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Abstract

The limit theory of a Gaussian Quasi-likelihood Ratio (QLR)-type test for the hypothesis of Optimal Predictive Ability is developed in the present note. This hypothesis which generalizes the Superior Predictive Ability hypothesis from a single given loss function to an entire class of loss functions was considered in APPK21 (1). The results are developed indicatively for the class of Convex Loss functions. As in APPK21 (1), the research hypothesis is formulated in terms of moment inequality conditions, the empirical versions of which are reducible to a set of piece-wise linear functions. A consistent and exact test is constructed based on a QLR-type test statistic, moment selection and subsampling.
Keywords: Forecast Comparison, Stochastic Dominance, QLR-type test, Moment Selection, Subsampling.

1 Introduction

The comparison of a multiple forecasting models potentially based on different statistical information and inference methods was facilitated by the formulation of the hypothesis of Superior Predictive Ability-see for example White (2000) (10) and Hansen (2005) (2). To improve robustness w.r.t. the choice of loss function, Jin, Corradi and Swanson (2017) (3), generalized the aforementioned hypothesis by introducing a stochastic dominance relation based on a class of loss functions, and considering the hypothesis of whether a given forecast model is a maximal element. Maximal elements rarely exist especially if the class of loss functions and/or the number of competing models is non-trivial, hence this generalization could suffer from lack of discriminatory power.

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In order to obtain robustness and improve discriminatory power, Arvanitis, Post, Poti and Karabati (2021-hereafter APPK21) (1) use an alternative generalization of the hypothesis of Superior Predictive Ability, which transliterates the criterion of SD Optimality (see for example Post (2017) (8) and the references therein) to the forecasting background, labeled as Optimal Predictive Ability (OPA). The ingredients of the OPA hypothesis are similar to the Jin, Corradi and Swanson (2017) (3) approach; a class of loss functions that define a stochastic dominance relation on the set of competing forecasting models. Optimality however is generally weaker than maximality; a model is considered optimal iff it is selected over the competing models by at least a loss function in the class in terms of expected loss. Maximality is quite stronger as it requires selection by every loss function in the class. Both concepts are stronger than (Pareto) efficiency; a model is efficient iff there exists a loss function and a competing model compared to which the efficient one is selected. Efficient models could be ubiquitous, maximal models are generally rare or non-existent, optimal models provide an application-wise attractive compromise; AAPK21 report significant reductions in the set of considered models if the researcher discards from analysis models that are not inferred optimal.

Given the latency of expected loss, AAPK21 use an Empirical Likelihood methodology in order to construct a statistical procedure that tests whether a given forecasting model is optimal; given the class of loss functions and the resulting dominance relation, as well as a time series of observable forecasting errors for each of the competing models, a Block Empirical Likelihood Ratio statistic is considered that is obtained from piece-wise linear approximation of the empirical moment inequalities that define the empirical version of the dominance relation and bi-convex optimization. Then, via a moment selection methodology that is based on a slack augmentation of the moment inequalities involved, an asymptotically conservative rejection region is obtained via a chi-squared distribution that dominates the latent asymptotic distribution of the test statistic under the null.

The conservative character of the APPK21 testing procedure could imply poor power properties on the boundary of the hypothesis of optimality and meager optimality reduction in the set of forecasting models. One way to circumvent such shortcomings, is via considering an asymptotically exact modification of the procedure above. In the present note this is formulated via a Gaussian Quasi-Likelihood Ratio statistic (QLR) that assumes the form of the infimum w.r.t. the loss functions involved of a quadratic form w.r.t. the selected empirical moment conditions; this is asymptotically equivalent to the Empirical Likelihood Ratio statistic of APPK21. In this approach and compared to APPK21, the moment selection procedure is also applied to the construction of the test statistic, while the estimation of a long-run covariance matrix is also required; this is achievable by the HAC estimator of Newey and West (1987) (6). This is something that APPK21 avoid due to their block structures. Given the above, an approximation of the asymptotic rejection region is achievable via subsampling. Pseudo-consistency of the estimators of the parameters associated with the forecasting models allow for the possibility that the long-run covariance matrix that appears in the QLR statistic is kept fixed at the original sample and not re-evaluated at each subsample. Also, the full sample optimal loss function involved in the construction of the statistic is also kept fixed at the subsampling phase; the above contributes to the computational simplicity of the procedure. The full sample optimizations involved for the evaluation of the QLR statistic have at worst the form of quadratic programming, while the ones associated with the subsampling phases are trivial. As long as the subsampling rate diverges to infinity at a slower rate compared to the sample size, and population dominance relations that hold as equalities exist for every optimal loss, the test that rejects the null of optimality iff the full sample sample QLR statistic lies inside the subsampling rejection region is consistent and asymptotically exact given assumptions that among others require non-singularity of relevant covariance matrices and an empirically innocuous restriction on the significance level.

The rest of the note is structured as follows: the next section introduces the forecasting framework, the dominance relation and its empirical version, the OPA hypothesis, the moment selection procedure and given those, the QLR statistic and the Monte Carlo procedure. The forecasting and dominance frameworks are identical to the ones in APPK21. Our results are indicatively presented for the class of Convex Loss Functions. They can be easily extended to other classes of losses as long as those satisfy uniform Lipschitz properties. The final section presents the assumption framework and the limit theory of the test as well as a brief concluding discussion; the proofs are heavily based on the proofs in the supplementary material of APPK21.

2 Framework and testing procedure

This section introduces the general forecasting framework, the loss function class indicatively used, the concepts of dominance and optimality, the empirical approximation of the moment inequalities associated with dominance by piece-wise linear inequalities, the moment selection procedure and the subsequent construction of the test statistic and the subsampling approximation of the rejection region.

2.1 Forecast errors and loss functions

The APPK21 framework and notation is more or less employed here; a random variable X is forecast using $M \geq 2$ distinct and fixed by the researcher forecast models, generating point forecasts $\boldsymbol{Y} := (Y_1 \cdots Y_M)$. Examples of forecast models could incorporate predictive regressions, means based on local estimation windows or past market prices of securities via martingale considerations. The forecast models could also include forecast combinations-i.e. convex mixures-of multiple base forecasts.

The researcher chooses one of the forecasting models to be compared with the remaining $(M-1)$ ones. The models are indexed such that the evaluated model takes the M-th position; the alternatives are collected in the set $\mathcal{I} := \{1, \cdots, M-1\}$.

The-potentially latent-forecast errors of the models are given by $\boldsymbol{E} := (E_1 \cdots E_M)$, $E_i := X - Y_i$, $i = 1, \cdots, M$. The joint cumulative distribution function (CDF) of the errors is denoted by $\mathbb{P} : \mathcal{X}^M \to [0,1]$, where $\mathcal{X} := [a, b]$, $-\infty < a < 0 < b < +\infty$. The compactness of the support is used without much loss of generality. The results below would also hold at the cost of strengthening several assumptions to incorporate for example the existence of suffcient moments for the random elements involved in the constructions that follow.

Predictive ability is measured using expected loss $\mathbb{E}_{\mathbb{P}}[\ell(E_i)]$ based on a loss function $\ell : \mathcal{X} \to \mathbb{R}_+$. The class of permissible loss functions employed in the present note is denoted-as in APPK21-by $\mathcal{L}_1$. This set contains the totality of the convex non-negative real functions definable on $\mathcal{X}$. The results below are easily extendible to the other two classes considered in APPK21-the General Loss functions, and the Symmetric Convex Loss functions, as well as in any other function class that satisfies the uniform Lipschitz conditions that appear in the Supplement of APPK21.

2.2 Stochastic dominance, optimality and hypothesis structure

The loss function class adopted above permits the definition of a stochastic dominance relation between the forecasting models employed in the analysis-as noted in APPK21-this is closely related to the SSD relation in the expected utility paradigm: model $i \in \mathcal{I}$ stochastically dominates model M for loss function class $\mathcal{L}_1$, or $E_i \succeq_{\mathcal{L}_1,\mathbb{P}} E_M$, iff $\mathbb{E}_{\mathbb{P}}[\ell(E_M)] \leq \mathbb{E}_{\mathbb{P}}[\ell(E_i)]$ for all $\ell \in \mathcal{L}$; non-dominance occurs iff the moment inequalities' system above is violated for some loss function in the class.

The concept of dominance can be extended in several distinct ways to a joint analysis of all models via joint properties of the order. The concept of optimality is considered here-the interested reader is referred to APPK21 for the definitions and comparisons between optimality, admissibility based on Pareto efficiency and superiority based on maximum elements.

Thus, OPA occurs iff the evaluated model minimizes expected loss for some permissible loss function over all competing forecasting models, i.e. $\exists \ell \in \mathcal{L}_1, \forall i \in \mathcal{I}$: $\mathbb{E}_{\mathbb{P}}\left[\ell(E_i) - \ell(E_M)\right] \geq 0$, or equivalently and due to the finiteness of $\mathcal{I}$:

$$\sup_{\ell \in \mathcal{L}_1} \inf_{i \in \mathcal{I}} \mathbb{E}_{\mathbb{P}}\left[\ell(E_i) - \ell(E_M)\right] \geq 0. \tag{1}$$

If the considered model is non optimal, then it can be discarded from the analysis; every loss function would prefer a competing model. Thus the OPA concept constitutes a criterion for refining the set of considered forecasting models.

Usually, $\mathbb{P}$ is latent and yet estimable using empirical data. This allows for statistical inference on optimality; given the null hypothesis of optimality statistical tests can be formulated using the empirical information. In this context, Definition (1) implies that the null hypothesis of optimality can be formulated as $(M-1)$ moment inequalities over the infinite-dimensional parameter space $\mathcal{L}_1$:

$$\boldsymbol{H}_0(\mathcal{L}_1, \mathbb{P}) : \left(\mathbb{E}_{\mathbb{P}}\left[\ell(E_i) - \ell(E_M)\right] \geq 0,\ i = 1, \cdots, M-1\right),\ \forall \ell \in \mathcal{L}_1. \tag{2}$$

Dually, the alternative is formulated as

$$\boldsymbol{H}_1(\mathcal{L}_1, \mathbb{P}) : \left(\mathbb{E}_{\mathbb{P}}\left[\ell(E_i) - \ell(E_M)\right] < 0,\ \exists i = 1, \cdots, M-1\right),\ \exists \ell \in \mathcal{L}_1. \tag{3}$$

2.3 Time series data

It is assumed that the available data lie in the time series' context; the analyst has at her disposal the time series realizations X_t, and point forecasts $\hat{\boldsymbol{y}}_t := (\hat{y}_{1,t} \cdots \hat{y}_{M,t})$, for $t = 1, \ldots, T$.

The analysis allows for the existence of latent point forecasts $\boldsymbol{y}_t := (y_{1,t} \cdots y_{M,t})$; those are functions $m_i\left(\boldsymbol{Z}_{i,t}, \boldsymbol{\theta}_{0_i}\right)$ of a random vector of predictive variables, $\boldsymbol{Z}_{i,t} \in \mathbb{R}^{d_i}$, and a latent parameter vector $\boldsymbol{\theta}_{0_i} \in \mathrm{Int}\Theta_i$ from the parameter space $\Theta_i \subseteq \mathbb{R}^{d_i}$. The forecasts at time t are constructed as $\hat{y}_{i,t} := m_i\left(\boldsymbol{Z}_{i,t}, \boldsymbol{\theta}_{t_i}\right)$ for realizations $\boldsymbol{Z}_{i,t}$ and parameter estimators $\boldsymbol{\theta}_{t_i}$.

Given X_t, the unobservable error is $\boldsymbol{u}_t := X_t \mathbf{1}'_M - \boldsymbol{y}_t$ and the observed error is $\boldsymbol{\varepsilon}_t := X_t \mathbf{1}'_M - \hat{\boldsymbol{y}}_t$, where $\boldsymbol{y}_t := \left[m_1\left(\boldsymbol{Z}_{1,t}, \boldsymbol{\theta}_{1_0}\right) \cdots m_M\left(\boldsymbol{Z}_{M,t}, \boldsymbol{\theta}_{M_0}\right)\right]'$ and $\hat{\boldsymbol{y}}_t := \left[m_1\left(\boldsymbol{Z}_{1,t}, \boldsymbol{\theta}_{1_t}\right) \cdots m_M\left(\boldsymbol{Z}_{M,t}, \boldsymbol{\theta}_{M_t}\right)\right]'$.

Given the observable data $\boldsymbol{\varepsilon}_t$, $t = 1, \ldots, T$, the latent $\mathbb{P}$ is approximated by its empirical version:

$$\mathbb{P}_T(E) := T^{-1} \sum_{t=1}^{T} \mathbb{I}\left(\varepsilon_t \leq E\right), \tag{4}$$

where $\mathbb{I}$ denotes the relevant indicator functions.

Several of the procedures and the results below are extendable to other estimators for $\mathbb{P}$ like kernels, polynomial approximations, parametric estimators, etc.

2.4 Empirical moment piece-wise linear approximations

Given $\mathbb{P}_T$, the OPA conditions that appear in the null hypothesis can be empirically approximated by a finite system of linear inequalities. APPK21 show that this system is obtainable by replacing the infinite-dimensional parameter $\ell \in \mathcal{L}_1$ by a permissible piecewise-linear loss function, along the lines of Post (2003, Thm 2). Following their derivations it is obtained that for $\mathcal{L}_1$, and if $\{z_t\}_{t=1}^{T+1}$ represent the ranked values of $\{\varepsilon_{M,t}\}_{t=1}^{T} \cup \{0\}$, so that $z_1 \leq \cdots \leq z_{T+1}$. Let $T_0 := \sup\{t : z_t < 0\}$, so that $z_{T_0+1} = 0$. For $\mathcal{L}_2$, let $\{z_t\}_{t=1}^{T+1}$ represent the ranked values of $\{|\varepsilon_{M,t}|\}_{t=1}^{T} \cup \{0\}$. Then for an arbitrary Convex Loss function $\ell \in \mathcal{L}_1$, let $\sigma_s := (\ell(z_{s+1}) - \ell(z_s)) / (z_{s+1} - z_s)$, $s = 1, \cdots, T$, be slopes of chords between two consecutive points, and $\beta_s := \sigma_{s+1} - \sigma_s$, $s = 1, \cdots, T_0 - 1$; $\beta_{T_0} := -\sigma_{T_0}$; $\beta_{T_0+1} := \sigma_{T_0+1}$; $\beta_s := \sigma_s - \sigma_{s-1}$, $s = T_0 + 1, \cdots, T$, increments of the slopes (recall that the slope at $E = 0$ is zero). A convex piecewise-linear loss function is then given by:

$$\ell_{1,\boldsymbol{\beta}}(E) := \begin{cases} +\infty & E < z_1 \\ \sum_{s=1}^{T_0} \beta_s \left(z_{s+1} - E\right)_+ + \sum_{s=T_0+1}^{T} \beta_s \left(E - z_s\right)_+ & z_1 \leq E \leq z_{T+1}. \\ +\infty & E > z_{T+1} \end{cases} \tag{5}$$

Then for any $i \in \mathcal{I}$, define the $T \times T$ coefficient matrix $\mathbf{M}_{1,i}$ with the following elements for $s, t = 1, \cdots, T$:

$$\left(\mathbf{M}_{1,i}\right)_{t,s} := \begin{cases} \left(z_{s+1} - \varepsilon_{i,t}\right)_+ - \left(z_{s+1} - \varepsilon_{M,t}\right)_+ & s = 1, \cdots, T_0 \\ \left(\varepsilon_{i,t} - z_s\right)_+ - \left(\varepsilon_{M,t} - z_s\right)_+ & s = T_0{+}1, \cdots, T \end{cases}. \tag{6}$$

Using the probability vector $\boldsymbol{p}$ associated with the atoms of $\mathbb{P}_T$, and the fact that the dominance relation is invariant to renormalization of the loss functions, APPK21 finally obtain that an empirical linear system that approximates the system of moment inequalities that appear in the null hypothesis of OPA is:

$$\begin{gathered} \boldsymbol{p}'\mathbf{M}_{1,i}\boldsymbol{\beta} \geq 0,\ i = 1, \cdots, M-1; \\ \boldsymbol{\beta} \in \Delta^T. \end{gathered}$$

2.5 Empirical moment selection

Moment selection is implemented in order to estimate the set of moment inequalities in the null that hold as equalities; those are usually referred as contacts. This is important due to the fact that the test statistic to be considered below is constructed as a quadratic form over this set. To implement moment selection, as in APPK21, the present study uses the following set of forecast models which are approximately equivalent to the evaluated model for a given loss function $\ell \in \mathcal{L}$, that is represented by the parameter $\boldsymbol{\beta}$-see the previous paragraph:

$$\mathrm{CS}(\ell, \mathbb{P}_T, c_T) := \{i = 1, \ldots, M-1 : |\boldsymbol{p}'\mathbf{M}_{1,i}\boldsymbol{\beta}| \leq c_T\},$$

where, $c_T > 0$ is a sample-dependent slack parameter which converges to zero at an appropriate rate. The number of moment conditions which are approximately binding amounts to:

$$N(\ell, \mathbb{P}_T, c_T) := \#\mathrm{CS}(\ell, \mathbb{P}_T, c_T).$$

2.6 QLR-type statistic

The moment selection procedure above, and the orientation of the moment inequalities employed in the null hypothesis for OPA enable the consideration of a QLR-type statistic. In order to define it consider $\boldsymbol{v}_T(\ell, c_T) := \sqrt{T}(\boldsymbol{p}'\mathbf{M}_{1,i}\boldsymbol{\beta})_{i\in \mathrm{CS}(\ell,\mathbb{P}_T,c_T)}$, for ℓ any loss function representable as in (5), and V_T is the Newey-West matrix

$$\begin{gathered}\tfrac{1}{T}\textstyle\sum_{l=0}^{L}\sum_{t=l+1}^{T}\left(1-\tfrac{l}{L+1}\right)\boldsymbol{v}_t(\ell, c_T)\boldsymbol{v}_t'(\ell, c_T)\\ -\tfrac{1}{T}\textstyle\sum_{l=0}^{L}\sum_{t=l+1}^{T}\left(1-\tfrac{l}{L+1}\right)\tfrac{1}{T}\sum_{t=1}^{T}\boldsymbol{v}_t(\ell, c_T)\tfrac{1}{T}\sum_{t=1}^{T}\boldsymbol{v}_{t-l}'(\ell, c_T),\end{gathered}$$

for the bandwidth parameter $0 < L \leq T-1$, where $\boldsymbol{v}_t(\ell, c_T) := \ell(\boldsymbol{\varepsilon}_{t,i})_{i\in \mathrm{CS}(\ell,\mathbb{P}_T,c_T)}$.

Then, provided that the matrix above is almost everywhere invertible, the statistic considered here has the form:

$$\mathrm{QLR}_T := \inf_{\boldsymbol{\beta}\in\Delta^T}\inf_{\boldsymbol{v}\in\mathbb{R}_+^{N(\ell,\mathbb{P}_T,c_T)}}(\boldsymbol{v}(\ell, c_T)-\boldsymbol{v})'V_T^{-1}(\boldsymbol{v}(\ell, c_T)-\boldsymbol{v}). \tag{7}$$

The statistic is first order asymptotically equivalent under the null to the BELR statistic considered in APPK21-see the following section. Its computation is not particularly involved: given an $\ell \in \mathcal{L}_1$, the inner optimization is trivial. The piecewise linear constructions above imply that the outer approximation is a standard problem of quadratic programming over the standard $T-1$ simplex.

2.7 Subsampling inference

A rejection region is constructed, based on the null limit theory of QLR_T derived in the following section, via subsampling. Let $0 < b_T \leq T$, and consider the subsamples from the original observations $(\boldsymbol{\varepsilon}_j)_{j=t,\ldots t+b_T-1}$ for all $t = 1, 2, \ldots, T - b_T + 1$. For $\alpha \in (0,1)$, denote with $q_{b_T}(1-\alpha)$ the $1-\alpha$ quantile of the subsample empirical distribution of the subsample realizations of the statistic

$$\left\{ \inf_{\boldsymbol{v} \in \mathbb{R}_+^{N(\ell, \mathbb{P}_{t,b_T}, c_{b_T})}} (\boldsymbol{v}_{b_T}(\ell_T^\star, c_{b_T}) - \boldsymbol{v})' V_{t,b_T}^{-1} (\boldsymbol{v}_{b_T}(\ell_T^\star, c_{b_T}) - \boldsymbol{v}); t = 1, \ldots, T - b_T + 1 \right\},$$

where $\mathbb{P}_{t,b_T}$ denotes the empirical distribution of the relevant subsample, and $\ell_T^\star$ is the optimal loss obtained from the optimization for the evaluation of the statistic in the full sample. V_{t,b_T} is the relevant Newey-West matrix associated with b_T-conformable bandwidth. The parameter estimators in the relevant forecasting models are likewise set fixed at their full sample values for simplicity. Then the null hypothesis of OPA is rejected iff $\mathrm{QLR}_T > q_{b_T}(1-\alpha)$.

3 Limit Theory

The limit theory of the testing procedure described above is derived in this section. The following paragraph presents the relevant statistical and probabilistic framework. This is almost identical to the one in APPK21-the differences lie in that in the present situation there are no blocking parameters, yet there are parameters associated with bandwidths and subsampling rates. Section 3.2 derives the null limiting behavior of the test statistic, along with the exactness and consistency of the testing procedure. The final paragraph provides with a brief discussion of the results.

Some additional notation is needed in what follows; $\|\cdot\|$ denotes the Euclidean norm, $\ell^\infty(A)$ the space of real-valued bounded functions on a set A equipped with the sup norm, and $\rightsquigarrow$ convergence in distribution. $\bar{B}_{\boldsymbol{\lambda}}(\eta)$ denotes the closed Euclidean ball in $\mathbb{R}^M$ centered at $\boldsymbol{\lambda}$ with radius equal to $\eta > 0$.

3.1 Assumption framework

As mentioned above the assumption utilized is almost identical to APPK21. It corresponds to (a) stationarity and dependence properties of the predictive variables; (b) smoothness properties of functions of the unknown parameters; (c) limiting representations for the estimators of the unknown parameters in the forecasting models, and their sample sizes; (d) the asymptotic rates of the slacks, the bandwidths and the subsampling rates that appear in the constructions of the test statistic and the subsampling rejection region. Those are exemplified in:

Assumption 1. *The following conditions hold:*

i. *For $r_T > 0$, as $T \to \infty$, $r_T \to \infty$ and $\frac{r_T}{T} \to \gamma \in (0, \infty]$.*

ii. *For all $i = 1, \ldots, M$, and any $t = 1, \ldots, T$, as $T \to \infty$, $\boldsymbol{\theta}_{i_t} = \boldsymbol{\theta}_{i_0} + H_{i_{r_T}} \left(\frac{1}{r_T} \sum_{j=t-r_T}^{t} h_{i,j} + o_{a.s.} \left(\frac{1}{\sqrt{r_T}} \right) \right)$, $H_{i_{r_T}} \rightsquigarrow H_{0_i}$ which is a non-singular $d_i \times d_i$ matrix, $\mathbb{E}\left[h_{i,j}\right] = 0_{d_i \times 1}$ and $\mathbb{E}\left[\|h_{i,j}\|^{2+\delta}\right] < +\infty$ for some $\delta > 0$.*

iii. *The vector process $\mathbf{Z}_t := \left[X_t, (\boldsymbol{Z}_{i,t}, h_{i,t})_{i=1,\ldots,M}\right]_{t \in \mathbb{Z}}$ is strictly stationary and absolutely regular with mixing coefficients $(\beta_k)_{k \in \mathbb{N}}$ that satisfy $\beta_k = O\left(k^{-r}\right)$ for $r > 1$. The joint distribution of $\mathbf{Z}_0$ has continuous marginals.*

iv. *For some $\eta > 0$, such that for $\boldsymbol{\theta} := (\boldsymbol{\theta}_1, \ldots, \boldsymbol{\theta}_M)$ restricted to $\bar{B}_{\boldsymbol{\theta}_0}(\eta) \subset \mathbb{R}^{\sum_{i=1}^{M} d_i}$, and $\boldsymbol{\theta}_0 = (\boldsymbol{\theta}_{1_0}, \ldots, \boldsymbol{\theta}_{M_0})$, the function $\boldsymbol{\theta} \to u\left(\mathbf{Z}_0, \boldsymbol{\theta}\right) := X_0 \mathbf{1}'_M - \left[m_1\left(Z_{1,0}, \boldsymbol{\theta}_1\right) \cdots m_M\left(Z_{M,0}, \boldsymbol{\theta}_M\right)\right]$ is almost surely Lipschitz continuous with respect to $\boldsymbol{\theta}$, with Lipschitz coefficient $l\left(\mathbf{Z}_0\right)$, that satisfies $\mathbb{E}\left[l\left(\mathbf{Z}_0\right)\right] < +\infty$. Furthermore, $\mathbb{E}\left[\sup_{\boldsymbol{\theta} \in \bar{B}_{\boldsymbol{\theta}_0}(\eta)} \|u\left(\mathbf{Z}_0, \boldsymbol{\theta}\right)\|^p\right] < +\infty$ for some $p \geq 3$, and for all t, the random variable $x_t - m_M\left(Z_{M,t}, \boldsymbol{\theta}_{M_t}\right)$ has a density, that is uniformly in t bounded away from zero.*

v. *The functions $\boldsymbol{\theta}_M \to \mathbb{E}_{\mathbb{P}}\left[u_M\left(\mathbf{Z}_0, \boldsymbol{\theta}_M\right)\right]$, and $(\ell, \boldsymbol{\theta}) \to \mathbb{E}_{\mathbb{P}}\left[\ell\left(u_i\left(\mathbf{Z}_0, \boldsymbol{\theta}\right)\right)\right]$ are continuously differentiable with respect to $\boldsymbol{\theta}_M$ on $Proj_M \bar{B}_{\boldsymbol{\theta}_0}(\eta)$, and $\boldsymbol{\theta}$ on $\bar{B}_{\boldsymbol{\theta}_0}(\eta)$, for all $\ell \in \mathcal{L}_1$ and $\sup_{Proj_M \bar{B}_{\boldsymbol{\theta}_0}(\eta)} \|D_{\boldsymbol{\theta}_M} \mathbb{E}_{\mathbb{P}}\left[u_M\left(\mathbf{Z}_0, \boldsymbol{\theta}_M\right)\right]\| < +\infty$ and furthermore $\sup_{\{1,\ldots,M\} \times \mathcal{L}_1 \times \bar{B}_{\boldsymbol{\theta}_0}(\eta)} \|D_{\boldsymbol{\theta}} \mathbb{E}_{\mathbb{P}}\left[\ell\left(u_i\left(\mathbf{Z}_0, \boldsymbol{\theta}\right)\right)\right]\| < +\infty$, where κ_i denotes the i^{th}-coordinate of κ, $Proj_i$ denotes projection to the i^{th}-coordinate, and $D_{\boldsymbol{\theta}_M} \mathbb{E}_{\mathbb{P}}\left[u_M\left(\mathbf{Z}_0, \boldsymbol{\theta}_M\right)\right]$, $D_{\boldsymbol{\theta}} \mathbb{E}_{\mathbb{P}}\left[\ell\left(u_i\left(\mathbf{Z}_0, \boldsymbol{\theta}\right)\right)\right]$ denote the relevant gradients w.r.t. $\boldsymbol{\theta}_M$ and $\boldsymbol{\theta}$ respectively.*

vi. *There exists some $\epsilon > 0$ such that*

$$\inf_{\ell \in \mathcal{L}_1^*(\mathbb{P}),\, CS(\ell, \mathbb{P}, 0) \neq \emptyset} \lambda_{\min}\left(\mathbb{E}_{\mathbb{P}}\left[\left(\ell\left(u_{i,0}\right) - \ell\left(u_{M,0}\right)\right)_{i \in CS}\left(\ell\left(u_{i,0}\right) - \ell\left(u_{M,0}\right)\right)'_{i \in CS}\right]\right) > \epsilon,$$

where $\lambda_{\min}(A)$ denotes the minimum eigenvalue of the positive-definite matrix A, and $\mathcal{L}_1^*(\mathbb{P})$ denotes the subset of $\mathcal{L}_1$ containing the loss functions that lie in the null hypothesis.

vii. *The slacks satisfy $c_T \to 0$ and for any subsequence $(T_\star)$, $\sqrt{T_\star} c_{T_\star} \to +\infty$ almost surely.*

viii. *The subsampling rate satisfies $b_T \to +\infty$ and $\frac{b_T}{T} = o(1)$. Furthermore, the bandwidth L satisfies $L \to +\infty$ and $\frac{L}{b_T} = o(1)$.*

Assumption 1.i-vii is identical to Assumption 4.1.1 of APPK21. For a detailed commentary of it and a comparison to assumption frameworks that appear in the literature the interested reader is referred to APPK21. It is remarked though that: i. the framework is restricted to unital forecasting horizons and rolling windows-both restrictions are generalizable to arbitrary horizons and/or fixed or moving estimation windows. ii. Stationarity of the process of the forecast errors is assumed. This is not harmless, as it for example disallows recursive estimation of latent model parameters using expanding estimation windows. iii. A plethora of regular M-estimators for the latent parameters of the forecasting models is allowed.

Assumption 1.viii is standard. It employs slower than the sample size divergence of the subsampling rates, and slower than the subsampling rates divergence for the bandwidths. The latter accommodates the use of the Newey-West estimators inside the subsampling phase.

3.2 Null limit theory and test properties

The main result below establishes the limiting properties of the empirical processes associated with the moment conditions, the null limiting distribution of the QLR test statistic, as well as the first order limiting properties of the associated testing procedure.

Theorem 1. *Under Assumption 1.i-v, and as $T \to \infty$, the following limiting distribution is obtained for the empirical process*

$$\sqrt{T}\left[\mathbb{E}_{\mathbb{P}_T}\left[\ell\left(\varepsilon_{i,t}\right)\right] - \mathbb{E}_{\mathbb{P}}\left[\ell\left(u_{i,0}\right)\right]\right] \rightsquigarrow \mathbb{G}_1\left(i,\ell\right) \text{ in } \ell^\infty\left(\{1,\ldots,M\} \times \mathcal{L}_1\right), \tag{8}$$

where $\mathbb{G}_1$ is a zero-mean Gaussian process with covariance kernel:

$$\begin{array}{c} K_{\mathbb{G}_1}\left(\left(i,\ell\right),\left(i^*,\ell^*\right)\right) := \sum_{t=0}^{\infty} \kappa_t Cov\left(\ell\left(u_{i,0}\right), \ell^*\left(u_{i^*,t}\right)\right) \\ +\varrho \sum_{t=0}^{\infty} \kappa_t Cov\left(\ell\left(u_{i,0}\right), D_{\boldsymbol{\theta}}\mathbb{E}_{\mathbb{P}}\left[\ell\left(u_i\left(\mathbf{Z}_0,\boldsymbol{\theta}_0\right)\right)\right] \boldsymbol{H}\boldsymbol{h}_t\right) \\ +\varrho \sum_{i=0}^{\infty} \kappa_t Cov\left(\ell^*\left(u_{i^*,0}\right), D_{\boldsymbol{\theta}}\mathbb{E}_{\mathbb{P}}\left[\ell^*\left(u_i\left(\mathbf{Z}_0,\boldsymbol{\theta}_0\right)\right)\right] \boldsymbol{H}\boldsymbol{h}_t\right) \\ +\varrho_\star D_{\boldsymbol{\theta}}\mathbb{E}_{\mathbb{P}}\left[\ell\left(u_i\left(\mathbf{Z}_0,\boldsymbol{\theta}_0\right)\right)\right] \boldsymbol{H} V_h \boldsymbol{H}' D_{\boldsymbol{\theta}}\mathbb{E}_{\mathbb{P}}\left[\ell^*\left(u_i\left(\mathbf{Z}_0,\boldsymbol{\theta}_0\right)\right)\right]' \end{array}, \tag{9}$$

$i, i^* \in \{1, \ldots, M\}$, $\ell, \ell^* \in \mathcal{L}_1$, and $\kappa_t = \begin{cases} 1, & t=0 \\ 2, & t>0 \end{cases}$; in addition, $\boldsymbol{H}$ is the $\sum_{i=1}^M d_i \times \sum_{i=1}^M d_i$ block diagonal matrix $diag_{1 \le i \le \sum_{i=1}^M d_i}(H_{0_i})$, $\boldsymbol{h}_t := (h_{i,t})'_{i=1,\ldots,M}$, $V_h := \sum_{t=0}^{\infty} \kappa_t \mathbb{E}[\boldsymbol{h}_0 \boldsymbol{h}'_t]$, and $\varrho = \begin{cases} 1 - \frac{\gamma}{2}, & \gamma < 1 \\ \frac{1}{2\gamma}, & \gamma \in [1, +\infty] \end{cases}$, $\varrho_\star = \begin{cases} 1 - \frac{\gamma}{3}, & \gamma < 1 \\ \frac{1}{\gamma} - \frac{1}{3\gamma^2}, & \gamma \in [1, +\infty] \end{cases}$.
Furthermore, if also Assumption 1.vi-vii holds, and under $\boldsymbol{H}_0(\mathcal{L}_1, \mathbb{P})$ and as $T \to \infty$, the limit distribution of the test statistic can be characterized as follows: i) if $\forall \ell \in \mathcal{L}_1^*(\mathbb{P})$, $\mathrm{CS}(\ell, \mathbb{P}, 0) \neq \emptyset$,

$$\mathrm{QLR}_T \rightsquigarrow \inf_{\ell \in \mathcal{L}_1^*(\mathbb{P})} \inf_{\boldsymbol{v} \in \mathbb{R}_+^{N(\ell,\mathbb{P},0)}} (\boldsymbol{v}(\ell, M) - \boldsymbol{v})' \operatorname{Var}^{-1}(\boldsymbol{v}(\ell, M)) (\boldsymbol{v}(\ell, M) - \boldsymbol{v}), \tag{10}$$

where $\boldsymbol{v}(\ell, M) := (\mathbb{G}_1(i, \ell) - \mathbb{G}_1(M, \ell))_{i \in \mathrm{CS}(\ell,\mathbb{P},0)}$, ii) if $\exists \ell \in \mathcal{L}_1^*(\mathbb{P})$, $\mathrm{CS}(\ell, \mathbb{P}, 0) = \emptyset$,

$$\mathrm{QLR}_T \rightsquigarrow 0. \tag{11}$$

Finally, if also Assumption 1.viii holds, then, for any $\alpha \in (0,1)$, and as $T \to \infty$, the testing procedure has the following properties:
A. Under $\boldsymbol{H}_0(\mathcal{L}_1, \mathbb{P})$, if i) above holds and $\alpha < 0.5$, then

$$\limsup_{T \to \infty} \mathbb{P}(\mathrm{QLR}_T \ge q_{b_T}(1-\alpha)) = \alpha, \tag{12}$$

while if ii) above holds, then

$$\lim_{T \to \infty} \mathbb{P}(\mathrm{QLR}_T \ge q_{b_T}(1-\alpha)) = 0. \tag{13}$$

B. Under $\boldsymbol{H}_1(\mathcal{L}_1, \mathbb{P})$,

$$\lim_{T \to \infty} \mathbb{P}(\mathrm{QLR}_T \ge q_{b_T}(1-\alpha)) = 1. \tag{14}$$

Proof. (8) follows directly from the proof of (21) in Theorem 4.2.1 of APPK21 for $B = 1$. Also from the proof of the aforementioned theorem it follows that $\mathrm{CS}(\ell_T, \mathbb{P}_T, c_T) \rightsquigarrow \mathrm{CS}(\ell, \mathbb{P}, 0)$ in the Painleve-Kuratowski topology, for any $\mathcal{L}_1 \ni \ell_T \to \ell \in \mathcal{L}_1^*(\mathbb{P})$ uniformly, when $\mathcal{L}_1^*(\mathbb{P}) \neq \emptyset$. Skorokhod representations, applicable due to Knight (1999) (4) and the fact that $\mathcal{L}_1^*(\mathbb{P})$ is compact in the uniform topology-see Paragraph A.2 in the Supplement of APPK21, Theorem 3.4 of Molchanov (2006) (5), the uniform in $\bar{B}_{\boldsymbol{\theta}_0}(\eta)$ convergence of the Newey-West estimator (as a function of $\boldsymbol{\theta}$), the pseudo consistency of the estimators, the non-degeneracy of the limiting variance matrix, and the previous imply (10) and (11). Assumption 1.vi implies the applicability of

Theorem 3.5.1.i of Politis et al. (1999) (7) to obtain (12), since the null limiting distribution of the statistic cannot have an atom at zero of probability greater than 0.5, due to the zero mean Gaussianity and the non degeneracy of the associated covariance kernel. (13) is obtained from (11) and the slower than the sample size divergence of the subsampling rate. (14) follows from that when $\mathcal{L}_1^*(\mathbb{P}) = \emptyset$, $\mathrm{QLR}_T \rightsquigarrow +\infty$ and the slower than the sample size divergence of the subsampling rate. □

The results along with Theorem 4.2.1 of APPK21 show directly that the first order limiting behavior of the QLR statistic is identical to the one of the BELR statistic used in the aforementioned paper as claimed before. This implies that both testing procedures are thus consistent, and very conservative when under the null there exist loss functions inside the null without contacts; conservativeness is due to the limiting degeneracy of the statistic. Absense of contacts for some loss function inside the null hypothesis is not unusual due to the finiteness of the number of moment conditions.

However, when every loss function in the null hypothesis attains contacts, there is a stark difference between the current testing procedure and the BELR conservative procedure of APPK21. The subsampling construction of the rejection region here, and the fact that the null limiting distribution of the QLR statistic cannot have an atom of probability greater than 0.5 at the origin, under the empirical relevant restriction that the significance level is less than 0.5, implies that the procedure is exact. Thus the current subsampling QLR procedure could have better local power properties under sequences of alternative hypotheses that converge on the boundary of the null. As explained in APPK21, this non-degeneracy for the limiting statistic is natural in several important cases. For example, if the set of alternative forecasting models includes nesting specifications and the null holds, then the contact sets are non-empty by construction and the procedure is asymptotically exact.

Similarly to what is remarked in APPK21, the limit theory can be extended to hold uniformly in the underlying distribution at least in iid settings. Under the appropriate topology on a class of underlying distributions containing $\mathbb{P}$, and via results like Theorem 2.8.9 of van der Vaart and Wellner (1996) (9), it is possible to extend (10) and (11) to hold locally uniformly in this class, and thus derive locally uniform exactness (consistency) for the test using convergent sequences of distributions for which the null (the alternative) holds.

3.3 Discussion

Besides the possible extensions mentioned in APPK21 that are also relevant here, an interesting path of further research could involve the extension of the present results in cases where pseudo-consistency for the estimators that live inside the forecasting models. The results concerning the limiting behavior of the test statistic per se are extendable to this case, under the appropriate extension of Assumption 1.i-v to hold for every possible accumulation point of the $\boldsymbol{\theta}_t$. The issue of tracking of the resulting sub-sequential behavior of the test statistic by the subsampling phase of the procedure seems more involved. The potential modification of the construction of the rejection region so as to accommodate the sub-sequential behavior of the test statistic while resulting into a procedure that is asymptotically exact and conservative seems important given the plausibility of misspecification for the forecasting models employed.

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