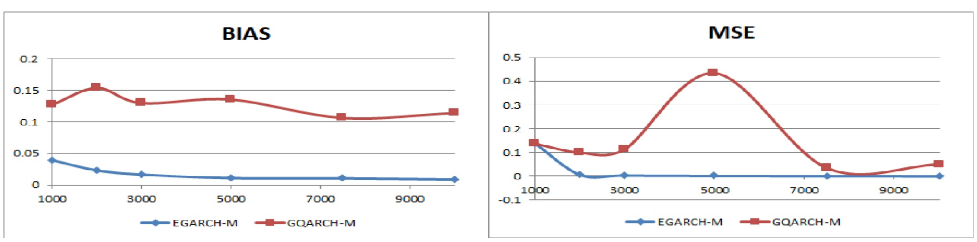


Fig. 3: Param. Set 3; $\delta_1 = 0.07$, $\delta_2 = 0.08$, $\omega = -0.1$, $\beta = 0.9$, $\rho = -0.9$ and $\sigma = 0.36$.



Supplement to "Gaussian Stochastic Volatility, Misspecified Volatility Filters and Indirect Inference Estimation"

Stelios Arvanitis and Antonis Demos

November 27, 2024

This an extended Technical Appendix for the main, above named, paper.

1 The SVM Model

The normal Autoregressive Stochastic Volatility in Mean models are given by:

$$y_t = \delta_1 + \delta_2 \sigma_t + z_t \exp\left(\frac{v_t}{2}\right) \quad where, \tag{1.1}$$

$$v_t = \omega + \beta \ln \sigma_{t-1}^2 + \sigma u_{t-1} \; and \tag{1.2}$$

$$\begin{pmatrix} \varepsilon_t \\ \eta_t \end{pmatrix} \overset{iid}{\sim} N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix} \right).$$

Let us call $\theta = (\delta_1, \delta_2, \omega, \beta, \sigma, \rho)^T$ the vector of the true parameters, and concentrate now to the estimation procedure.

2 Estimation

The Gallant and Tauchen (1996) estimator is defined as, for the vector of parameters θ,

$$\widehat{\theta} = \arg\min_{\theta} \left(\sum_{s=1}^{S} \sum_{t=1}^{T} \frac{\partial l_t \left(y_t^s(\theta), \widehat{\phi} \right)}{\partial \phi} \right)^T \Sigma \left(\sum_{s=1}^{S} \sum_{t=1}^{T} \frac{\partial l_t \left(y_t^s(\theta), \widehat{\phi} \right)}{\partial \phi} \right),$$

where $\{y_t^s(\theta)\}_{t=1}^T$ are the simulated paths of $y_t(\theta)$ via the simulated values of $(z_t^s, u_t^s)'$, and $\widehat{\phi} = \left(\widehat{\delta}_1, \widehat{\delta}_2, \widehat{\omega}, \widehat{\alpha}, \widehat{\gamma}, \widehat{\beta}\right)^T$ is the first step estimator, i.e. the maximiser of the approximate conditional Gaussian quasi log-likelihood function

$$l_T(\phi) = -\frac{T}{2}\ln 2\pi - \frac{1}{2}\sum_{t=1}^{T}\left(\ln h_t + \frac{\left(y_t - \delta_1 - \delta_2\sqrt{h_t}\right)^2}{h_t}\right) = \sum_{t=1}^{T} l_t(\phi) \quad (2.1)$$

where

$$l_t(\phi) = -\frac{1}{2}\ln 2\pi - \frac{1}{2}\left(\ln h_t + \varepsilon_t^2\right), \ and\ \varepsilon_t = \frac{y_t - \delta_1 - \delta_2\sqrt{h_t}}{\sqrt{h_t}}.$$

Now for $\circ = \{\delta_1, \delta_2, \omega, \alpha, \gamma, \beta\}$ we have that (see Demos and Kyriakopoulou (2013) for the EGARCH model) :

$$l_\circ = \frac{\partial l_T(\phi)}{\partial \circ} = -\frac{1}{2}\sum_{t=1}^{T}\frac{\partial \ln h_t}{\partial \circ} - \frac{1}{2}\sum_{t=1}^{T}\frac{\partial \varepsilon_t^2}{\partial \circ} = -\frac{1}{2}\sum_{t=1}^{T} h_{t;\circ} - \sum_{t=1}^{T}\varepsilon_t\frac{\partial \varepsilon_t}{\partial \circ},$$
$$where\ \ h_{t;\circ} = \frac{\partial \ln h_t}{\partial \circ}.$$

Now, we assume that the process h_t is given by either

$$\ln h_t = \omega + \gamma\varepsilon_{t-1} + \alpha\left|\varepsilon_{t-1}\right| + \beta \ln h_{t-1},$$
$$\varepsilon_{t-1} = \frac{y_{t-1} - \delta_1 - \delta_2\sqrt{h_{t-1}}}{\sqrt{h_{t-1}}}$$

for the EGARCH-M auxiliary or

$$h_t = \omega + \alpha\varepsilon_{t-1}^2 h_{t-1} + \gamma\varepsilon_{t-1}\sqrt{h_{t-1}} + \beta h_{t-1}$$

for GQARCH-M one. $h_{t;\circ} = \frac{\partial \ln h_t}{\partial \circ}$ and $\frac{\partial \varepsilon_t}{\partial \circ}$ can be found in the following subsections.

Notice that as the number of auxiliary parameters is the same as the number of parameters, six, Σ is irrelevant, at least asymptotically, and consequently it is set to the Identity matrix (see e.g. Gourieroux and Monfort (1996)). Consequently, $\widehat{\theta}$ is given as

$$\widehat{\theta} = arg\min_\theta\left(\sum_{s=1}^{S}\sum_{t=1}^{T}\frac{\partial l_t\left(y_t^s(\theta), \widehat{\phi}\right)}{\partial \phi}\right)^T\left(\sum_{s=1}^{S}\sum_{t=1}^{T}\frac{\partial l_t\left(y_t^s(\theta), \widehat{\phi}\right)}{\partial \phi}\right).$$

2.1 EGARCH(1,1)-M Auxiliary

The EGARCH(1,1)-M class of models of Nelson (1991)) is given by:

$$y_t = \delta_1 + \delta_2\sqrt{h_t} + \varepsilon_t\sqrt{h_t}, \;\; t = 1, \ldots, n, \quad where \quad \varepsilon_t \overset{iid}{\sim} (0,1) \quad and$$
$$\ln h_t = \omega + \gamma\varepsilon_{t-1} + \alpha\left|\varepsilon_{t-1}\right| + \beta \ln h_{t-1}$$

with

$$\ln h_0 = E\left(\ln h_t\right) = \frac{\omega + \alpha E\left|\epsilon\right|}{1-\beta},$$

for $|\beta| < 1$, and

$$\ln h_1 = \omega + \gamma\varepsilon_0 + \alpha\left|\varepsilon_0\right| + \beta\ln h_0 = \omega + \beta \ln h_0 = \frac{\omega + \beta\alpha E\left|\varepsilon\right|}{1-\beta}$$

assuming that $\varepsilon_0 = 0$.

The Quasi normal log-likelihood is given by:

$$l\left(\phi\right) = -\frac{T}{2}\ln 2\pi - \frac{1}{2}\sum_{t=1}^{T}\left(\ln h_t + \frac{\left(y_t - \delta_1 - \delta_2\sqrt{h_t}\right)^2}{h_t}\right) = -\frac{T}{2}\ln 2\pi - \frac{1}{2}\sum_{t=1}^{T}\left(\ln h_t + \varepsilon_t^2\right).$$

Now for $\circ = \{\delta_1, \delta_2, \omega, \alpha, \gamma, \beta\}$ we have that (see Demos and Kyriakopoulou (2013) for the EGARCH model) :

$$l_\circ = \frac{\partial l}{\partial \circ} = -\frac{1}{2}\sum_{t=1}^{T}\frac{\partial \ln h_t}{\partial \circ} - \frac{1}{2}\sum_{t=1}^{T}\frac{\partial \varepsilon_t^2}{\partial \circ} = -\frac{1}{2}\sum_{t=1}^{T} h_{t;\circ} - \sum_{t=1}^{T}\varepsilon_t\frac{\partial \varepsilon_t}{\partial \circ},$$
$$where \quad h_{t;\circ} = \frac{\partial \ln h_t}{\partial \circ}$$

where

$$\frac{\partial \varepsilon_t}{\partial \delta_1} = \frac{\partial\left(y_t - \delta_1 - \delta_2 e^{\frac{1}{2}\ln h_t}\right)e^{-\frac{1}{2}\ln h_t}}{\partial \delta_1} = -\frac{1}{2}\varepsilon_t h_{t;\delta_1} - \frac{1}{2}\delta_2 h_{t;\delta_1} - e^{-\frac{1}{2}\ln h_t} = -\frac{1}{2}\left(\varepsilon_t + \delta_2\right)h_{t;\delta_1} - \frac{1}{\sqrt{h_t}},$$

$$\frac{\partial \varepsilon_t}{\partial \delta_2} = \frac{\partial\left(y_t - \delta_1 - \delta_2 e^{\frac{1}{2}\ln h_t}\right)e^{-\frac{1}{2}\ln h_t}}{\partial \delta_2} = -\frac{1}{2}\left(y_t - \delta_1 - \delta_2 e^{\frac{1}{2}\ln h_t}\right)e^{-\frac{1}{2}\ln h_t}h_{t;\delta_2} - \frac{1}{2}\delta_2 h_{t;\delta_2} - 1$$
$$= -\frac{1}{2}\left(\varepsilon_t + \delta_2\right)h_{t;\delta_2} - 1,$$

and for $@ = \{\omega, \alpha, \gamma, \beta\}$, the conditional variance parameters

$$\frac{\partial \varepsilon_t}{\partial @} = \frac{\partial \left(y_t - \delta_1 - \delta_2 e^{\frac{1}{2}\ln h_t} \right) e^{-\frac{1}{2}\ln h_t}}{\partial @} = \frac{\partial \left(y_t - \delta_1 \right) e^{-\frac{1}{2}\ln h_t} - \delta_2}{\partial @}$$
$$= -\frac{1}{2}\left(y_t - \delta_1 \right) e^{-\frac{1}{2}\ln h_t} h_{t;@} = -\frac{1}{2}\left(\varepsilon_t + \delta_2 \right) h_{t;@}.$$

Now the derivative of the conditional variance with respect to the parameters are given:

$$h_{t;\delta_1} = \frac{\partial \left(\omega + \gamma \varepsilon_{t-1} + \alpha \left| \varepsilon_{t-1} \right| + \beta \ln h_{t-1} \right)}{\partial \delta_1}$$
$$= \frac{\partial \left(\gamma \varepsilon_{t-1} + \alpha \left(I\left(\varepsilon_{t-1} \geq 0 \right) - I\left(\varepsilon_{t-1} < 0 \right) \right) \varepsilon_{t-1} + \beta \ln h_{t-1} \right)}{\partial \delta_1}$$
$$= \left(\gamma + \alpha \left(I\left(\varepsilon_{t-1} \geq 0 \right) - I\left(\varepsilon_{t-1} < 0 \right) \right) \right) \frac{\partial \left(\varepsilon_{t-1} \right)}{\partial \delta_1} + \beta h_{t-1;\delta_1} =$$
$$= \left(\gamma + \alpha \left(I\left(\varepsilon_{t-1} \geq 0 \right) - I\left(\varepsilon_{t-1} < 0 \right) \right) \right) \left(-\frac{1}{2}\left(\varepsilon_{t-1} + \delta_2 \right) h_{t-1;\delta_1} - \frac{1}{\sqrt{h_{t-1}}} \right) + \beta h_{t-1;\delta_1}$$
$$= -\left\{ \gamma + \alpha \left[I\left(\varepsilon_{t-1} \geq 0 \right) - I\left(\varepsilon_{t-1} < 0 \right) \right] \right\} \frac{1}{\sqrt{h_{t-1}}}$$
$$+ \left[\beta - \frac{1}{2}\left(\gamma \varepsilon_{t-1} + \alpha \left| \varepsilon_{t-1} \right| \right) - \frac{1}{2}\delta_2 \left[\gamma + \alpha \left(I\left(\varepsilon_{t-1} \geq 0 \right) - I\left(\varepsilon_{t-1} < 0 \right) \right) \right] \right] h_{t-1;\delta_1}$$

with

$$h_{1;\delta_1} = 0,$$

as

$$\ln h_1 = \frac{\omega + \beta \alpha E \left| \varepsilon \right|}{1 - \beta}.$$

Now

$$
\begin{aligned}
h_{t;\delta_2} &= \frac{\partial \left(\omega + \gamma\varepsilon_{t-1} + \alpha\left|\varepsilon_{t-1}\right| + \beta \ln h_{t-1}\right)}{\partial \delta_2} \\
&= \theta \frac{\partial \varepsilon_{t-1}}{\partial \delta_2} + \alpha\left[I\left(\varepsilon_{t-1} \geq 0\right) - I\left(\varepsilon_{t-1} < 0\right)\right] \frac{\partial \varepsilon_{t-1}}{\partial \delta_2} + \beta h_{t-1;\delta_2} \\
&= \left\{\gamma + \alpha\left[I\left(\varepsilon_{t-1} \geq 0\right) - I\left(\varepsilon_{t-1} < 0\right)\right]\right\}\left(-\frac{1}{2}\left(\varepsilon_{t-1} + \delta_2\right) h_{t-1;\varphi} - 1\right) + \beta h_{t-1;\delta_2} \\
&= -\left[\gamma + \alpha I\left(\varepsilon_{t-1} \geq 0\right) - \alpha I\left(\varepsilon_{t-1} < 0\right)\right] \\
&+ \left[\beta - \frac{1}{2}\gamma\varepsilon_{t-1} - \frac{1}{2}\alpha\left|\varepsilon_{t-1}\right| - \frac{1}{2}\delta_2\left[\gamma + \alpha I\left(\varepsilon_{t-1} \geq 0\right) - \alpha I\left(\varepsilon_{t-1} < 0\right)\right]\right] h_{t-1;\delta_2}
\end{aligned}
$$

with

$$
h_{1;\delta_2} = 0.
$$

$$
\begin{aligned}
h_{t;\omega} &= 1 + \gamma \frac{\partial \varepsilon_{t-1}}{\partial \omega} + \alpha \frac{\partial \varepsilon_{t-1}}{\partial \omega}\left[I\left(\varepsilon_{t-1} \geq 0\right) - I\left(\varepsilon_{t-1} < 0\right)\right] + \beta h_{t-1;\omega} \\
&= 1 + \gamma\left(-\frac{1}{2}\left(\varepsilon_{t-1} + \delta_2\right) h_{t-1;\omega}\right) \\
&+ \left[\alpha I\left(\varepsilon_{t-1} \geq 0\right) - \alpha I\left(\varepsilon_{t-1} < 0\right)\right]\left(-\frac{1}{2}\left(\varepsilon_{t-1} + \delta_2\right) h_{t-1;\omega}\right) + \beta h_{t-1;\omega} \\
&= 1 + \left(\beta - \frac{1}{2}\gamma\varepsilon_{t-1} - \frac{1}{2}\alpha\left|\varepsilon_{t-1}\right| - \frac{1}{2}\delta_2\left[\gamma + \alpha I\left(\varepsilon_{t-1} \geq 0\right) - \alpha I\left(\varepsilon_{t-1} < 0\right)\right]\right) h_{t-1;\omega}
\end{aligned}
$$

$$
h_{1;\omega} = \frac{1}{1-\beta}.
$$

as

$$
\ln h_1 = \frac{\omega + \beta\alpha E\left|\varepsilon\right|}{1-\beta}.
$$

Now for $\circ = \{\alpha\}$ the derivatives are:

$$
\begin{aligned}
h_{t;\alpha} &= \frac{\partial\left(\omega + \gamma\varepsilon_{t-1} + \alpha\left|\varepsilon_{t-1}\right| + \beta\ln h_{t-1}\right)}{\partial\alpha} \\
&= \left|\varepsilon_{t-1}\right| + \left[\gamma + \alpha I\left(\varepsilon_{t-1}\geq 0\right) - \alpha I\left(\varepsilon_{t-1}<0\right)\right]\frac{\partial\left(\varepsilon_{t-1}\right)}{\partial\alpha} + \beta h_{t-1;\alpha} \\
&= \left|\varepsilon_{t-1}\right| + \left[\gamma + \alpha I\left(\varepsilon_{t-1}\geq 0\right) - \alpha I\left(\varepsilon_{t-1}<0\right)\right]\left(-\frac{1}{2}\left(\varepsilon_{t-1}+\delta_2\right)h_{t-1;\alpha}\right) + \beta h_{t-1;\alpha} \\
&= \left|\varepsilon_{t-1}\right| + \left\{\beta - \frac{1}{2}\gamma\varepsilon_{t-1} - \frac{1}{2}\alpha\left|\varepsilon_{t-1}\right| - \frac{1}{2}\delta_2\left[\gamma + \alpha I\left(\varepsilon_{t-1}\geq 0\right) - \alpha I\left(\varepsilon_{t-1}<0\right)\right]\right\}h_{t-1;\alpha}
\end{aligned}
$$

with

$$
h_{1;\alpha} = \frac{\beta E\left|\varepsilon\right|}{1-\beta}.
$$

Now for $\circ = \{\gamma\}$ the derivatives are:

$$
\begin{aligned}
h_{t;\gamma} &= \frac{\partial\left(\omega + \gamma\varepsilon_{t-1} + \alpha\left|\varepsilon_{t-1}\right| + \beta\ln h_{t-1}\right)}{\partial\theta} \\
&= \varepsilon_{t-1} + \left[\gamma + \alpha\left(I\left(\varepsilon_{t-1}\geq 0\right) - I\left(\varepsilon_{t-1}<0\right)\right)\right]\frac{\partial\left(\varepsilon_{t-1}\right)}{\partial\gamma} + \beta h_{t-1;\gamma} \\
&= \varepsilon_{t-1} - \frac{1}{2}\left(\varepsilon_{t-1}+\delta_2\right)h_{t-1;\gamma}\left[\gamma + \alpha\left(I\left(\varepsilon_{t-1}\geq 0\right) - I\left(\varepsilon_{t-1}<0\right)\right)\right] + \beta h_{t-1;\gamma} \\
&= \varepsilon_{t-1} + \left\{\beta - \frac{1}{2}\left(\gamma\varepsilon_{t-1} + \alpha\left|\varepsilon_{t-1}\right|\right) - \frac{1}{2}\delta_2\left[\gamma + \alpha I\left(\varepsilon_{t-1}\geq 0\right) - \alpha I\left(\varepsilon_{t-1}<0\right)\right]\right\}h_{t-1;\gamma}
\end{aligned}
$$

with

$$
h_{1;\gamma} = 0.
$$

Now for $\circ = \{\beta\}$ the derivatives are:

$$h_{t;\beta} = \frac{\partial\left(\omega + \gamma\varepsilon_{t-1} + \alpha\left|\varepsilon_{t-1}\right| + \beta\ln h_{t-1}\right)}{\partial\beta}$$

$$= \theta\frac{\partial\left(\varepsilon_{t-1}\right)}{\partial\beta} + \left(\alpha I\left(\varepsilon_{t-1} \geq 0\right) - \alpha I\left(\varepsilon_{t-1} < 0\right)\right)\frac{\partial\left(\varepsilon_{t-1}\right)}{\partial\beta} + \ln h_{t-1} + \beta h_{t-1;\beta}$$

$$= \ln h_{t-1} + \gamma\left(-\frac{1}{2}\left(\varepsilon_{t-1} + \delta_2\right)h_{t-1;\beta}\right)$$

$$+\left(\alpha I\left(\varepsilon_{t-1} \geq 0\right) - \alpha I\left(\varepsilon_{t-1} < 0\right)\right)\left(-\frac{1}{2}\left(\varepsilon_{t-1} + \delta_2\right)h_{t-1;\beta}\right) + \beta h_{t-1;\beta}$$

$$= \ln h_{t-1} + \left[\beta - \frac{1}{2}\gamma\varepsilon_{t-1} - \frac{1}{2}\alpha\left|\varepsilon_{t-1}\right| - \frac{1}{2}\delta_2\left(\gamma + \alpha I\left(\varepsilon_{t-1} \geq 0\right) - \alpha I\left(\varepsilon_{t-1} < 0\right)\right)\right]h_{t-1;\beta}$$

with

$$h_{1;\beta} = \frac{\omega + \alpha E\left|\varepsilon\right|}{\left(1-\beta\right)^2}.$$

2.2 The GQARCH(1,1)-M Auxiliary

The GQARCH(1,1) process of Sentana (1995) is given by:

$$y_t = \delta_1 + \delta_2\sqrt{h_t} + \sqrt{h_t}\varepsilon_t, \quad \varepsilon_t \overset{iid}{\sim} N(0,1)$$
$$h_t = \omega + \alpha\varepsilon_{t-1}^2 h_{t-1} + \gamma\varepsilon_{t-1}\sqrt{h_{t-1}} + \beta h_{t-1}$$

with

$$h_0 = \frac{\omega}{1-(\alpha+\beta)} \;\; and \;\; h_1 = \frac{\omega\left(1-\alpha\right)}{1-(\alpha+\beta)}.$$

Then for $\circ = \{\delta_1, \delta_2, \omega, \gamma, \alpha, \beta\}$ we have that :

$$l_\circ = \frac{\partial l}{\partial\circ} = -\frac{1}{2}\sum_{t=1}^{T}\frac{\partial\ln h_t}{\partial\circ} - \frac{1}{2}\sum_{t=1}^{T}\frac{\partial\varepsilon_t^2}{\partial\circ} = -\frac{1}{2}\sum_{t=1}^{T}h_{t;\circ} - \sum_{t=1}^{T}\varepsilon_t\frac{\partial\varepsilon_t}{\partial\circ},$$

as with the EGARCH-M auxiliary, where

$$\frac{\partial \varepsilon_t}{\partial \delta_1} = \frac{\partial \left(y_t - \delta_1 - \delta_2 \sqrt{h_t}\right)\left(\sqrt{h_t}\right)^{-1}}{\partial \delta_1} = y_t \frac{\partial \left(e^{-\frac{1}{2}\ln h_t}\right)}{\partial \delta_1} - \frac{\partial \left(\delta_1 e^{-\frac{1}{2}\ln h_t}\right)}{\partial \delta_1}$$

$$= -\frac{1}{2} y_t e^{-\frac{1}{2}\ln h_t} h_{t;\delta_1} - e^{-\frac{1}{2}\ln h_t} + \frac{1}{2}\delta_1 e^{-\frac{1}{2}\ln h_t} h_{t;\delta_1} = -\frac{1}{2}\left(y_t - \delta_1\right)\frac{1}{\sqrt{h_t}} h_{t;\delta_1} - \frac{1}{\sqrt{h_t}}$$

$$= -\frac{1}{2}\left(\varepsilon_t + \delta_2\right) h_{t;\delta_1} - \frac{1}{\sqrt{h_t}},$$

$$\frac{\partial \varepsilon_t}{\partial \delta_2} = \frac{\partial \left(y_t - \delta_1 - \delta_2 \sqrt{h_t}\right)\left(\sqrt{h_t}\right)^{-1}}{\partial \delta_2} = \frac{\partial \left(y_t - \delta_1\right) e^{-\frac{1}{2}\ln h_t} - \delta_2}{\partial \delta_2}$$

$$= -\frac{1}{2}\left(y_t - \delta_1\right)\frac{1}{\sqrt{h_t}} h_{t;\delta_2} - 1 = -\frac{1}{2}\left(\varepsilon_t + \delta_2\right) h_{t;\delta_2} - 1,$$

and for $\circ = \{\omega, \gamma, \alpha, \beta\}$ we have that :

$$\frac{\partial \varepsilon_t}{\partial \circ} = \frac{\partial \left(y_t - \delta_1 - \delta_2 \sqrt{h_t}\right)\left(\sqrt{h_t}\right)^{-1}}{\partial \circ} = \frac{\partial \left(y_t - \delta_1\right)\left(\sqrt{h_t}\right)^{-1}}{\partial \circ}$$

$$= -\frac{1}{2}\left(y_t - \delta_1\right)\frac{1}{\sqrt{h_t}} h_{t;\circ} = -\frac{1}{2}\left(\varepsilon_t + \delta_2\right) h_{t;\circ}. OK$$

The conditional variance derivatives, for $\circ = \{\delta_1, \delta_2, \omega, \gamma, \alpha, \beta\}$, are:

$$h_{t;\circ} = \frac{\partial \ln h_t}{\partial \circ} = \frac{1}{h_t}\frac{\partial h_t}{\partial \circ} = \frac{1}{h_t}\frac{\partial \left(\omega + \alpha \varepsilon_{t-1}^2 h_{t-1}^2 + \gamma \varepsilon_{t-1}\sqrt{h_{t-1}} + \beta h_{t-1}^2\right)}{\partial \circ}$$

$$= \frac{1}{h_t}\left[\frac{\partial \omega}{\partial \circ} + \frac{\partial \alpha}{\partial \circ}\varepsilon_{t-1}^2 h_{t-1}^2 + 2\alpha \varepsilon_{t-1}\frac{\partial \left(\varepsilon_{t-1}\right)}{\partial \circ} h_{t-1} + \alpha \varepsilon_{t-1}^2 \frac{\partial h_{t-1}}{\partial \circ}\right]$$

$$+\frac{1}{h_t}\left[\gamma \frac{\partial \left(\varepsilon_{t-1}\right)}{\partial \circ}\sqrt{h_{t-1}} + \gamma \varepsilon_{t-1}\frac{\partial \sqrt{h_{t-1}}}{\partial \circ} + \frac{\partial \gamma}{\partial \circ}\varepsilon_{t-1}\sqrt{h_{t-1}} + \beta \frac{\partial h_{t-1}}{\partial \circ} + \frac{\partial \beta}{\partial \circ} h_{t-1}^2\right]$$

$$= \frac{1}{h_t}\left[\frac{\partial \omega}{\partial \circ} + \frac{\partial \alpha}{\partial \circ}\varepsilon_{t-1}^2 h_{t-1}^2 + 2\alpha \varepsilon_{t-1}\frac{\partial \left(\varepsilon_{t-1}\right)}{\partial \circ} h_{t-1} + \alpha \varepsilon_{t-1}^2 h_{t-1} h_{t-1;\circ}\right]$$

$$+\frac{1}{h_t}\left[\gamma \frac{\partial \left(\varepsilon_{t-1}\right)}{\partial \circ}\sqrt{h_{t-1}} + \frac{1}{2}\gamma \varepsilon_{t-1}\sqrt{h_{t-1}} h_{t-1;\circ} + \frac{\partial \gamma}{\partial \circ}\varepsilon_{t-1}\sqrt{h_{t-1}} + \beta h_{t-1} h_{t-1;\circ} + \frac{\partial \beta}{\partial \circ} h_{t-1}^2\right]$$

It follows that for $\circ = \delta_1$ we get

$$h_{t;\delta_1} = \frac{1}{h_t}\left[\beta h_{t-1} h_{t-1;\delta_1} - \left(2\alpha \varepsilon_{t-1}\sqrt{h_{t-1}} + \gamma\right)\left(\frac{1}{2}\delta_2 \sqrt{h_{t-1}} h_{t-1;\delta_1} + 1\right)\right],$$

with

$$h_{1;\delta_1} = 0.$$

For $\circ = \delta_2$

$$h_{t;\delta_2} = \frac{1}{h_t}\left[\beta h_{t-1} h_{t-1;\delta_2} - \left(2\alpha\varepsilon_{t-1}h_{t-1} + \gamma\sqrt{h_{t-1}}\right)\left(\frac{1}{2}\delta_2 h_{t-1;\delta_2} + 1\right)\right]$$

with

$$h_{1;\delta_2} = 0.$$

Now for $\circ = \omega$ the derivatives are:

$$h_{t;\omega} = \frac{1}{h_t}\left\{1 + \left[\beta h_{t-1} - \frac{1}{2}\delta_2\left(2\alpha\varepsilon_{t-1}h_{t-1} + \gamma\sqrt{h_{t-1}}\right)\right] h_{t-1;\omega}\right\}$$

with

$$h_{1;\omega} = \frac{1}{\omega}.$$

For $\circ = \alpha$ the derivative is:

$$h_{t;\alpha} = \frac{1}{h_t}\left[\varepsilon_{t-1}^2 h_{t-1}^2 + \left(\beta h_{t-1} - \alpha\varepsilon_{t-1}\delta_2 h_{t-1} - \frac{1}{2}\gamma\delta_2\sqrt{h_{t-1}}\right) h_{t-1;\alpha}\right]$$

with

$$h_{1;\alpha} = \frac{\beta}{\left(1 - (\alpha + \beta)\right)(1 - \alpha)}.$$

For $\circ = \gamma$ the derivatives are:

$$h_{t;\gamma} = \frac{1}{h_t}\left[\varepsilon_{t-1}\sqrt{h_{t-1}} + \left(\beta h_{t-1} - \alpha\varepsilon_{t-1}\delta_2 h_{t-1} - \frac{1}{2}\gamma\delta_2\sqrt{h_{t-1}}\right) h_{t-1;\gamma}\right]$$

with

$$h_{1;\gamma} = 0,$$

and for $\circ = \beta$ the derivatives are:

$$h_{t;\beta} = \frac{1}{h_t}\left[h_{t-1}^2 + \left(\beta h_{t-1} - \alpha\varepsilon_{t-1}\delta_2 h_{t-1} - \frac{1}{2}\gamma\delta_2\sqrt{h_{t-1}}\right) h_{t-1;\beta}\right]$$

with

$$h_{1;\beta} == \frac{1}{1 - (\alpha + \beta)}.$$

3 Monte Carlo Simulations

To compare the properties in terms of bias and Mean Squared Error for the two estimators we perform two Monte Carlo exercises. One where the mean parameters are set to zero and not estimated, where the EGARCH and GQARCH processes are employed as auxiliary ones, and one where the full model SV(1)-M model is simulated.

3.1 EGARCH and GQARCH

The results in Jacquier, Polson and Rossi (1994) and in Calzolari, Fiorentini and Sentana (2004) imply that the most important determinant of the performance of the different estimators is the unconditional coefficient of variation of the unobserved volatility level σ_t^2, say CV, where

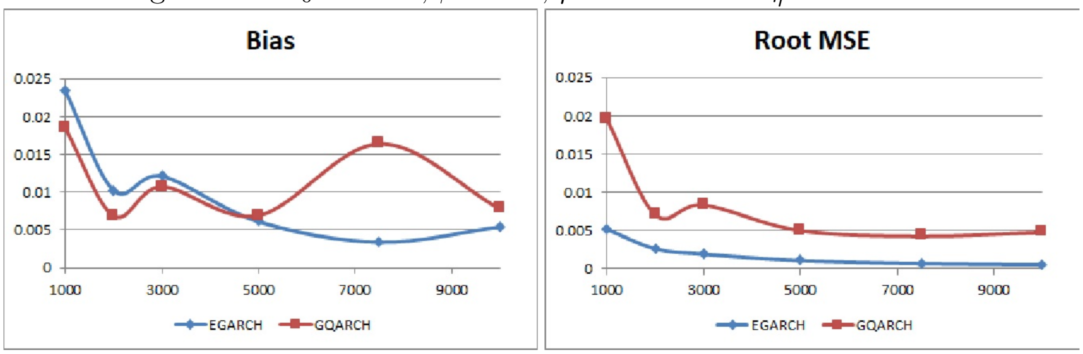
$$CV^2 = \frac{Var\left(\sigma_t^2\right)}{E^2\left(\sigma_t^2\right)} = \exp\left(\frac{\sigma_\eta^2}{1-\psi^2}\right) - 1.$$

Notice, that when CV^2 is low, the observed process is close to Gaussian white noise, and consequently the estimation of the stochastic volatility parameters is difficult. Furthermore, CV is independent of ρ.

The simulated paths were generated employing 3 sets of parameter values. For the the first one we set $\omega = -0.1$,$\beta = 0.9$,$\rho = -0.8$ and$\sigma = 0.3629$ getting $CV^2 = 1$, for the second one we chose $\omega = 0.0$, $\beta = 0.9$,$\rho = -0.95$ and $\sigma = 0.31623$ with $CV^2 = 0.693$ as in Monfardini (1998), and for the third one we chose $\omega = -0.736$, $\beta = 0.9$,$\rho = -0.95$ and $\sigma = 0.363$ with $CV^2 = 1.0$ as in Jacquier et al. (1994). Notice that the third set of parameters values has been employed by among others Andersen and Sorensen (1996), as well. However, the previous articles are dealing with symmetric SV models, i.e. $\rho = 0$.

In all simulations we choose $S = 200$ for $T = 1000$, 2000 and 3000, and $S = 150$ for $T = 5000$, 7500 and 10000, and perform 500 Monte Carlo simulations for each score generator. The choice of S is based mainly in time considerations, as higher value of S results in smaller asymptotic variance of the estimators and consequently increases the stability of the estimation but increases the time needed for the program to converge. These values of S are far smaller than the ones employed in the application with real data section. The norm of the estimated biases of the 3 parameters sets and

Figure 3.1: $\omega_0 = -0.1$, $\beta = 0.9$, $\rho = -0.8$ and $\sigma_\eta = 0.3629$.



the root MSEs (the Frobenius norm of the MSE matrix) are presented and discussed in the main article, where we also discuss the convergence failures of the optimization procedure.

In the following Figure we present the bias and root MSE for the first set of parameters. The norm bias of the GQARCH auxiliary estimator is smaller for T=1000, T=2000 and T=3000, whereas the EGARCH auxiliary has smaller bias for T=5000, T=7500 and T=10000. The root MSE of the EGARCH auxiliary is smaller for all Ts.

Monfardini (1998) employed an Indirect Inference estimator using as first step estimators AR and ARMA models, capitalizing the autocorrelation function of the squared residuals of a symmetric SV(1) model. In Tables 1 and 2 we present the biases and the root MSE's of the two estimators of Monfardini (1998) together with ours. Of course in our case we estimate, apart from the presented parameters, the dynamic asymmetry parameter ρ, as well. For the two sample sizes considered in that Monfardini (1998), $T = 1000$ and $T = 2000$ it is obvious that the EGARCH score generator is less biased and has smaller root MSE.

The bias and root MSE for the second set of parameters are presented in the following Figure. Either in terms of bias or of root MSE the EGARCH auxiliary has better properties than the GQARCH one.

Let us turn our attention to the third parameter set. From the following Figure it is obvious that the EGARCH score generator is uniformly, over all examined sample sizes, superior to the GQARCH one in terms of bias and root MSE.

For this parameter set it is fruitful to compare our results with the ones in articles where symmetric SV(1) models have been estimated. In Table 3

Figure 3.2: $\omega_0 = 0.0$, $\beta = 0.9$, $\rho = -0.95$ and $\sigma_\eta = 0.31623$.

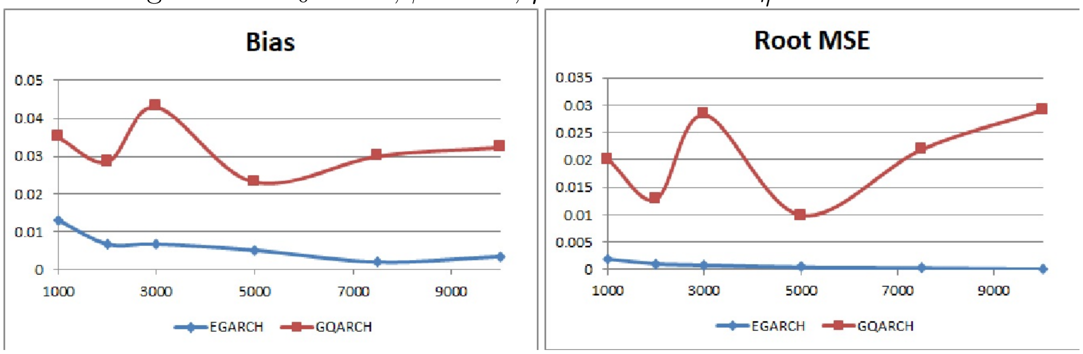


Figure 3.3: $\omega = -0.736$, $\beta = 0.9$, $\rho = -0.95$ and $\sigma_\eta = 0.363$.

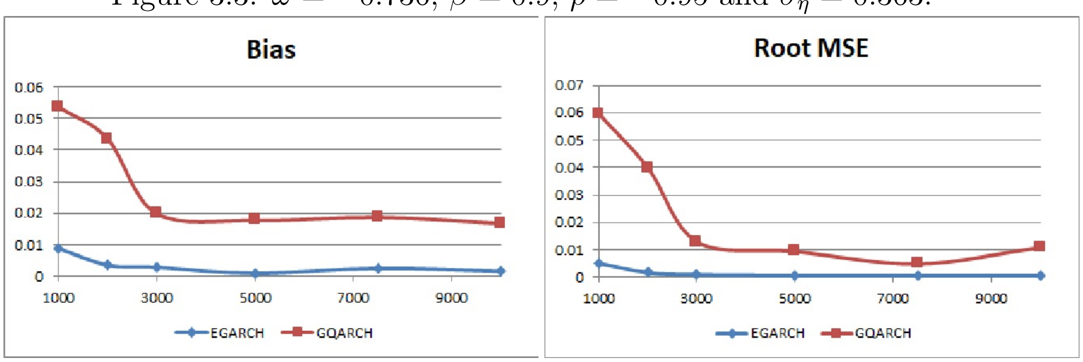


Table 1: Biases and Root MSE's (in parenthesis) of the two II Estimators in Monfardini (1998), and EGARCH and GQARCH score generators T=1000

$Method/param.$	$\omega_0 = 0.0$	$\psi_0 = 0.9$	$\sigma_{\eta 0} = 0.31623$
$Ind.Inf.1 - AR$	0.0014 (0.0197)	−0.0314 (0.1036)	0.0170 (0.1557)
$Ind.Inf.2 - ARMA$	−0.0055 (0.0239)	−0.0363 (0.1013)	0.0496 (0.160)
QML	−	−0.0327 (0.1047)	0.0319 (0.1577)
$BAYES$	−	−0.0213 (0.0540)	0.0194 (0.0941)
SEM	−	−0.0010 (0.0400)	−0.0129 (0.0570)
$GQARCH$	0.0351 (0.12098)	0.0185 (0.0518)	−0.0289 (0.1116)
$EGARCH$	**0.0002 (0.0063)**	**0.0010 (0.0143)**	**−0.0004 (0.0357)**

Monfardini (1998)

Table 2: Biases and Root MSE's (in parenthesis) of the 2 II Estimators in Monfardini (1998), and EGARCH and GQARCH score generators T=2000

$Method/param.$	$\omega_0 = 0.0$	$\psi_0 = 0.9$	$\sigma_{\eta 0} = 0.31623$
$Ind.Inf.1 - AR$	0.0006 (0.0108)	−0.0124 (0.0598)	0.0029 (0.1090)
$Ind.Inf.2 - ARMA$	0.0021 (0.0112)	−0.0133 (0.0600)	0.0194 (0.1104)
SEM	−	−0.0009 (0.02407)	−0.0168 (0.0438)
$GQARCH$	0.0133 (0.0712)	0.0027 (0.0415)	−0.0241 (0.1059)
$EGARCH$	**0.0003 (0.0042)**	**0.0002 (0.0103)**	**0.0012 (0.0243)**

we compare, in terms of bias and root MSE, various estimators for only the three parameters, i.e. ω, β and σ_η.

It seems that the EGARCH auxiliary II estimation is performing quite well, at least for the β and σ_η parameters. Notice that in our case the estimated biases and root MSEs are the ones when at the same time we estimated the ρ parameter, i.e. our third parameter set.

To further check our routines we repeated the Monte Carlo experiment of Harvey and Shephard (1996), and Yu (2005). For T=1000 and T=3000 the two Indirect estimators we consider are performing quite well, whereas for T=6000 the EGARCH auxiliary outperforms the other two estimators.

Finally, we repeat the simulations in Jacquier, Polson and Rossi (2004) (JPR04). However, notice that in JPR04 a fat tailed distribution is chosen, and the tail thickness is estimated, as opposed to our normal one. Again the two considered II estimators are performing quite well.

Table 3: Bias and Root MSE (in parenthesis) of MM, QML, Bayes, GQARCH and EGARCH score Generators, T=2000

Method/param.	$\omega = -0.736$	$\beta = 0.9$	$\sigma_\eta = 0.363$
$MM^{1)}$	0.124 (0.420)	0.020 (0.060)	0.053 (0.100)
$QML^{1)}$	0.117 (0.460)	0.020 (0.060)	−0.020 (0.110)
$Bayes^{1)}$	−0.026 (0.150)	−0.004 (0.020)	−0.004 (0.034)
$QML^{2)}$	**0.000 (0.010)**	−0.012 (0.050)	0.018 (0.100)
$MCL^{2)}$	−0.009 **(0.010)**	0.013 (0.020)	−0.046 (0.030)
$GMM^{3)}$	0.151 (0.311)	0.020 (0.043)	−0.086 (0.117)
$EMM^{4)}$	−0.057 (0.224)	−0.007 (0.030)	−0.004 (0.049)
GQARCH	0.003 (0.110)	−0.033 (0.057)	−0.026 (0.191)
EGARCH	−0.001 (0.038)	**0.000 (0.005)**	**−0.003 (0.022)**

1) Jacquier, Polson and Rossi (1994) Table 9; 2) Sandmann and Koopman (1998) Table 3; 3) Andersen and Sorensen (1996) Table 5; 4) Andersen, Chung and Sorensen (1999) Table 5

Table 4: Bias and Root MSE (in parenthesis) of QML, GQARCH and EGARCH score Generators

Method/param.	$\beta = 0.975$	$\rho = -0.9$	$ln\left(\sigma_\eta^2\right) = -4.605$
		$T = 1000$	
QML^*	−0.007 (0.034)	**−0.009** (0.132)	0.045 (0.708)
GQARCH	−0.006 (0.000)	0.024 (0.022)	0.135 (0.783)
EGARCH	**−0.002 (0.009)**	−0.029 **(0.075)**	**−0.025 (0.390)**
		$T = 3000$	
QML^*	−0.001 (0.007)	−0.011 (0.079)	−0.012 (0.353)
$MCMC^{**}$	0.002 (0.005)	0.019 **(0.045)**	**−0.010 (0.209)**
GQARCH	0.000 (0.005)	−0.010 (0.480)	−0.048 (0.280)
EGARCH	**0.000 (0.004)**	**−0.009** (0.046)	−0.010 (0.223)
		$T = 6000$	
QML^*	0.000 (0.005)	−0.007 (0.058)	−0.007 (0.249)
GQARCH	−0.001 (0.004)	0.010 (0.033)	0.051 (0.233)
EGARCH	**0.000 (0.003)**	**−0.004 (0.032)**	**−0.002 (0.153)**

* Harvey and Shephard (1996) Table 1; ** Yu (2005) Table 5

Table 5: Bias and Root MSE (in parenthesis) of Bayes, GQARCH and EGARCH score Generators

Method/param.	$\beta = 0.95$	$\rho = -0.6$	$\sigma_\eta = 0.26$
		$T = 1000$	
*Bayes**	0.010 (0.025)	-0.180 (0.190)	-0.010 **(0.039)**
GQARCH	**0.000 (0.002)**	-0.034 (0.122)	-0.040 (0.092)
EGARCH	0.001 (0.012)	**0.024 (0.107)**	**0.002** (0.048)

* JPR04 Table 1

3.2 EGARCH-M and GQARCH-M

For the mean specifications, we set $(\delta_1, \delta_2) = (0.0, 0.111)$ for the first set, $(\delta_1, \delta_2) = (0, 0.044)$ for the second one, and $(\delta_1, \delta_2) = (0.07, 0.08)$ for the third one. The norm of the estimated biases of the 3 parameters sets and the root MSEs (the Frobenius norm of the MSE matrix) are presented and discussed in the main article, where we also discuss the convergence failures of the optimization procedure.

In Appendix we present all biases and root MSEs of all parameters, for all three parameter sets. In terms of estimated biases, in almost all cases, the auxiliary EGARCH estimates are closer to the true ones. The same applies for the root MSEs, i.e. in almost all cases the ones of the EHARCH-M auxiliary model are smaller than the equivalent for the GQARCH ones.

4 Application to International Markets.

We apply the developed methods of estimation to weekly excess returns of four Indecies of International Markets, i.e. the S&P, the FTSE, the DAX and the Nikkey. In this way we estimate two aglosaxon markets, a European and an Asian one. In the following Table we present some descriptive statistics for the 4 indecies, along with the period of estimation and the number of observations. It is obvious that the standard deviation of returns is almost 22 times higher than the average return. Further, in all markets the skewness and kurtosis coefficients are far from the corresponding of the normal distribution ones. The asymptotic confidence interval for the autocorrelations is $(-0.041, 0.041)$ for the 3 markets and $(-0.049, 0.049)$ for FTSE, indicating that, apart from Nikkey, either the 1st or the second order autocorrelations are significant. However, it is known that in the presents of GARCH-type effects the asymptotic distribution of the correlation coefficients are affected (see e.g. Diebold (1986), Weiss (1984), and Milhoj (1985)). $Q(4)$ is the 4th order Ljng-Box statistic, distributed as χ^2_4 under the null of no-autocorrelation up to order 4.

Table 6: Statistics Weekly Excess Returns

$Index$	$S\&P$	$FTSE$	DAX	$Nikkey$
$Period$	$1973-2017$	$1987-2017$	$1973-2017$	$1973-2017$
$No.\,of\,Obs.$	2299	1621	2300	2299
$Average$	0.103	0.104	0.134	0.037
$Stand.\,Dev.$	2.299	2.299	2.767	2.485
$Skewness$	-0.542	-0.541	-0.592	-0.614
$Kurtosis$	8.309	8.304	8.003	7.578
$Jarque-Bera$	2811.4	2807.5	2533.9	2151.9
$\widehat{\rho}\,(y_t, y_{t-1})$	-0.063	-0.064	-0.005	0.000
$\widehat{\rho}\,(y_t, y_{t-2})$	0.038	0.037	0.058	0.038
$Q(4)$	15.359	15.408	13.635	4.220
$\widehat{\rho}\left(y_t^2, y_{t-1}^2\right)$	0.267	0.267	0.203	0.217
$\widehat{\rho}\left(y_t^2, y_{t-2}^2\right)$	0.168	0.168	0.252	0.143
$Q^{(2}\,(4)$	363.53	363.78	425.78	201.19
$Dyn.Asym.(1)$	-0.198	-0.198	-0.176	-0.137

The 1st and 2nd order autocorrelation of the squared returns is significant indicating strong volatility clustering effects. This is justified by the 4th Ljung-Box statistic for the squared returns $Q^{(2}\,(4)$. The estimated Dynamic Asymmetry, $\widehat{\rho}\,(y_t^2, y_{t-1})$, is negative and significant in all cases. Notice that the theoretical dynamic asymmetry depends on the leverage effect parameter ρ as well as the parameter δ_2 (see Demos 2023 and Bollerslev and Zhou (2006)).

Let us turn our attention to the estimation of the model. First, the asymptotic variance-covariance matrix is evaluated employing the formulae in Gourieroux, Monfort, and Renault (1993). The asymptotic distribution of the II estimator of θ, $\widehat{\theta}$ is given

$$\sqrt{T}\left(\widehat{\theta}-\theta\right) \underset{T\to\infty}{\overset{d}{\longrightarrow}} N\left(0, W\right),$$

where

$$W=\left(1+\frac{1}{S}\right)\left(\frac{\partial^2 l_\infty}{\partial\theta\partial\varphi'} I_0^{-1} \frac{\partial^2 l_\infty}{\partial\varphi\partial\theta'}\right)^{-1}.$$

As the objective functions of the auxiliary estimator, either for the EGARCH-M or GQARCH-M, is the sum of individual observations of the quasi-normal log-likelihood function (see equation 2.1) and there are not exogenous variables we have that

$$I_0 = \lim_{T \to \infty} V_0 \left[\frac{1}{\sqrt{T}} \sum_{t=1}^{T} \frac{\partial l_t(\varphi)}{\partial \varphi} \right]$$

where $V_0[\bullet]$ is the variance under the assumed true model. Employing Newey and West (1987) I_0 can be consistently estimated by

$$\widehat{\Gamma} = \widehat{\Gamma}_0 + \sum_{k=1}^{K} \left(1 - \frac{k}{K+1}\right) \left(\widehat{\Gamma}_k + \widehat{\Gamma}_k'\right)$$

where

$$\widehat{\Gamma}_k = \frac{1}{T} \sum_{t=k=1}^{T} \frac{\partial l_{t-k}}{\partial \varphi}(\widehat{\varphi}) \frac{\partial l_t}{\partial \varphi'}(\widehat{\varphi}).$$

Further, $\frac{\partial^2 l_\infty}{\partial \theta \partial \varphi'}$ can be evaluated numerically at $\widehat{\theta}$, i.e. by $\frac{\partial^2 l_T(\widehat{\theta})}{\partial \theta \partial \varphi'}$. Additionally, as $\dim(\theta) = \dim(\varphi) = 6$, $\frac{\partial^2 l_T(\widehat{\theta})}{\partial \theta \partial \varphi'}$ is a square non-singular matrix and it follows that the estimated asymptotic variance matrix is given by:

$$W = \left(1 + \frac{1}{S}\right) \left(\frac{\partial^2 l_T\left(\widehat{\theta}\right)}{\partial \varphi \partial \theta'} \right)^{-1} \widehat{\Gamma} \left(\frac{\partial^2 l_T\left(\widehat{\theta}\right)}{\partial \theta \partial \varphi'} \right)^{-1}.$$

In the main paper we present the estimated values of the model in equations 1.1 and 1.2 together with the asymptotic z-statistics (in parentheses). To avoid inflating the estimator variances we have chosen $S = 99000$.

Table 7: Estimation

	$S\&P$		$FTSE$		DAX		$NIKKEY$	
	$EGARCH-M$	$GQARCH-M$	$EGARCH-M$	$GQARCH-M$	$EGARCH-M$	$GQARCH-M$	$EGARCH-M$	$GQARCH-M$
$Parameter$								
c	-0.125	-0.145	-0.176	-0.205	-0.021	0.010	0.056	0.101
	(-0.678)	(-0.601)	(-0.424)	(-0.807)	(-0.087)	(0.047)	(0.239)	(0.773)
λ	0.119	0.128	0.110	0.134	0.079	0.068	-0.014	-0.019
	(1.249)	(0.954)	(0.584)	(1.058)	(0.804)	(0.651)	(-0.142)	(0.279)
ω	0.084	0.115	0.095	0.100	0.077	0.102	0.066	0.038
	(1.156)	(1.222)	(0.863)	(1.077)	(0.844)	(1.152)	(0.830)	(0.893)
ψ	0.934	0.911	0.937	0.935	0.951	0.937	0.958	0.980
	(6.000)	(4.872)	(4.756)	(5.353)	(5.087)	(5.593)	(5.692)	(8.007)
ρ	-0.606	-0.558	-0.695	-0.600	-0.438	-0.367	-0.348	-0.251
	(-2.484)	(-2.513)	(0.279)	(-2.340)	(-1.395)	(-1.465)	(-1.112)	(-0.726)
σ_η	0.259	0.315	0.249	0.276	0.235	0.284	0.231	0.215
	(2.700)	(2.688)	(2.052)	(2.346)	(2.103)	(2.577)	(2.175)	(2.917)

It is obvious that the mean constant c is highly insignificant in all cases. Consequently, we estimated the SV-M model with EGARCH-M as an auxiliary imposing the constraint that $c = 0$, but we have chosen $S = 90000$, to conserve time. The results are presented in the main paper.

APPENDIX

Figure 4.1: Biases Parameter δ_1.

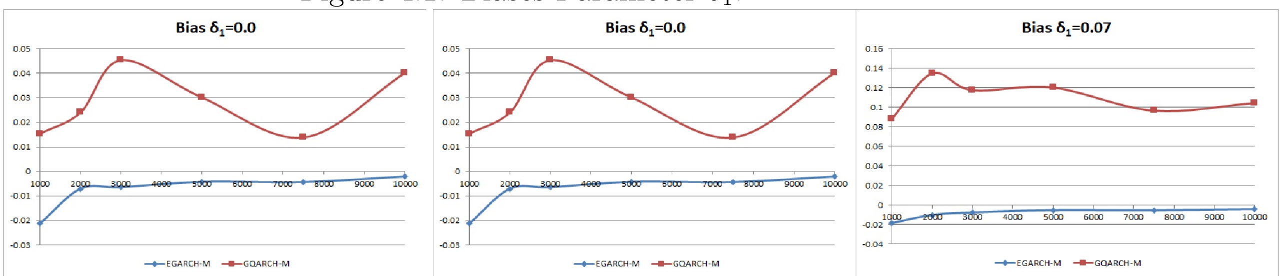


Figure 4.2: Root MSE Parameter δ_1.

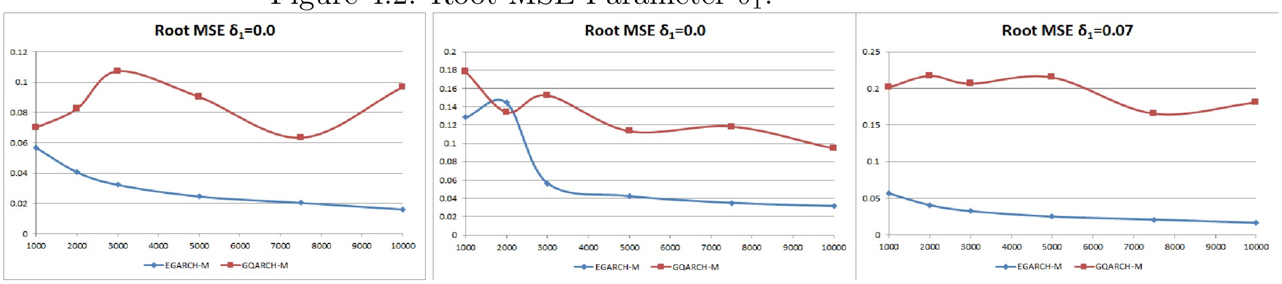


Figure 4.3: Biases Parameter δ_2.

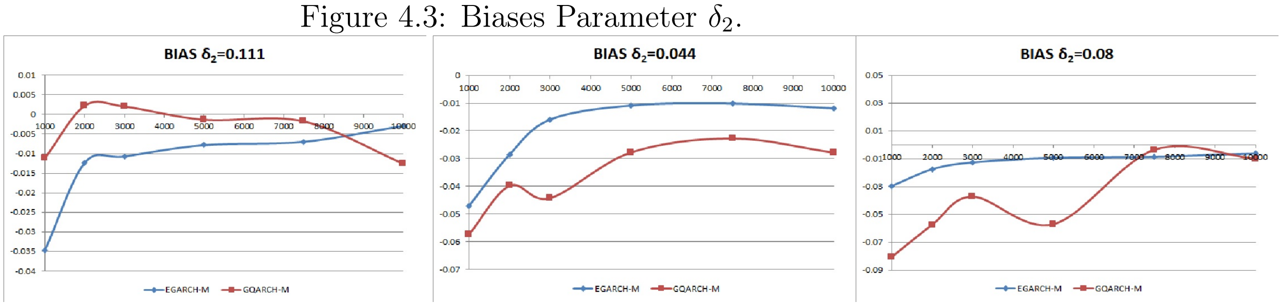


Figure 4.4: Root MSE Parameter δ_2.

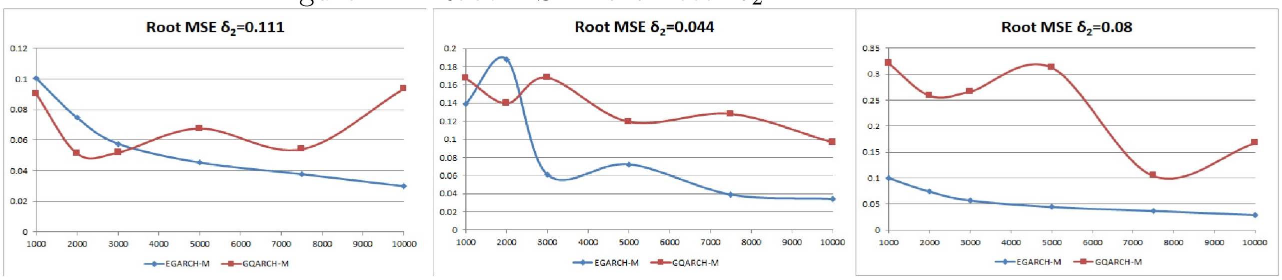


Figure 4.5: Biases Parameter ω.

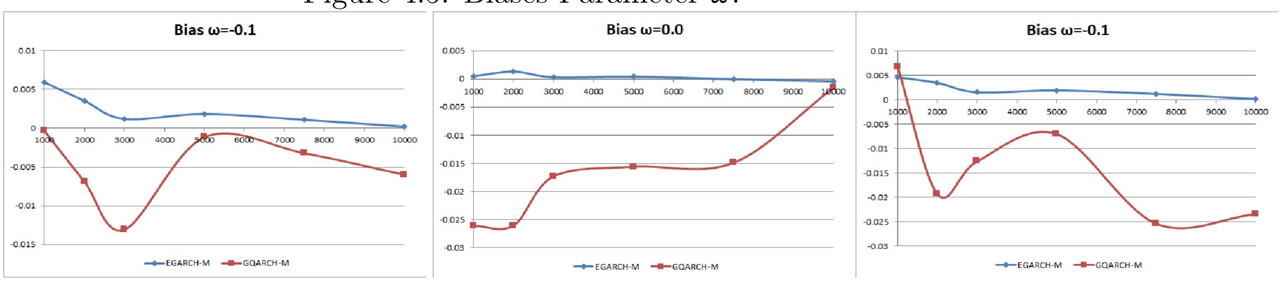


Figure 4.6: Root MSE Parameter ω.

μσεὸ

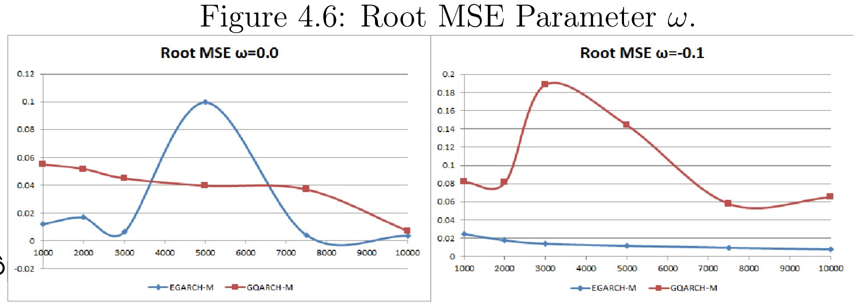


Figure 4.7: Biases Parameter β.

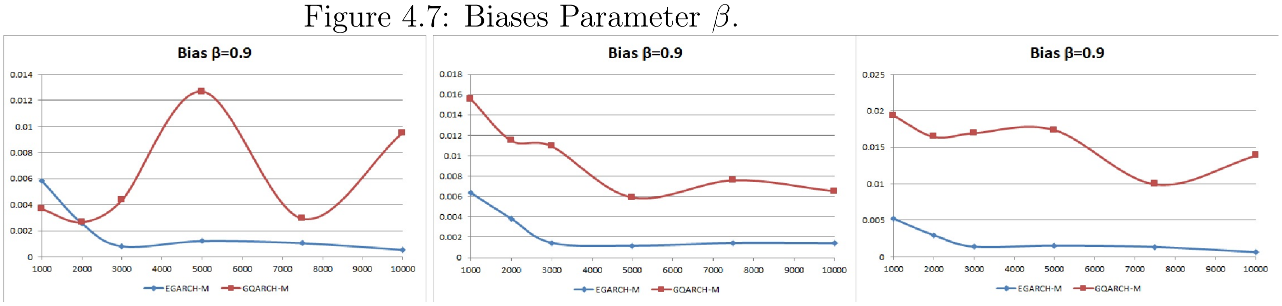


Figure 4.8: Root MSE Parameter β.

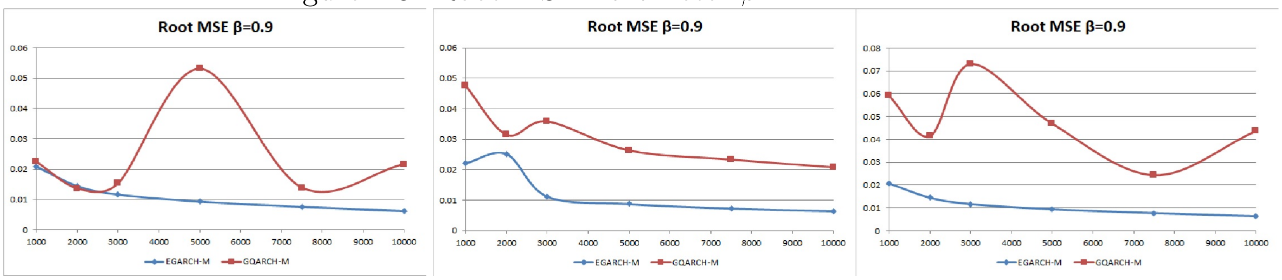


Figure 4.9: Bias Parameter ρ.

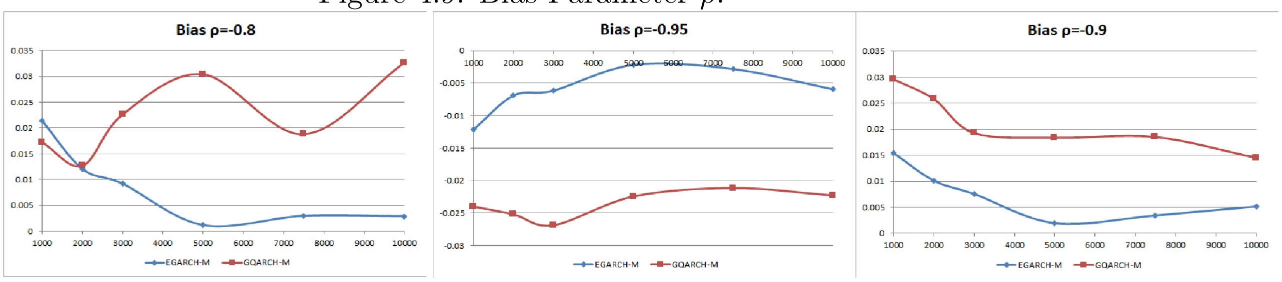


Figure 4.10: Root MSE Parameter ρ.

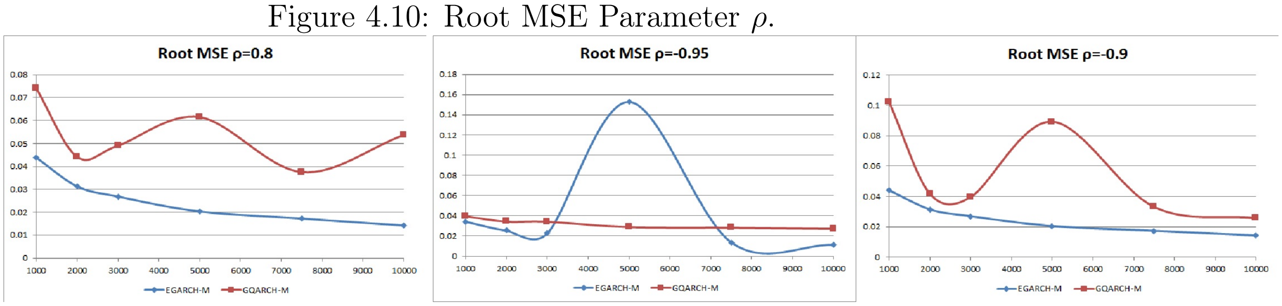


Figure 4.11: Bias Parameter σ.

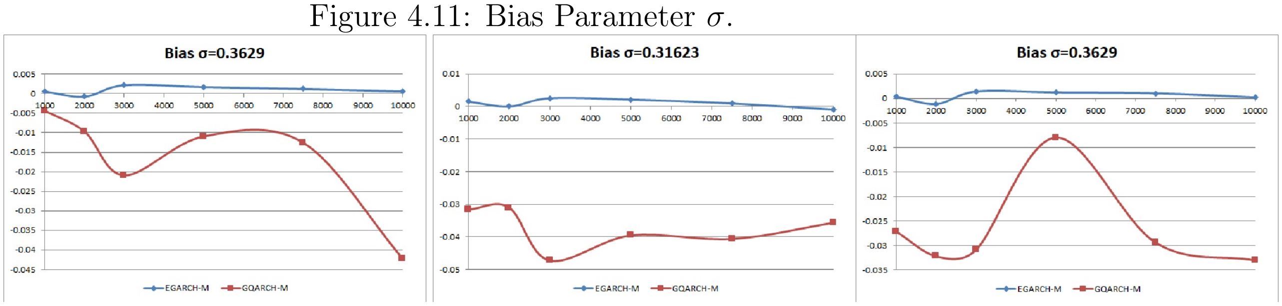
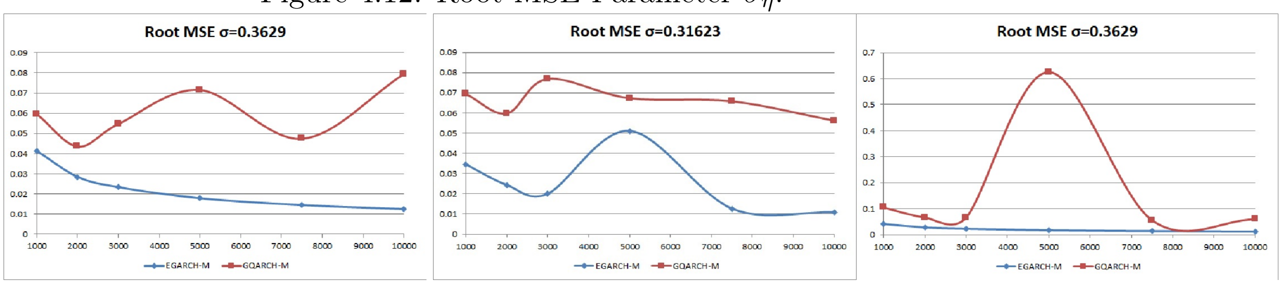


Figure 4.12: Root MSE Parameter σ_η.



References

[1] Andersen, T.G. H-J. Chung and B. E. Sorensen (1999) Efficient method of moments estimation of a stochastic volatility model: A Monte Carlo study, *Journal of Econometrics* 91, 61-87.

[2] Andersen, T.G. and B.E. Sorensen (1996), GMM Estimation of a Stochastic Volatility Model: a Monte Carlo Study, *Journal of Business and Economic Statistics* 14, 328–352.

[3] Arvanitis, S. and A. Demos (2004), Time Dependence and Moments of a family of Time-Varying parameter $GARCH$ in Mean Models, *Journal of Time Series Analysis* 25, 1-25.

[4] Bollerslev, T. and H. Zhou (2006), Volatility puzzles: a simple framework for gauging return-volatility regressions, *Journal of Econometrics* 131, 123–150.

[5] Calzolari, G., G. Fiorentini and E. Sentana (2004), Constraint Indirect Estimation, *Review of Economic Studies* 71, 945-973.

[6] Demos, A. (2023), Statistical Properties of Two Asymmetric Stochastic Volatility in Mean Models, DP 23-03, Dpt. IEES, AUEB

[7] Demos, A. and D. Kyriakopoulou (2013), Edgeworth and moment approximations: The case of MM and QML estimators for the $MA(1)$ models, *Communications in Statistics-Theory and Methods* 42, 1713-1747.

[8] Diebold, F.X. (1986), Testing for Serial Correlation in the Presence of ARCH, in *Proceedings of the American Statistical Association, Business and Economics Statistics Section*, 323–328.

[9] Gallant, R. A. and G. Tauchen (1996), Which Moments to Match, *Econometric Theory* 12, 657-81.

[10] Gourieroux, C. and A. Monfort (1996), *Simulation-Based Econometric Methods*, Oxford University Press.

[11] Gourieroux, C., A. Monfort, and E. Renault (1993), Indirect Inference, *Journal of Applied Econometrics* 8 , S85-S118.

[12] Harvey, A. and N. Shephard (1996), Estimation of an Asymmetric Stochastic Volatility Model for asset Returns, *Journal of Business and Economic Statistics* 14, 429-434.

[13] Jacquier, E., N.G. Polson and P.E. Rossi (1994), Bayesian Analysis of Stochastic Volatility Models, *Journal of Business and Economic Statistics* 12, 371-417.

[14] Jacquier, E., N.G. Polson and P.E. Rossi (2004), Bayesian Analysis of Stochastic Volatility Models with Fat-tails and Correlated Errors, *Journal of Econometrics* 122, 185-212.

[15] Milhoj, A. (1985), The Moment Structure of ARCH Processes, *Scandinavian Journal of Statistics* 12, 281-292.

[16] Monfardini, C. (1998), Estimating Stochastic Volatility Models through Indirect Inference, *Econometrics Journal* 1, 113-128.

[17] Nelson, D.B. (1991), Conditional Heteroskedasticity in Asset Returns: A New Approach, *Econometrica* 59, 347-370.

[18] Newey, W.K. and K.D. West (1987), A Simple Positive Semi-Definite Heteroskedasticity and Autocorrelation-Consistent Covariance Matrix, *Econometrica* 55, 703-708.

[19] Sandmann, G. and S. J. Koopman (1998), Estimation of stochastic volatility models via Monte Carlo maximum likelihood, *Journal of Econometrics* 87, 271-301.

[20] Sentana, E. (1995), Quadratic ARCH Models, *Review of Economic Studies* 62, 639-661.

[21] Tai, C.S. (2000), Time Varying Market, Interest Rates and Exchange Rates Risk Premia in the US Commercial Bank Stock Returns, *Journal of Multinational Financial Management* 10 , 397-420.

[22] Weiss, A.A. (1984), ARMA Models with ARCH Errors, *Journal of Time Series Analysis* 5, 129-43.

[23] Yu, J. (2002), $MCMC$ Methods for Estimating Stochastic Volatility Models with Leverage Effects: comments on Jacquier, Polson and Rossi (2002), discussion paper University of Auckland.

[24] Yu, J. (2002), On Leverage in a Stochastic Volatility Model, *Journal of Econometrics* 127, 165-178.

Department of Economics
Athens University of Economics and Business

List of Recent Working Papers

2022

01-22 Is Ireland the most intangible intensive economy in Europe? A growth accounting perspective, Ilias Kostarakos, KieranMcQuinn and Petros Varthalitis
02-22 Common bank supervision and profitability convergence in the EU, Ioanna Avgeri, Yiannis Dendramis and Helen Louri
03-22 Missing Values in Panel Data Unit Root Tests, Yiannis Karavias, Elias Tzavalis and Haotian Zhang
04-22 Ordering Arbitrage Portfolios and Finding Arbitrage Opportunities, Stelios Arvanitis and Thierry Post
05-22 Concentration Inequalities for Kernel Density Estimators under Uniform Mixing, Stelios Arvanitis
06-22 Public Sector Corruption and the Valuation of Systemically Important Banks, Georgios Bertsatos, Spyros Pagratis, Plutarchos Sakellaris
07-22 Finance or Demand: What drives the Responses of Young and Small Firms to Financial Crises? Stelios Giannoulakis and Plutarchos Sakellaris
08-22 Production function estimation controlling for endogenous productivity disruptions, Plutarchos Sakellaris and Dimitris Zaverdas
09-22 A panel bounds testing procedure, Georgios Bertsatos, Plutarchos Sakellaris, Mike G. Tsionas
10-22 Social policy gone bad educationally: Unintended peer effects from transferred students, Christos Genakos and Eleni Kyrkopoulou
11-22 Inconsistency for the Gaussian QMLE in GARCH-type models with infinite variance, Stelios Arvanitis and Alexandros Louka
12-22 Time to question the wisdom of active monetary policies, George C. Bitros
13-22 Investors' Behavior in Cryptocurrency Market, Stelios Arvanitis, Nikolas Topaloglou and Georgios Tsomidis
14-22 On the asking price for selling Chelsea FC, Georgios Bertsatos and Gerassimos Sapountzoglou
15-22 Hysteresis, Financial Frictions and Monetary Policy, Konstantinos Giakas
16-22 Delay in Childbearing and the Evolution of Fertility Rates, Evangelos Dioikitopoulos and Dimitrios Varvarigos
17-22 Human capital threshold effects in economic development: A panel data approach with endogenous threshold, Dimitris Christopoulos, Dimitris Smyrnakis and Elias Tzavalis
18-22 Distributional aspects of rent seeking activities in a Real Business Cycle model, Tryfonas Christou, Apostolis Philippopoulos and Vanghelis Vassilatos

Πατησίων 76, 104 34 Αθήνα. Τηλ.: 210 8203303-5. E-mail: econ@aueb.gr / www.aueb.gr
76, Patission Street, Athens 104 34 Greece. Tel.: (+30) 210 8203303-5

2023

01-23 Real interest rate and monetary policy in the post Bretton Woods United States, George C. Bitros and Mara Vidali
02-23 Debt targets and fiscal consolidation in a two-country HANK model: the case of Euro Area, Xiaoshan Chen, Spyridon Lazarakis and Petros Varthalitis
03-23 Central bank digital currencies: Foundational issues and prospects looking forward, George C. Bitros and Anastasios G. Malliaris
04-23 The State and the Economy of Modern Greece. Key Drivers from 1821 to the Present, George Alogoskoufis
05-23 Sparse spanning portfolios and under-diversification with second-order stochastic dominance, Stelios Arvanitis, Olivier Scaillet, Nikolas Topaloglou
06-23 What makes for survival? Key characteristics of Greek incubated early-stage startup(per)s during the Crisis: a multivariate and machine learning approach, Ioannis Besis, Ioanna Sapfo Pepelasis and Spiros Paraskevas
07-23 The Twin Deficits, Monetary Instability and Debt Crises in the History of Modern Greece, George Alogoskoufis
08-23 Dealing with endogenous regressors using copulas; on the problem of near multicollinearity, Dimitris Christopoulos, Dimitris Smyrnakis and Elias Tzavalis
09-23 A machine learning approach to construct quarterly data on intangible investment for Eurozone, Angelos Alexopoulos and Petros Varthalitis
10-23 Asymmetries in Post-War Monetary Arrangements in Europe: From Bretton Woods to the Euro Area, George Alogoskoufis, Konstantinos Gravas and Laurent Jacque
11-23 Unanticipated Inflation, Unemployment Persistence and the New Keynesian Phillips Curve, George Alogoskoufis and Stelios Giannoulakis
12-23 Threshold Endogeneity in Threshold VARs: An Application to Monetary State Dependence, Dimitris Christopoulos, Peter McAdam and Elias Tzavalis
13-23 A DSGE Model for the European Unemployment Persistence, Konstantinos Giakas
14-23 Binary public decisions with a status quo: undominated mechanisms without coercion, Efthymios Athanasiou and Giacomo Valletta
15-23 Does Agents' learning explain deviations in the Euro Area between the Core and the Periphery? George Economides, Konstantinos Mavrigiannakis and Vanghelis Vassilatos
16-23 Mild Explocivity, Persistent Homology and Cryptocurrencies' Bubbles: An Empirical Exercise, Stelios Arvanitis and Michalis Detsis
17-23 A network and machine learning approach to detect Value Added Tax fraud, Angelos Alexopoulos, Petros Dellaportas, Stanley Gyoshev, Christos Kotsogiannis, Sofia C. Olhede, Trifon Pavkov
18-23 Time Varying Three Pass Regression Filter, Yiannis Dendramis, George Kapetanios, Massimiliano Marcellino
19-23 From debt arithmetic to fiscal sustainability and fiscal rules: Taking stock, George Economides, Natasha Miouli and Apostolis Philippopoulos
20-23 Stochastic Arbitrage Opportunities: Set Estimation and Statistical Testing, Stelios Arvanitis and Thierry Post
21-23 Behavioral Personae, Stochastic Dominance, and the Cryptocurrency Market, Stelios Arvanitis, Nikolas Topaloglou, and Georgios Tsomidis
22-23 Block Empirical Likelihood Inference for Stochastic Bounding: Large Deviations Asymptotics Under *m*-Dependence, Stelios Arvanitis and Nikolas Topaloglou

23-23 **A Consolidation of the Neoclassical Macroeconomic Competitive General Equilibrium Theory via Keynesianism (Part 1 and Part 2), Angelos Angelopoulos**
24-23 **Limit Theory for Martingale Transforms with Heavy-Tailed Noise, Stelios Arvanitis and Alexandros Louka**

<u>2024</u>

01-24 **Market Timing & Predictive Complexity, Stelios Arvanitis, Foteini Kyriazi, Dimitrios Thomakos**
02-24 **Multi-Objective Frequentistic Model Averaging with an Application to Economic Growth, Stelios Arvanitis, Mehmet Pinar, Thanasis Stengos, Nikolas Topaloglou**
03-24 **State dependent fiscal multipliers in a Small Open Economy, Xiaoshan Chen, Jilei Huang, Petros Varthalitis**
04-24 **Public debt consolidation: Aggregate and distributional implications in a small open economy of the Euro Area, Eleftherios-Theodoros Roumpanis**
05-24 **Intangible investment during the Global Financial Crisis in the EU, Vassiliki Dimakopoulou, Stelios Sakkas and Petros Varthalitis**
06-24 **Time will tell! Towards the construction of instantaneous indicators of different agent-types, Iordanis Kalaitzoglou, Stelios Arvanitis**
07-24 **Norm Constrained Empirical Portfolio Optimization with Stochastic Dominance: Robust Optimization Non-Asymptotics, Stelios Arvanitis**
08-24 **Asymptotics of a QLR-type test for optimal predictive ability, Stelios Arvanitis**
09-24 **Strongly Equitable General Equilibrium Allocations, Angelos Angelopoulos**
10-24 **The Greek macroeconomy: A note on the current situation and future outlook, Apostolis Philippopoulos**
11-24 **Evolution of Greek Tax System, A Survey of Legislated Tax Changes from 1974 to 2018, Panagiotis Asimakopoulos**
12-24 **Macroeconomic Impact of Tax Changes, The case of Greece from 1974 to 2018, Panagiotis Asimakopoulos**
13-24 **`Pareto, Edgeworth, Walras, Shapley' Equivalence in a Small Economy, Angelos Angelopoulos**
14-24 **Stimulating long-term growth and welfare in the U.S, James Malley and Apostolis Philippopoulos**
15-24 **Distributionally Conservative Stochastic Dominance via Subsampling, Stelios Arvanitis**
16-24 **Before and After the Political Transition of 1974. Institutions, Politics, and the Economy of Post-War Greece, George Alogoskoufis**
17-24 **Gaussian Stochastic Volatility, Misspecified Volatility Filters and Indirect Inference Estimation, Stelios Arvanitis, Antonis Demos**

Department of Economics
Athens University of Economics and Business

The Department is the oldest Department of Economics in Greece with a pioneering role in organising postgraduate studies in Economics since 1978. Its priority has always been to bring together highly qualified academics and top quality students. Faculty members specialize in a wide range of topics in economics, with teaching and research experience in world-class universities and publications in top academic journals.

The Department constantly strives to maintain its high level of research and teaching standards. It covers a wide range of economic studies in micro-and macroeconomic analysis, banking and finance, public and monetary economics, international and rural economics, labour economics, industrial organization and strategy, economics of the environment and natural resources, economic history and relevant quantitative tools of mathematics, statistics and econometrics.

Its undergraduate program attracts high quality students who, after successful completion of their studies, have excellent prospects for employment in the private and public sector, including areas such as business, banking, finance and advisory services. Also, graduates of the program have solid foundations in economics and related tools and are regularly admitted to top graduate programs internationally. Three specializations are offered:1. Economic Theory and Policy, 2. Business Economics and Finance and 3. International and European Economics. The postgraduate programs of the Department (M.Sc and Ph.D) are highly regarded and attract a large number of quality candidates every year.

For more information:

https://www.dept.aueb.gr/en/econ/