Department of Economics

Athens University of Economics and Business

WORKING PAPER no. 13-2024

`Pareto, Edgeworth, Walras, Shapley' Equivalence in a Small Economy

Angelos Angelopoulos

October 2024

Πατησίων 76, 104 34 Αθήνα. Τηλ.: 210 8203303-5. E-mail: econ@aueb.gr / www.aueb.gr
76, Patission Street, Athens 104 34 Greece. Tel.: (+30) 210 8203303-5

Πατησίων 76, 104 34 Αθήνα. Τηλ.: 210 8203303-5. E-mail: econ@aueb.gr / www.aueb.gr
76, Patission Street, Athens 104 34 Greece. Tel.: (+30) 210 8203303-5

‘Pareto, Edgeworth, Walras, Shapley’ Equivalence in a Small Economy

Angelos Angelopoulos[1]

Abstract

Heretofore, equivalence among allocative general equilibria, which may be viable without coalitions or may be classified as cooperative, and which may be supportable by prices or not, has profoundly been a privilege of economies with large populations. This paper shows that, with agents’ convex preferences, the same type of - even more powerful - bonds or coincidences can be yielded in economies with a small population scale, as long as the initial endowments allocation of the agents is (socially) efficient.

Key Words: General Equilibrium, Allocations, Pareto Optimal, Edgeworth Core, Walras Competition, Shapley Value, Equivalence, Finite Population, Initial Endowment, Efficiency, Convex Preferences.

Classification: D5.

Acknowledgements: The author acknowledges both a financial support from Erasmus+ for a research visit at the University of Salamanca and the University of Vigo and independent financial reimbursements from the two Universities, provided for the same reason. Accordingly, the paper has strongly benefited from discourses, suggestions and comments with and by Emma Moreno-García and Carlos Hervés-Beloso.

[1]Adjunct Professor, Department of Economics, Athens University of Economics and Business.

Prologue

In an economy of pure exchange, or of more sophisticated trade into markets, which is set forth as a consumption economy exclusively, and which is populated by a large number of agents, the Edgeworth (1881) core allocation, the Walras (1874) competitive allocation and the Shapley (1969) value allocation coincide in general equilibrium. *Inter alia*, see in Shapley and Shapiro (1960), Debreu and Scarf (1963), Shapley and Shubik (1969), Shubik (1969), Aumann (1964, 1975), Vind (1964, 1973), Hildenbrand (1974), Bewley (1974), Aumann and Shapley (1974), Grodal (1975), Champsaur (1975), Hart (1977), Mas-Colell (1977) and Anderson (1978). The three previous general equilibrium solutions, which are originally priced or not, and coalitional or not, validate, together with their Pareto (1906) optimal allocations counterparts, the most outstanding allocative general equilibria for a trade-to-consumption economy.

It is expedient to work with concise large economies. In them, unless uncertainty or some other exogenous factor creates accountable - and impossible to be eliminated - frictions to markets and/or traders, all participants are indistinguishable (equal) stake holders of the economy's estate and coffer, general equilibria exist under the weakest possible assumptions for the decision makers' tastes, sets of alternatives and initial inheritances and, most importantly, all general equilibria are indistinguishable (equal) as well, while they are automatically (or, in the end, unavoidably) normative: socially efficient, egalitarian-equitable and ethical. Size erases inefficiency, deletes distributional injustice and wipes off immorality. So in practice there is no much left to say or to do with such economies, apart from checking ones' aptitude in elevating the challenging indeed mathematical machinery that is parcelled into them.

This conveniency, however, is no longer appealing when (actual) small market economies, accommodating (plausible) unequal lucrative opponents, are put under the analytical microscope. In this case, all the previous virtues of the economy's allocative general equilibria are elusive. Ergo, they have to be earned, since they are not any more simply bestowed to the economy.

Remarkably now, the prefaced allocative coincidence can be retrieved - upgraded - in an economy with a finite number of participants, which is more tractable in analytical terms. For this reason, in turn, such an economy becomes immediately susceptible to empiricism and computations in the discipline of Economics. Besides, ostensibly, such an economy is more palpable and pragmatic, in this way facilitating the modeller's intention to make an artisan economic device stay *vis-à-vis* with a real life economy.

To robustly craft its argument, on the long road towards the fulfilment of this task, a bunch of subsidiary parameters, auxiliary leverages and ancillary collaterals get entangled with (and eventually subsumed into) the paper's analysis. In the end, however, it simply boils down to two sufficient conditions for the occurrence of the overlap in reference, a mild and a heavy one. In particular, agents' preferences have to satisfy convexity (this is the light assumption), while the agents' initial endowments allocation has to be Pareto optimal (this is the ponderous assumption).

A Universal Economy

To get down to this project, consider the (pulled back from production) economy

$$\mathcal{E} = \{B; X_i, \precsim_i, \omega_i : i \in C \subseteq I = \{1, 2, ..., n\}\}.$$

After each agent's preferences and initial endowment get properly intertwined, agents take up two distinctive roles in $\mathcal{E}$: they are traders (buyers and sellers of commodities) and consumers of commodities' quantities. Trade, consequently, refers to the commercial transactions that are executed by the agents into $\mathcal{E}$'s markets. Consumption, on the other hand, refers to the investment action of (demanding) and purchasing commodities' quantities. The return (or payoff, or gain) to the agents (or investors) from this investment activity is dubbed utility.

B is extraneous to $\mathcal{E}$. It is all the agents' extended action or investment space. It determines the number of markets or commodities in $\mathcal{E}$. Predominantly, the determinacy of the general equilibria that are associated with $\mathcal{E}$ relies heavily on the algebraic, ordering and topological structure of B. To this end, B is a pre-ordered separable Banach space, almost always of infinite dimension. B contains both consumption vectors and price vectors. B is Cauchy and Dedekind complete. The scalar field on B is $\mathbb{R}$, which is all the agents' extended re-action or outcome space. All properties for the mathematical items that take up an instrumental role in $\mathcal{E}$ hold, when required, with respect to some useful and suitable (metric induced) topology on B. The weak topology, the weak* topology or the Mackey topology of uniform convergence of weakly compacta of B are the exemplar such topologies.

B_+ is the positive cone of B and $X_i \subset B_+$ is the subjective (convex and closed) consumption set of agent $i \in I$[1]. The last inclusion implies that gross trading agents are seen as adding (and never extracting) commodities' quantities or assets into (out of, respectively) their consumption baskets or portfolios. X_i, $i \in I$, represents the Marshallian demand of agent $i \in I$, which becomes the Walrasian demand in general equilibrium. The Walrasian demand function $\{x_i(p, \omega_i) := x_i \mid x_i \in X_i\} \subset B_+$ of agent $i \in I$, where p is a (relative) prices' vector of B and $\omega_i \in B_+$ is the exogenous (demonetised) initial endowment of agent $i \in I$, satisfies the so called cash invariance (aka money or monetary neutrality and inessentiality) property: fix an $i \in I$; given any endogenous p and any exogenous ω_i, then the Walrasian demand function satisfies $\{x_i(p, \omega_i) = x_i(ap, a\omega_i) := x_i \mid x_i \in X_i, a \in R\} \subset B_+$ (scalar homogeneity of degree zero with respect to prices and income vectors). As usual, X_i, $i \in I$, is determined or formed by agent's i optimisation of utility function(s), or preferences, $\precsim_i$ (whose elucidation is pending). Subsequently, $\omega_i \in X_i \setminus \{\mathbf{0}\}$, $i \in I$. Foremostly, with this condition, it is secured that $X_i \neq \emptyset$ for every $i \in I$. At the same time, the original absolute exclusion of an agent from $\sum\limits_{i \in I} \omega_i = \mathbf{w} \in [\sum\limits_{i \in I} X_i \setminus \{\mathbf{0}\}] \subset B_+$, which is the aggregate initial resources of the agents (or of the economy), is dodged. A choice (or individual

[1] When adopting the practice of the formation of coalitions, the personalised consumption sets of agents can be ultimately consolidated to a public decision or social choice set; see in the sequel.

allocation) of an agent $i \in I$ is indicated with $x_i \in X_i$ and then with $\chi_i = (x_i - \omega_i) \in B$ we notate the net trade of the same agent i.

A (general equilibrium) allocation or distribution of $\mathcal{E}$'s aggregate income $\mathbf{w}$ (here, not necessarily specified to product or output) across $\mathcal{E}$' population $I \neq \emptyset$ is notated with the n-tuple $x = (x_1, x_2, ..., x_n) \in \prod_{i \in I} X_i := X$. This is a (foreground) gross trade allocation of $\mathcal{E}$. A (background) net trade allocation for $\mathcal{E}$ is denoted with $\chi \in X$. Gross trade allocations will be being typically manipulated, unless the context explicitly requires the conscription of net trade allocations.

Interchangeably, a (gross trade) allocation of $\mathcal{E}$ can be also stated with the (discrete here) map $x : I \to B_+$, such that $x(i) := x_i \in X_i$, for all $i \in I$. This notational option allows us to both (i) apprehend $\mathcal{E}$ as an 'agents onto markets system' and (ii) alternatively symbolise with $I^{B_+} = \{x | x : I \to B_+\}$ the function space of all allocations for $\mathcal{E}$, instead of employing the less instructive and informative symbol X. Both notations are deployable in $\mathcal{E}$. If $\mathcal{E}$ is a finite economy, reserving both a finite number of agents and a finite number of markets (or commodities) for its pursuits, then the neater and more declarative representation for the space of all allocations of $\mathcal{E}$, $B_+^{|I|} = B_+^n \supset X$, makes also sense.

X or I^{B_+} preponderantly contains allocations that ought to convey the generic conceptualisation and contextualisation, the one that revolves around the agents' axiomatic rationality, i.e., utility maximisation. To make sure that such an allocation exists, i.e., that $X = I^{B_+} \neq \emptyset$, $X_i \subset B_+$, $i \in I$, is assumed to be compact[2], so the space $X = I^{B_+}$ is also compact, so $\mathcal{E}$ has allocative general equilibria, which may be perceived as locally unique (practically, of a finite number) under appropriate assumptions (see in Debreu, 1970).

If $(\mathbf{x} =) \sum_{i \in I} x_i = \sum_{i \in I} \omega_i$, then x is a feasible allocation for $\mathcal{E}$. In this case, the markets of $\mathcal{E}$ clear without free disposal, i.e., without excess supply or, equivalently, with the ordinary (prices delivering) supply equals demand condition. The previous expression may be also indicated as $\sum_{i \in I} \chi_i = 0$, out of which $p \sum_{i \in I} \chi_i = 0$ is implied, for some price vector p of B, which is tantamount to the celebrated *Walras law*: the value of the sum of all excess demands of all markets must be zero in an economy. In the sequel, prices of $\mathcal{E}$ will be naturally restricted to B_+.

Define the partial order (thus, pre-order as well) $\leq^*$ that is globally attached to B. Less rigidly thinking, feasibility or admissibility of an allocation $x \in X$ could mean that $\sum_{i \in I} x_i \leq^* \sum_{i \in I} \omega_i$, on the proviso of course that the two former vectors do get binarily comparable and ranked. Note that for every agent $i \in I$ it holds that $x_i \leq^* \mathbf{w}$, to wit, $\mathbf{w} \in B_+$ is the least upper bound of every agent's consumption set, but then it also holds that $\mathbf{x} \leq^* |I|\mathbf{w} = n\mathbf{w}$. If one restrictively admits from the beginning that $x_i \leq^* \omega_i (\leq^* \mathbf{w})$ for every $i \in I$, then it is ensued that $\mathbf{x} \leq^* \mathbf{w}$.

[2]If the closed consumption set of each agent is (equi-potently) order bounded, norm bounded or finite radius ball bounded, then its compactness follows from the Heine-Borel property that B retains as a metric space.

To remedy the previous analytical restraints and complications, an allocation is feasible (acceptable or allowable) in the previous weaker sense if $\sum_{i \in I} x_i \leq^* \mathbf{0}$. It then follows that $\sum_{i \in I} x_i \leq^* \sum_{i \in I} \omega_i$. To evade potential analytical turbulences, it then has to be satisfied that $p \sum_{i \in I} x_i = 0$, for a price vector p of B, so that the *Walras law* is still valid. To emphasise, if $\sum_{i \in I} x_i <^* \mathbf{0} \Rightarrow$ (i) $\sum_{i \in I} x_i <^* \sum_{i \in I} \omega_i$ and (ii) $p = \mathbf{0}$ for the Walras law to be motionless, then the markets of $\mathcal{E}$ clear with excess supply or free disposal, quantities of commodities are left undisposed in $\mathcal{E}$, so they have to be freely (zero priced) disposed to the traders of the markets. In advance, besides, the weak feasibility of allocations highlights the merits of $p \in B_+$. Given some p of B_+, the implied expression $p \sum_{i \in I} x_i \leq p \sum_{i \in I} \omega_i$ provides the (normative) surplussed or balanced public budget of $\mathcal{E}$. With monotone preferences the previous deliverable is just the price supportability of allocations. Feasibility of allocations with strict inequality (strict equality, respectively) represent the idea that the feasible re-distributions of ω are leaky (not leaky respectively). In general, given the particularities upon which the model is parameterised with, feasibility of an allocation with or without free disposal may become a *sine qua non*.

By normalisation however, solely (stronger) feasibility with strict equality is generically acknowledged for $\mathcal{E}$ herein, irregardless of the (forthcoming) axiomatisation of the agents' rational preferences. To resume, I^{B_+} packages only rational and feasible allocations. Denote then with $\omega \in X$ the initial endowments allocation of $\mathcal{E}$, which is feasible by design.

The constructive formation of any $x \in X$, provided that x exists, is (distributively) conditional on ω: x is a feasible re-allocation or re-distribution of the (feasible) ω. This trait of allocations gives rise to a critical intrinsic property of $\mathcal{E}$: all allocations of $\mathcal{E}$ admit feasible re-allocations, so side payments or transfers of commodities among the agents are by default permitted in $\mathcal{E}$.

A priori, before any further (more specified) knowledge[3], the endogenous (optimal) existence of a rational $x \in X$ is subject to the feasibility condition $\mathbf{x} = \mathbf{w}$, which is the economy's recourses constraint, and then, upon this fundamental restriction, the existence of x becomes also fundamentally contingent onto the objective constraint $x_i \in [\mathbf{0}, \mathbf{w}] \cap X_i \neq \emptyset$[4], for every agent $i \in I$. Then,without loss in generality, each agent's terminating consumption set may be simply uniformly identified with the former (non-empty, convex and compact) order interval of B_+, whilst $I^{B_+} = X = [\mathbf{0}, \mathbf{w}]^n$. The rectangle $[\mathbf{0}, \mathbf{w}]$ is

[3]For example, information regarding the personal affordability of allocations, given agents' wherewithals.

[4]This is certainly true when $\mathbf{0} \in X_i$, $i \in I$, from the beginning, that is, when $\mathbf{0}$ is the greater lower bound of every agent's consumption set with respect to $\leq^*$. This translates as: given an agent's preferences, which determine the same agent's consumption set, this agent might end up with 'nothing' even if he begun with 'something'.

(i) a Dedekind complete lattice set inside B_+ and (ii) a comprehensive subset of B_+[5]. So its desirability with respect to accounting for the agents' order-theoretic rationality of preferences is eminent. Furthermore, with it, the following two prospects are applicable: (a) some conversion of the Weierstrass extreme value theorem onto Banach spaces (see for example in Sandine, 2024) and (b) the Schauder's fixed point theorem on convex and compact subsets of infinite dimensional normed spaces. So its desirability with respect to accounting for the agents' welfare-theoretic rationality of preferences is glaring.

A posteriori, the endogenous (optimal) attainment of a rational and feasible $x \in [\mathbf{0}, \mathbf{w}]^n$ is discretionarily customised to meet the case and context dependent guidelines that are exogenously set, which in turn agree with the modeller's beliefs.

Each agent in $\mathcal{E}$ develops subjective tastes in order to satisfy her private needs. Thereby, $\precsim_i$ are the (extrinsic to $\mathcal{E}$) idiosyncratic rational preferences of agent $i \in I$, which obey to objective norms and principles.

Primitively, i.e., order theoretically, they are complete, reflexive and transitive. Irreflexive and/or non-transitive, or even un-ordered preferences, are irrational *per se* with respect to order theory, and need to be rationalised for $\mathcal{E}$'s purposes, by being first of all ordered. Incomplete (reflexive and transitive) preferential pre-orderings, in the sense of being restrictively equipped (so as to totally operate) only onto the (closed and) bounded $X_i \subset B_+$ of each agent $\in I$ instead of the whole (unbounded) potential B_+, can be also conceptually viable in $\mathcal{E}$. Such incomplete pre-orders would condemn agents with bounded (instead of full) rationality.

Secondarily, welfare-theoretically, only because agents' preferences are continuous on the agents' compact X_i, $i \in I$, they are representable by a continuous utility function $u_i : X_i \to \mathbb{R}$, for each agent $i \in I$, which function is unique up to any, affine or not, monotone transformation; and conversely, continuous utility functions represent continuous preferences (see in Wold, 1943, Dedreu, 1954, 1959, 1964, 1972, Bowen, 1968, Arrow and Hahn, 1971, Chipman et al, 1971, Kreps, 1988, among plenty more other articles and surveys). On this account, the separability of B is crucial: if B does not contain a countable dense subset, then there can exist continuous preferences on B that are not representable by real valued utility functions (see in Beloso and Estevez, 1995). Continuity of preferences renders them eligible for global satiation or saturation. Equivalently, real valued continuous utility functions that represent them (defined upon compact consumption sets) are enabled to attain a global maximum.

On a third level of behavioural compatibility with rationalism, agents' preferences (loaded onto the agents' decision sets) are additionally monotone and convex. Monotone preferences are represented by increasing (on each argument separately) utility functions (and increasing functions are also quasi-concave); and *vice versa*, increasing utility functions represent monotonic preferences. Strictly monotone preferences transmit signals for unique existence of general equilibria and imply the feasibility of general equilibrium allocations as herein, with strict equality. A weaker assumption that could replace the monotonicity

[5]Consider an agent $i \in I$. X_i is a comprehensive subset of B_+ iff for any $y_i \in X_i$, $x_i \leq^* y_i$ (thus, x_i is less preferable than y_i according to the monotonicity of preferences) implies that $x_i \in X_i$.

axiom of preferences is the local non satiation of preferences. Convex preferences are represented by quasi-concave utility functions. Inversely, concave utility functions, which are automatically quasi-concave, imply (or represent) convex preferences. In the same fashion, strictly convex preferences are tied with strictly concave utility functions. For more delicate details on the relationship between concave utility functions and convex preferences see in Moulin (1973).

At large, the existence of general equilibria in a trade-to-consumption economy as $\mathcal{E}$, with a finite number of agents and an infinite number of markets or commodities, may occur without convex and monotone - and even without continuous - preferences on each agent's action set. Same case as with their existence without order-theoretically rational preferences, i.e., with preferences not being complete pre-orders on each agent's set of alternatives. For the most recent and generalised advancement in this theme, see in Podczeck and Yannelis (2024). For the purposes of this paper, however, $\mathcal{E}$ remains a standard economy, with sticky continuous, monotone and convex - and of course complete, reflexive and transitive - preferences on the agents' consumption sets X_i, $i \in I$. Anyhow, the fusion of the last two preferential axioms in $\mathcal{E}$, monotonicity and convexity, reflects the law of declining positive marginal utility for the agents of this economy: utility always increases with the marginal addition of a consumption unit, but at a decreasing rate. Roughly speaking, 'more' is always better than 'less', but from some point and on the diversity or quality (and not only the quantity) of commodities matters as well

Convexity of preferences proves to be an axiom of paramount importance for the purposes and proceedings of this paper, thus, calls in advance for extra attention. So below are several well known and firmly established in the literature definitions concerning this notion.

Fix an agent $i \in I$ and consider the consumption arrays $x_i, y_i, z_i \in X_i$ and the (non-empty) set $V_i(x_i) = \{y_i \in X_i : y_i \succsim_i x_i\} \subseteq X_i \subset B_+$ of i[6]. Then the preferences $\succsim_i$ ($\iff \succ_i$) of agent i (which are defined up-onto agent's i X_i) are declared:

• convex iff $V_i(x_i)$ is convex in B_+ iff $y_i \succsim_i x_i \Rightarrow ay_i + (1-a)x_i \succsim_i x_i$, $a \in [0,1]$ iff $x_i \succsim_i z_i$, $y_i \succsim_i z_i \Rightarrow ax_i + (1-a)y_i \succsim_i z_i$, $a \in [0,1]$.

• strictly convex iff $y_i \succ_i x_i \Rightarrow ay_i + (1-a)x_i \succ_i x_i$, $a \in (0,1)$ iff $x_i \succsim_i z_i$, $y_i \succsim_i z_i$, $x_i \neq y_i$ $\Rightarrow ax_i+(1-a)y_i \succ_i z_i$, $a \in (0,1)$ iff $x_i \sim_i y_i$, $x_i \neq y_i \Rightarrow ax_i+(1-a)y_i \succ_i x_i(\sim_i y_i)$, $a \in (0,1)$.

• semi strictly convex iff (i) $y_i \succ_i x_i \Rightarrow ay_i + (1-a)x_i \succ_i x_i$, $a \in (0,1]$ and (ii) $y_i \sim_i x_i \Rightarrow$ $ay_i + (1-a)x_i \succsim_i x_i(\sim_i y_i)$, $a \in [0,1]$.

[6]Recall that $x_i \in X_i = [\mathbf{0}, \mathbf{w}]$. Then, $V_i(\mathbf{w}) = \{\mathbf{w}\}$ and $V_i(\mathbf{0}) = X_i$. This is true because $\mathbf{0}$ is the infimum (minimum) and $\mathbf{w}$ is the supremum (maximum) of X_i with respect to $\leq^*$, while i's preferences are monotone on X_i.

In the absence of externalities, transaction costs and any other 'bads' that provide negative utility, the agents' utilitarian welfare space may be simply reduced from $\mathbb{R}$ to $\mathbb{R}_+$[7]. Next, since each market's commodity is a 'good' (not a 'bad'), it will be definitely evaluated and priced strictly positively (not strictly negatively) by the agents. So, by also making allowances for the fact that there always exist commodities that can be price-freely dispatched to and enjoyed by the whole population simultaneously[8], price vectors of $\mathcal{E}$ dwell inside B_+. Henceforward, either latently or explicitly, allocations in $\mathcal{E}$ may be supported by general equilibrium prices (or not).

There may exist severe tension and polarisation between the miscellaneous general equilibria of $\mathcal{E}$. In general, however, the conventional neoclassical wisdom mandates general equilibria in $\mathcal{E}$ to take the form of prices-allocation schedules $(p, x) \in B_+ \times X$. The desirable (or at least indicative) case is when a unique price vector accompanies an allocation, i.e., the *law of one price* holds. $\mathcal{E}$ is externally commissioned by the modeller (or internally destined by nature) to foster, avail and service general equilibria of that ilk. So in $\mathcal{E}$, the most highly ranked task is to constitute all general equilibrium allocations supportable by the overlying prices, by either centralising or de-centralising them. With this procedure, the disparities among $\mathcal{E}$'s general equilibria are mitigated, without their normativeness being lost.

Normativeness of an allocative general equilibrium is a matter of stability for this general equilibrium. As a testimony, once a prices-allocation pair has been attained with precision in $\mathcal{E}$, and has been then characterised with rigor as normative (socially efficient, equitable and ethical), it becomes from within steady, stable or sustainable, because it possesses qualities that can be hardly outflanked by any other general equilibrium pair that is stripped from these properties. At the same time, normative priced allocations demand market stability as well, in the sense that they must not be sluggish when they need to (re)adjust to the movements of the supply-versus-demand market forces. This kind of stability of general equilibria has been robustly insured by the literature. To quote several only relevant essays, the reader is initially referred to the hallmark treatise of Hicks (1939), and then to Arrow and Hurwicz (1958), Arrow et all (1959) and Negishi (1962), who generalise the idea of Hicksian stability.

In the thin or tight $\mathcal{E}$ there exists a small quantity of sufficiently large individuals. In such a framework, agents have inbuilt heterogenous market powers and differential comparative advantages. With them, they can disturb and divert the natural occurrence of the market outcomes, and lessen the degree of contestability in the economy's markets, even if the number of markets (or commodities) is large, i.e., countably infinite. There are always moral hazards and agents' incentives do not necessarily display compatibility

[7]The controversial Coase Theorem, attributed to Coase (1960) by Stigler (1966) who propagated it in the literature as an actual Theorem, can partially provide a (not impervious to criticism) resolution to the issue of the existence of (Marshallian to Pigouvian) utilitarian welfare externalities within the agents' trade.

[8]Such as the economy's public goods, or the freely disposed commodities when the economy's markets balance with a general glut.

with ethics. To recite a couple only of manifestations of the agents' opportunistic or rent-seeking (immoral) behaviours that may upset $\mathcal{E}$'s allocative general equilibria, agents can influence the market prices, or agents informed of the forthcoming general equilibria can manipulate them using either their utility functions (see in Hurwicz, 1972) or their initial endowments (see in Postlewaite, 1979). This weak *status quo* for $\mathcal{E}$ can be fluently shifted to a stronger one, via the employment of a thick or spacious analogue of $\mathcal{E}$, one with a large volume of arbitrarily small individuals. Markets then acquire homogenous participants and become perfectly competitive *per se*, since all agents' personal asymmetries and behavioural imperfections are depleted. Coincidences between the general equilibria of $\mathcal{E}$ are then encouraged.

Certainty (thus, non randomness) and agents' full and public information about everything prevails in $\mathcal{E}$. $\mathcal{E}$, however, can be rigorously reverted to a weaker non-deterministic *ex ante - ex post* economic environment with Dworkin's (1981a,b) uniform across the agents and outwardly sourced luck, which is brute or involuntary, not option, provoked or induced luck. Option luck extinguishes the need for egalitarianism in $\mathcal{E}$. The favourable or unfavourable exterior events of this luck are measured or evaluated by prior subjective probability measures of the agents, when exogenous informational (dis)advantages for the agents are also framed and underpinned by $\mathcal{E}$. Coincidences between the general equilibria of $\mathcal{E}$ are then discouraged.

A coalition $C \subseteq I$ is a community of individuals with an inwardly and uniformly arranged etiquette and, thereafter, with interior mandatory operational protocols. It may be envisaged as a club of exclusive rights and liabilities for its members. Its formation is incentivised by the undeniable force of unity. A coalition is always observable and co-appears as a single (distinguishable) economic entity in $\mathcal{E}$, since atomic actions (decisions or choices) are taken co-existentially inside a coalition. It is, therefore, a discrete economic unit, of separate units or decision makers that get synthesised together in a unified economic body. A coalition, in other words, may be broadly understood as a company, a partnership or a (trade) union of individuals, with a commonly agreed code of attitude, which in turn projects united customs, laws, rules, stances and judgements for all its members[9]. Agents, however, join coalitions with the intention to reinforce their own (and only) personal allocative positions, by enhancing the mutual or joint position of the coalition they take part into. They do share the same values and a communal vision or objective, to contribute to the growth or augmentation of the coalition's equity, but they do so by caring for and servicing their personal interests and benefits.

Now an agent (voluntarily) enters into a coalition of $\mathcal{E}$ with the transparent intention to cooperate in the aforementioned sense, transmitting therefore clear and sincere signals of companionship. Same as with $\mathcal{E}$' markets, there are no entry barriers to the coalitions of $\mathcal{E}$, so an agent can (and does eventually) participate into all the teams of $\mathcal{E}$, even if he does so nully or dummilly, by managing to make zero contribution to the coalitions he becomes a member of.

[9]For instance, a household (i.e., a family) or a firm (i.e., a productive entrepreneurial enterprise) are coalitions of agents.

Each time an agent enters into a coalition, the agent exchanges priors and posteriors with all the rest of the agents in this coalition. With this process, agents of the same coalition share first of all their consumption sets: an agent $i \in C$ reforms its consumption set into the convex and compact box $[\mathbf{0}, \sum_{i \in C} \omega_i] \cap X_i \neq \emptyset$ of B_+. When no cooperation of agents is visualised, the coalition I is solely portrayed in $\mathcal{E}$.

In sum, in the coalitional regime of $\mathcal{E}$, upon their exchangeable consumption sets, both consumption bundles (i.e., individual allocations) and utilities are coalitionally transferable (re-distributable or re-allocated) among the agents, either when agents are trading cooperatively (namely, in/within all coalitions $C \subseteq I$) or when they are trading solo (or else, in isolation, inside I only).

Let the vector $\lambda \in \Delta$, where Δ is the standard (probability or unit) $(n-1)-$simplex of $\mathbb{R}^n$. Each coordinate of this vector, λ_i, $i \in I$, is a personal factor for every agent, pinpointing to the agent's weight (strength or market power) to $\mathcal{E}$ and, in particular, inside $\mathcal{E}$'s coalitions. The same number is also a tool that enables, in the outset, agent $i \in I$ to perform utilitarian welfare interpersonal comparisons and listings inside any coalition so as to, subsequently, voluntarily circulate his utility (that is, pass his utility to other persons) inside this coalition. In other words, instrumental transferability of utility (accordingly, of utilitarian side payments) is perceivable and captured in $\mathcal{E}$. For some $\lambda_i \in [0,1]$ that agent $i \in I$ founds herself with, and for some utility function that represents agent's i preferences, $\lambda_i u_i(x_i) \geq 0$[10], $x_i \in [\mathbf{0}, \sum_{i \in C} \omega_i]$, is the weighted and transferable utility this agent i inside the coalition $C \subseteq I$. Together with $\lambda \in \Delta$, finally, consider the agglomeration of all agents' preferences inside a set $\precsim = \{\precsim_i : i \in I\}$. This set gives rise to alternative sets $u = \{u_i : i = 1, 2, ..., n\}$.

Had a perfectly competitive $\mathcal{E}$ been depicted under the continuum of agents, it would have been the case that $\lambda_i = 0$, for every $i \in I$, and only non-negligible coalitions would have been important or valuable (powerful or worthy), with non-null volume or magnitude (and weight and contribution) to $\mathcal{E}$. If an imperfectly competitive $\mathcal{E}$, on the other hand, aspires to be run under an individually equitable (fair and impartial) basis, then the market powers of all the asymmetric agents of $\mathcal{E}$, even if assigned with heterogenous or subjective priors, should be synchronised to the agents' equal (common, uniform, public or objective) weight $\lambda_i = \frac{1}{n}$, $n \neq 0$. Viewing now this equitable (just and neutral) situation the other way around: agents with the same private characteristics (initial endowments, hence, preferences and, hence, automatically consumption sets), who have equal weights and make exactly the same contribution to $\mathcal{E}$'s collations, end up with identical individual allocations in a general equilibrium, thus, with indistinguishable utilitarian welfares. This is the so called *equal treatment* (equitability) *property* of $\mathcal{E}$ (and its allocations).

Specify the set of preferences $\precsim$ into some set of utility functions u and employ then the set function $V_{\lambda,u,I^{B_+}} : 2^I \to \mathbb{R}_+$, which is defined by the formula $V_{\lambda,u,X}(C) = \sum_{i \in C} \lambda_i u_i(x_i)$,

[10]For some $i \in I$: $\lambda_i u_i(x_i) = 0$ iff at least one of the λ_i or $u_i(x_i)$ is zero. Note that it does not necessarily hold that $u_i(\mathbf{0}) = 0$, $i \in I$.

and call it the value or characteristic function of $\mathcal{E}$. This set function measures the (aggregate or coalitional) weighted utilitarian proceed or earning of any coalition $C \in \mathcal{P}(I)$, hence, the projected worth, value or power of this coalition in $\mathcal{E}$, under alternative consumption choices of all the agents inside the coalition C. It satisfies super-additivity and takes the zero value for the empty coalition. Any positive supper-additive set function is also monotone[11]. The functional format of $V_{\lambda,u,X}$ is simply the weighted average of the utilities of all agents that have joined an alternative coalition. When all agents have equal weights, the former collapses to the simple arithmetic mean of the inter-coalitional utilities.

Let next $(\emptyset \neq) S, T, .. \subseteq I$ symbolise coalitions of agents of I, disjoint or not, and recall that $|I| = n$. Define the map $Sh : I \times \{V_{\lambda,u,X}\} \to \mathbb{R}_+$, the formula of which is

$$Sh(i, V_{\lambda,u,X}) := Sh_i(V_{\lambda,u,X}) = \sum_{(\emptyset \neq) S \subseteq I, i \in S} \tfrac{(|S|-1)!(|I|-|S|)!}{|I|!} [V_{\lambda,u,X}(S) - V_{\lambda,u,X}(S \setminus \{i\})] (\geq 0).$$

This the Shapley (1953) value of agent $i \in I$. The Shapley value of $\mathcal{E}$, $Sh(\mathcal{E})$, is the vector

$$Sh(\mathcal{E}) = (Sh_i(V_{\lambda,u,X}) : i = 1, 2, ..., n)^T \in \mathbb{R}^n_+.$$

The mathematical economic substantiation and the interpretation of the subtle notion of an agent's Shapley value are pressing, because the $Sh(\mathcal{E})$ justifiably becomes a cooperative allocative general equilibrium solution for $\mathcal{E}$ of supreme elegancy and unique impact onto the equitable cake-cutting literature.

(A) Start with the clarification of the part $[V_{\lambda,u,X}(S) - V_{\lambda,u,X}(S \setminus \{i\})]$, $i \in S \subseteq I$, of the algebraic expression supra. This is, first of all, always a positive number due to the monotonicity of the $V_{\lambda,u,X}$. It is, afterwards, interpreted as the marginal (or unit) weighted utilitarian welfare contribution of agent $i \in I$ (when entering and/or being) inside the coalition $S \subseteq I$. One may alternatively restate the former difference by defining, since $S \ni i$, $S = T \cup \{i\}$ and $T = S \setminus \{i\}$ (not that when $S = \{i\}$ then $T = \emptyset$), so that $[V_{\lambda,u,X}(T \cup \{i\}) - V_{\lambda,u,X}(T)]$ (≥ 0) is now the new (equivalent to the previous) factor that interchangeably arises, carrying exactly the same interpretation as with its predecessor. Super-additivity then of the $V_{\lambda,u,X}$ implies that $V_{\lambda,u,X}(T \cup \{i\}) \geq V_{\lambda,u,X}(T) + V_{\lambda,u,X}(\{i\})$ or, equivalently, that $[V_{\lambda,u,X}(T \cup \{i\}) - V_{\lambda,u,X}(T)] \geq V_{\lambda,u,X}(\{i\})(\geq 0)$. Ultimately, the acquisition of this fundamental observation indicates that the λ-scaled utilitarian welfare of each agent is fortified and amplified inside any coalition, due to the powers of unity, association and solidarity inside any coalition, so the atomic weighted utilitarian welfare contribution of agent $i \in I$ inside the coalition $S \subseteq I$ is greater or equal to the individual and independent (non contributed to any coalition) utilitarian welfare of this agent. The Shapley value of agent $i \in I$ supersedes this idea. Indeed, since, for every $i \in S \subseteq I$, $[V_{\lambda,u,X}(S) - V_{\lambda,u,X}(S \setminus \{i\})] \geq V_{\lambda,u,X}(\{i\})$, it follows that

[11]Consider the specific set function $V_{\lambda,u,X}$ and take two coalitions $S, T \subseteq I$ with $S \subseteq T$, so that $T = S \cup F$, with $S \cap F = \emptyset$, $F \subseteq I$. Then, since $V_{\lambda,u,X}$ is super-additive, one has that $V_{\lambda,u,X}(T) = V_{\lambda,u,X}(S \cup F) \geq V_{\lambda,u,X}(S) + V_{\lambda,u,X}(F)$, thus, $V_{\lambda,u,X}(T) \geq V_{\lambda,u,X}(S) \geq 0$, for $V_{\lambda,u,X}(F) \geq 0$.

$Sh_i(V_{\lambda,u,X}) \geq V_{\lambda,u,X}(\{i\}) = \lambda_i u_i(x_i)$. This inequality is translated as: individualism or autarky (i.e., non cooperation) is definitely inefficient in terms of utilitarian welfare yields. As $S \subseteq I$, from the last inequality it is then implied that

$$\sum_{i \in S} Sh_i(V_{\lambda,u,X}) \geq \sum_{i \in S} V_{\lambda,u,X}(\{i\}) = V_{\lambda,u,X}(S),$$

whilst, in particular, it can be eventually deduced that

$$\sum_{i \in I} Sh_i(V_{\lambda,u,X}) = V_{\lambda,u,X}(I).$$

This equality is translated as: social cooperation, inside the grand coalition of all agents, is for sure efficient in terms of utilitarian welfare awards.
(B) Move now to the factor $\frac{(|S|-1)!(n-|S|)!}{n!} \in (0,1)$, for every $\{i\} \subseteq S \subseteq I$, of the Shapley value of an agent. The meaning of this ratio is enveloped into (and accrues from) a finite series of steps. More precisely, given that
(i) in total, there are $n!$ possible ways of ordering the $\{1, 2, ..., n\}$ agents (of coalition $I \ni i$, i.e., of) $\mathcal{E}$ (with all of these scenarios or cases being equally likely to happen),
(ii) in total, there are $(|S|-1)!$ (equiprobable) ways to rank the agents of $S \setminus \{i\}$
and
(iii) in total, there are $(n-|S|)!$ (equiprobable) prospects to list the agents of $I \setminus S$,
the specific ratio stands for the probability of the agent i to find himself into a/any random listing of the agents of any non-empty coalition $S \subseteq I$. This is the probability of the occurrence of a random event. More constructively and accurately arriving to this concept, this fraction can be understood as the probability that the agent $i \in I$ enters into the coalition $T = S \setminus \{i\}$, which consists of all the other (than i) agents being listed in any possible order within T, and who enter in T either strictly before or strictly after i so as to co-form with i the coalition S, when all the rest of the (possibly remaining) agents of $I \setminus S$ are free to potentially join $S \ni i$, also in the same random pattern that agent i obeys to. It is worth distinguishing, finally, the case of the two extremes in the quotient $\frac{(|S|-1)!(n-|S|)!}{n!}$, $\{i\} \subseteq S \subseteq I$,$S = \{i\}$ and $S = I$. The probability of i to join in any randomly ordered position the coalition $\{i\}$ is $\frac{1}{n}$, which is equal to the probability of i to enter in any randomly ordered location the coalition I.

By combining now (A) and (B) supra, the Shapley (1953) value of an agent $i \in I$ equals by design to: the varying probability of the agent $i \in I$ to place herself into a/any random ordering of the agents of any non-empty coalition $S \subseteq I$ *times* the marginal weighted utilitarian welfare contribution of agent i (when entering and/or being) inside each coalition S. Consequently, when $\mathcal{E}$ is solved in general equilibrium with the $Sh(\mathcal{E})$, and each agent attains her $Sh_i(V_{\lambda,u,X})$, $i \in I$, every agent is rewarded with the expected (or average) contribution that she makes to a coalition of $\mathcal{E}$. Therefore, $Sh(\mathcal{E})$is an equitable (fair and impartial) outcome for $\mathcal{E}$.

In an economy with massive population, the set function of the prequel analysis would be replaced by the atomless Lebesgue (probability) measure μ which would be equipped upon the agents' measure space $([0,1],\mathcal{B}([0,1]))$, where $\mathcal{B}([0,1])$ is the Borel σ-algebra of all the Borel (thus, Lebesgue as well) measurable subsets of I[12]. That being the case, all agents would be appearing again as (infinitesimal) units in any general equilibrium that transcends $\mathcal{E}$, but all the activities in $\mathcal{E}$ towards the fulfilment of this general equilibrium would be inescapably executed via (and inside) the important sets or coalitions of agents. Any mathematical-economic relationship concerning both the agents' actions (behaviours, choices or allocations) and re-actions (outcomes, utilities or welfare) would hold for every $i \in I$, μ-a.e. or a.s. (equivalently, for almost all agents). Equitability would not be an issue in such an economy and drops off the analytical radar, since size removes inequity (see in Hildenbrand and Kirman, 1973, but the interested reader is referred to all the general equilibrium literature pivoting around the consumption economies with large populations).

To close the presentation of $\mathcal{E}$, $\mathcal{E}$ is clearly a normalised economy. Evidently, for the accustomed instructive reasons of keeping a concise analytical facet, $\mathcal{E}$ admits elemental presentational simplifications and abstractions from reality.

Some of them are: $\mathcal{E}$ is a-political, a-spatial and a-temporal. $\mathcal{E}$ does not contain a public sector (although public goods are traversable by $\mathcal{E}$). $\mathcal{E}$ is not a barter economy, but is a demonetised economy, with pay-as-you-go (not over the counter) markets. $\mathcal{E}$ does not contain an underground or shadow sector of stealthy, unobservable and unregistered economic activities. $\mathcal{E}$ contains benevolent (moral) agents, with flat needs, acumen, sanity, phycology, emotional intelligence an so on. $\mathcal{E}$ does not contain intermediary agents, i.e., delegates or representatives of the principal agents. $\mathcal{E}$ authorises general equilibrium allocations that are (indifferently to and neutrally for the analysis) already and by command sustainable with respect to agents' well-being, quality of life or non-utilitarian welfare; or likewise, according to utilitarian welfare that is drawn by environment, health, education, culture, lifestyle, leisure versus work and the like standard of living terms, apart from consumption[13]. $\mathcal{E}$ does not explicitly accommodate qualitative pricing of commodities' qualities. $\mathcal{E}$ contains only purely divisible commodities. $\mathcal{E}$ abstains from extra-neoclassical irrational, but colourful as well, contrivances.

More or less laboriously, all these normalisations can be relinquished, since most of the conceptual enrichments or modifications that would come from these sides or aspects of the general equilibrium story would add rudimental, routine or even negligible mathematical complications to the model, in essence, to the cell of the tale that is meant to be told by it. For example, ethical, explicit pecuniary, governmentally facilitated, political, 'sustainable with respect to what makes life worth living' and 'vicinity and/or time dependent' allocations are tagged as big (and potentially highly complicated) themes in the general

[12]Let $\nu : \mathcal{B}([0,1]) \to \mathbb{R}_+$ be a Borel measure. Then $\mu : \mathcal{L}([0,1]) \to \mathbb{R}_+$ is the extension of ν (and ν is the restriction of μ). In other words, μ completes some Borel measure ν by agreeing with it on the Borel measurable sets of $[0,1]$. That is, the Lebesgue σ-algebra of $[0,1]$, $\mathcal{L}([0,1]) \supset \mathcal{B}([0,1])$, is the completion of $\mathcal{B}([0,1])$ (think the Lusin set in $[0,1]$). The second is the smallest σ-algebra generated by all the open (or closed) subsets of $[0,1]$, while the first is the smallest σ-algebra generated both by all the open (or closed) subsets of $[0,1]$ and by the null or negligible sets of $[0,1]$.

[13]This doctrine enables agents' utility or return from consumption of commodities' quantities to be more generally comprehended as hedonism, satisfaction, pleasure, happiness or joy.

equilibrium literature, but they do not (ordinarily) rotate the mathematical economic facade of $\mathcal{E}$.

Some of them however are intrinsically impactful, in the sense of bringing about non-trivial mathematical economic consequences with respect to $\mathcal{E}$'s original script. For instance, for the inclusion of (the suppressed in $\mathcal{E}$) indivisible commodities, $\mathcal{E}$ must not be a convex economy, and convexities of consumption sets, preferences and spaces of allocations have to be dropped from $\mathcal{E}$'s affairs. For an exemplar text that matches $\mathcal{E}$'s mentality, see in Mas-Colell (1977). Furthermore, for the eradication of order-theoretically rational allocative general equilibria from $\mathcal{E}$, and the passage to less narrow equilibria which either are deficient in, or even entirely lack, order theoretic rationality, see in the seminal essays of Mas-Colell (1974) and Gale and Mas-Colell (1975, 1979). Additionally, for the explicit blending of $\mathcal{E}$'s agenda with qualitative consumption, and accordingly qualitative pricing of commodities' qualities, see in the influential paper of Lancaster (1966). Moreover, when agents' stated preferences are revealed and become testable, the relevant literature documents and suggests that the axiomatisation that is embedded into the agents' preferences might get deformed in denomination (but not in essence), so as to get adjusted to the new facts and conditions that arise. The intrigued reader may navigate herself through the associated bulky literature, which was initiated by Samuelson (1938, 1948). Concurrently, $\mathcal{E}$ can eloquently champion choices that are elicited using choice functions, which in turn are derived by preference relations of axiomatisation that is closely affiliated with the one in $\mathcal{E}$. Arrow (1951) was the first one to point out the implementation of this practice. Finally, if agents are termed players, whose action sets become then inter-dependent and joint, so that each agent possesses an inter-correlated preferences correspondence, while $\mathcal{E}$ is presented into its underlying (market) 'game in normal form', as the game theoretic jargon has it, say $\mathcal{G}$, then stronger (non-cooperative or cooperative) strategic solutions for $\mathcal{E}$ follow though. The cornerstone of them is the emblematic Nash (1950, 1951) equilibrium, which is labeled as non-cooperative, but for the transition to cooperative solutions see, for example, in Scarf (1971). Mechanism design theory can be then naturally amalgamated with $\mathcal{E}$, especially the Harsanyi's (1967) incomplete information approach.

To conclude, $\mathcal{E}$ transcends all domains of Economics. Arguably, although $\mathcal{E}$ is the most minimal economy one can perceive, it is the basal, benchmark and universal economy in Economics, especially when production is activated and restored back into $\mathcal{E}$. At the same time, $\mathcal{E}$ is an impeccable economy in all respects. To substantiate this last statement: if the feasibility of allocations (which is founded upon the iconic markets' *Walras law*) cannot be circumvented, and if the practice of maximisation of utility functions (which is built upon the agents' rationality prompting welfare-theoretic preferential axioms) cannot be bypassed, then (i) it is tough to conceptually falsify $\mathcal{E}$, while (ii) it is impossible to prove that the mathematical economic principles that span $\mathcal{E}$ are erroneous.

Definitions, Existence, Normative Properties and Relation of Allocations

The objective of the previous section was to create a universal body in Economics, the economy $\mathcal{E}$. Feasible allocations, that are founded upon the agents' rational state of minds, are the first-hand accountable ones in $\mathcal{E}$. *Ab ovo*, only such allocations are allowed or accepted to be attained or admitted by $\mathcal{E}$. Thereupon, when putting normative sensors in I^{B+} so as to reasonably restrain its magnitude, two definitions are needed for a first normative filtration and classification (or stratification) of the feasible (rationality purporting) allocations of $\mathcal{E}$ into accordingly suitable classes (or strata). Axiomatic choice theoretic or decision theoretic rationality of allocations will be silently taken for granted from now on. It will be a background binding condition that limits the size of the feasible allocations that can participate into a designated category of allocations, but no direct reference will be being made (in this sense) into this term.

Let $y \in X$ be a feasible re-allocation or re-distribution (among all the agents of I) of some feasible (original allocation or distribution) $x \in X$; this implies that $\sum\limits_{i \in I} y_i = \sum\limits_{i \in I} x_i = \sum\limits_{i \in I} \omega_i$. Then in $\mathcal{E}$:

• $x \in X$ is Pareto optimal (socially efficient) iff given $x \in X$, $\nexists\ y \in X$: (i) $y_i \succsim_i x_i$, $\forall i \in I$, with specifically (ii) $\exists\ i \in I$: $y_i \succ_i x_i$.
{in words: iff with the attainment of $x \in X$, all the Pareto improvements have been made within I iff there does not exist a $y \in X$ which strictly improves or benefits at least one agent, while leaving unaffected or indifferent (i.e., not harming or dis-benefiting) all the rest of the agents}

• $x \in X$ is individually rational (personally efficient) iff given $x \in X$, $x_i \succsim_i \omega_i$, $\forall i \in I$.

Claim. In $\mathcal{E}$, every Pareto optimal allocation is individually rational.

Proof. A Pareto optimal allocation $x \in X$ includes that case that $\nexists\ y \in X$: $y_i \succ_i x_i$, $\forall i \in I$. Then, for (the feasible) $\omega \in X$ it must hold that $x_i \succsim_i \omega_i$, $\forall i \in I$. Thus, the Pareto optimal x is individually rational.

Remark 1. The most general album of normatively accessible (feasible) allocations in $\mathcal{E}$ is the one encircling those allocations that are furnished with the feature of personal efficiency. This portfolio of allocations of $\mathcal{E}$ exhibits the least possible requirements for a (feasible) allocation to enter into. Now, upon a given (feasible) allocation $x \in X$, one may restrict the population of $\mathcal{E}$, I, to a non-empty group of collaborating agents $S \subseteq I$, namely, derive the $x|_S$. To test the stability of the pilot (feasible) allocation $x \in X$ one may then induce new (re)distributions with respect to x exclusively among and concerning the sub-population of I, S. These (re) allocations with respect to x have to be coalitionally feasible (c-feasible for short) with respect to the $\omega|_S \subseteq \omega$. A $x|_S$ may then simply collapse to a sub-allocation or sub-distribution of x, i.e., $x|_S \subseteq x$, but this is the extraordinary case. Obviously, with this protocol, every feasible allocation $x \in X$ can be constructively and for technical reasons converted to a c-feasible allocation. Albeit, to demand stronger c-feasibility of allocations in is the first place so as to prematurely serve normativeness in $\mathcal{E}$, to wit, to formally portray a desirable situation where an allocation $x \in X$ is by construction feasible within all its sub-allocations, is clearly a futile project. Nonetheless, given some feasible $x \in X$, the illustration and precise definition of a steady (invariant or stagnant with respect to x) position channeled through a scenario in which all the sub-populations of I consistent with provoked c-feasibility satisfy individual rationality is not void. In mathematical parlance: (i) a (feasible) individually rational allocation $x \in X$ implies that $x_i \succsim_i \omega_i$, $\forall i \in S$, for all $S \subseteq I$, but (typically) with $\sum\limits_{i \in S} x_i \neq \sum\limits_{i \in S} \omega_i$, $S \subseteq I$ and (ii) the feasible $x \in X$ is coalitionally individually rational (c-individually rational or c-personally efficient) iff given the feasible $x \in X$, for all $S \subseteq I$, $x_i \succsim_i \omega_i$, $\forall i \in S$ and $\sum\limits_{i \in S} x_i = \sum\limits_{i \in S} \omega_i$. In $\mathcal{E}$: (i) every c-individually rational allocation is individually rational, (ii) c-personal efficiency is a stronger notion than (and implies) the concept of personal efficiency, (iii) the (non vacuous) cohort of c-individually rational allocations is a strict subset of the one of individually rational allocations since the first is enriched with more attributes than the second and (iv) the genre of c-personal efficient allocations demonstrates more barriers for feasible allocations to become member of. Pareto optimal allocations do not fulfil c-individual rationality. They have to be dressed with more specifications for the realisation of this incidence. When conceptualising the Pareto optimality notion, the smaller the number of (immobile from I) agents that can shatter a given allocation by escaping from this allocation, the stronger (or the more solid) the idea of Pareto optimality is. If the compelling idea is that all the agents, that are stationary (i.e., invariantly situated) in I, have to flee from and abandon a given allocation so as to destroy it, then the weakest possible Pareto optimality arises: the feasible $x \in X$ is weakly Pareto optimal (weakly socially efficient) iff given the feasible $x \in X$, $\nexists\, y \in X$: $y_i \succ_i x_i$, $\forall i \in I$. This admission in turn renders the Pareto optimality that is defined supra strong (or firm). Strong Pareto optimality is stricter than (and implies) the weak Pareto optimality (as in the Claim above).

Remark 2. Upon the previous two definitions that normatively scan and filter X, let $\mathcal{A}(\mathcal{E}) \subset I^{B_+}$ be the class of all the (feasible) Pareto optimal (inherently supplied with individual rationality as well) allocations of $\mathcal{E}$. The (feasible) allocation ω is by definition individually rational. If it is Pareto optimal (individually rational) it belongs in $\mathcal{A}(\mathcal{E})$. All the canonically priced, i.e., in congruence with the *laissez faire* neoclassical creed, Pareto optimal allocative schemes are licensed and included in $\mathcal{A}(\mathcal{E})$. Notwithstanding, $\mathcal{A}(\mathcal{E})$ may be accrediting and engulfing other (close to the tenet of neo-liberalism) Pareto optimal allocative plans that are sensibly backed by prices. $\mathcal{A}(\mathcal{E})$ is the normative allocative kernel of $\mathcal{E}$, scilicet, the most magnetic field of general equilibrium allocations of $\mathcal{E}$. Inevitably, $\mathcal{A}(\mathcal{E})$ is convex in $I^{B_+} = X = [\mathbf{0}, \mathbf{w}]^n$.

Recall next the (feasible) Core, Competitive and Value allocations of $\mathcal{E}$. **Let**

(i) $p \in B_+$ **be** an endogenously attained, uniquely existing general equilibrium market-clearing price system of $\mathcal{E}$, ordinarily identified with a price simplex[14] and potentially pairing with $\mathcal{E}$'s general equilibrium allocations as follows: p supports $x \in X$ iff $\exists\, y \in X$: $y_i \succsim_i x_i\ \forall\, i \in I \Rightarrow py_i \geq px_i\ \forall\, i \in I$,
(ii) $\mathcal{B}_i(\omega_i, p) = \{x_i : px_i \leq p\omega_i\} \subseteq X_i$ **be** the non-empty, convex and compact budget set (of affordable consumption combinations) of agent $i \in I$,
(iii) $(\emptyset \neq)S \subseteq I$ is a coalition of agents of I,
(iv) for any $S \in \mathcal{P}(I) \setminus \{\emptyset\}$, $x|_S \in \prod\limits_{i\in S} X_i := X|_S$, with $x|_S \nsubseteq x$ in principle, **be** a c-feasible with respect to $\omega|_S \in X|_S$, $\omega|_S \subseteq \omega$, re-allocation or re-distribution (among all the agents of S) with respect to the feasible original allocation or distribution $x \in X$; $\sum\limits_{i\in S}(x|_S)_i = \sum\limits_{i\in S}(\omega|_S)_i$; $x|_I = x \in X|_I = X$,
(v) $u = \{u_i : i = 1, 2, ..., n\}$, $u_i \in \mathcal{U}_i$, $i \in I$, **be** a cluster of all the agents' some specification of their (ordinally equivalent) utility functions (see also Remark 3 down below) and
(vi) $\lambda_i \geq 0$ and $\sum\limits_{i\in I}\lambda_i = 1$ $(\Rightarrow \lambda_i \in [0,1], i \in I)$ **be** the endogenously attained weight (and utility transferability mechanism) of agent $i \in I$ inside $\mathcal{E}$'s coalitions.

Then, whenever reference is exclusively made to feasible allocations:

• $x \in \mathcal{C}(\mathcal{E}) \subset I^{B_+}$ is a core allocation for $\mathcal{E}$ iff given $x \in X$, $\nexists\, x|_S$, $S \subseteq I$: (i) $(x|_S)_i \succsim_i x_i$, $\forall i \in S$, with specifically (ii) $\exists\, i \in S$: $(x|_S)_i \succ_i x_i$.
{in words: iff with the attainment of $x \in X$, all the Pareto improvements have been made within all the coalitions of I}
• $x \in \mathcal{W}(\mathcal{E}) \subset I^{B_+}$ is a competitive allocation of $\mathcal{E}$ iff $\exists\, y \in X$: $y_i \succ_i x_i$, $\forall i \in I \Rightarrow py_i > p\omega_i \geq px_i$ $[\Rightarrow py_i > px_i]$, $\forall i \in I$.
{in words: iff whenever $x \in X$ is strictly supportable by p, which means that if some other allocation $(x \neq)y \in X$, which is, for every agent, strictly more desirable (or beneficial in utilitarian welfare) than x, then y may be strictly more valuable, expensive or pricy than x, but y is not affordable with respect to p and ω, as x is, and falls out of the budget set of every agent}
• $x \in \mathcal{V}(\mathcal{E}) \subset I^{B_+}$ is a value allocation of $\mathcal{E}$ if and only if
(1) $\forall i \in S \subseteq I$, $x_i \in x \in \mathcal{V}(\mathcal{E})$ maximises every $V_{\lambda,u,X}(S)$, $S \subseteq I$, s.t. $\sum\limits_{i\in S} \cancel{x_i} \leq^* \mathbf{0}$.
(2) $\lambda_i u_i(x_i) = Sh_i(V_{\lambda,u,X})$, $\forall i \in I$.

{in words: iff with her (typically unequal) individual allocation from x and (generically differential) individual weight and preferences (or utility function), each and every agent

[14]The infinite dimensional simplex of B may be defined to be the closure of the convex hull of the standard basis vectors in B. A price simplex has an economic translation: market prices in $\mathcal{E}$ are relative, not absolute.

contributes to the maximisation of the overall or additive welfare (i.e., the sum of independent weighted utilities) of every coalition she becomes a member of, subject to possibly leaky feasibility inside this coalition, when the (personal) scaled utility of each agent is corresponded to the (personal) Shapley value of this agent ($\Rightarrow$ when x is an equitable allocation)}.

Moreover, $\mathcal{C}(\mathcal{E})$ (the Allocative Core of $\mathcal{E}$ = the set of all core allocations of $\mathcal{E}$) is non empty. Relevant results can be found in Scarf (1962, 1967), Vind (1965), and elsewhere. $\mathcal{W}(\mathcal{E})$ (the set of all Walrasian or Competitive allocations of $\mathcal{E}$) is non empty. Sample works are: Arrow and Debreu (1954), MacKenzie (1954, 1959, 1981), Negishi (1960), Bewley (1972), Florenzano (1983), Aliprantis and Brown (1983) and Yannelis (1985). $\mathcal{V}((\mathcal{E})$ (the Allocative Value of $\mathcal{E}$ = the set of all the Shapley value allocations of $\mathcal{E}$) is non empty. Specimen parers are: Shafer (1980), Scafuri and Yannelis (1984) and Emmons and Scafuri (1985).

By and large, existential results for the herein examined allocative general equilibria involve usage of the theory of correspondences, separation theorems, fixed point theorems, the Gale (1955) – Nikaido (1956) – Debreu (1956, 1959) lemma for the excess demand correspondence, truncation of the infinite-dimensionality of B, in between other popular convex analysis optimisation methods on Banach spaces.

Remark 3. The first two of the previous three definitions (and actually all the definitions and results of this paper) can be equi-powerfully reverted to ones comprising (globally maximisable wherever appropriate) utility (functions). In particular, let $\mathcal{U}_i \ni u_i$ be a class of well-behaved ordinally equivalent (hence, cardinally equivalent as well) numerical utility functions $u_i : X_i \to \mathbb{R}_+$ of agent $i \in I$. Each one of the $u_i \in \mathcal{U}_i$ preserves agent's $i \in I$ preferential weightings (scalings) and rankings (orderings) under any increasing monotone (linear or not) transformation of this utility function. Then a $u_i \in \mathcal{U}_i$ represents $\precsim_i$ of agent $i \in I$ if and only if for two consumption profiles $x_i, y_i \in X_i$, if $x_i \precsim_i y_i$, then $u_i(x_i) \leq u_i(y_i)$.

Remark 4. It is a widely known result in the general equilibrium literature that all the three allocative general equilibria supra belong in $\mathcal{A}(\mathcal{E})$, and in particular that $\mathcal{V}(\mathcal{E}) \subseteq \mathcal{W}(\mathcal{E}) \subseteq \mathcal{C}(\mathcal{E}) \subseteq \mathcal{A}(\mathcal{E})[\subseteq \mathcal{IR}(\mathcal{E})^{15}] \subset I^{B_+}$ holds, so $\mathcal{A}(\mathcal{E}) \neq \emptyset$. Still, from this ordering by inclusion relationship, several fresh (viewed from somewhat novel lenses) insights and intricacies are propitious.
(a) One is that $\mathcal{C}(\mathcal{E})$ is simply a refinement of $\mathcal{A}(\mathcal{E})$. This means that a core allocation is simply a (stricter) coalitionally Pareto optimal (c-Parato optimal, or c-socially efficient) allocation for $\mathcal{E}$. The reason being that, in a $x \in \mathcal{C}(\mathcal{E})$, the blocking drill is tested inside any coalition, not simply inside the grand coalition of agents. In light of this fact, with a $x \in \mathcal{C}(\mathcal{E})$ none coalition or smaller community, not just the whole society, can Pareto improve (or dominate) the original allocation. c-Pareto optimality is stronger than Pareto

[15] This is the set of all individually rational allocations of $\mathcal{E}$.

optimality in the sense that it provides a stronger test for establishing social efficiency, by using all the coalitions of I. With it, more opportunities are provided to an agent to demolish a given allocation, by breaking out from this allocation, when deserting and evacuating I at the same time, since a mobile agent can resort to all the available coalitions of I in order to run away from a given allocation. c-Pareto optimality provides a double social efficiency insulation to a candidate allocation of $\mathcal{E}$. Concurrently, by virtue of the Claim in the prequel, every c-Pareto optimal allocation is c-individually rational. Same as with the Sh($\mathcal{E}$), the Core($\mathcal{E}$) is a cooperative non-strategic game theoretic solution for $\mathcal{E}$, which is borrowed by $\mathcal{E}$ by some superjacent to $\mathcal{E}$ cooperative non-strategically solved market game Γ. The theory of such games in then diffused into $\mathcal{E}$.

(b) Another is that, same as with the core and the Pareto optimal allocations, the competitive allocation and the value allocation of $\mathcal{E}$ are intuitively correlated and bear (less obvious this time) morphological similarity. After careful consideration, actually, one can easily realise that the second is an equitable-coalitional refinement of the first. To see that every Shapley value allocation, by admitting price supportability, is an equitable-coalitional (e-c for short) Walrasian allocation, one first needs to roll forward the definition of the Walrasian allocation into its utilitarian outfit. So restate the definition of a (feasible) competitive allocation as follows: $x \in \mathcal{W}(\mathcal{E}) \subset I^{B_+}$ is a competitive allocation of $\mathcal{E}$ iff $\forall i \in I$, $x_i \in x \in \mathcal{W}(\mathcal{E})$ maximizes any $u_i \in \mathcal{U}_i$, subject to $\mathcal{B}_i(\omega_i, p)$ iff $\forall i \in I$, $[\mathcal{W}(\mathcal{E}) \ni x \ni]x_i \in \arg \max_{x_i \in \mathcal{B}_i(\omega_i,p)} u_i(x_i)$, for any $u_i \in \mathcal{U}_i$. Consider now the definition of the value allocation under the particular case where $S = \{i\}$ and $\lambda_i = \frac{1}{n}$, for all $i \in I$. Condition (1) of this definition becomes then: $\forall i \in I$, $x_i \in x \in \mathcal{V}(\mathcal{E})$ maximises $V_{\lambda,u,X}(\{i\}) = \frac{1}{n}u_i(x_i)$, subject to $x_i \leq^* \omega_i$, (thence, subject to $x_i \precsim_i \omega_i$ by dint of the monotonicity of agents' preferences,) thus, subject to $px_i \leq p\omega_i$, for some $p \in B_+$. Also, by having set S and $\lambda_i, i \in I$, as before, condition (2) of the same definition becomes: $V_{\lambda,u,X}(\{i\}) = u_i(x_i)$. The concurrent consideration of the two newly and specifically derived conditions amounts simply to the *de jure* fact that the utilitarian welfare of the agent $i \in I$ is given by any of the two ordinally equivalent (thence, cardinally equivalent as well) utility functions u_i or $\frac{1}{n}u_i$ of i. To conclude: $\forall i \in I$, $x_i \in x \in \mathcal{V}(\mathcal{E})$ maximises $\frac{1}{n}u_i \iff u_i$, subject to $px_i \leq p\omega_i$, for some $p \in B_+$. So any value allocation of $\mathcal{E}$ is a (c-e) competitive allocation. To close with an introspective property of a (feasible) $x \in \mathcal{V}((\mathcal{E}) \subset I^{B_+}$, in a value allocation of $\mathcal{E}$ a (sub)allocation is at once decided and selected for a (sub)society, i.e., for a coalition, by all the associates of this community, in order for the aggregate-communal weighted utilitarian welfare of this coalition to be maximised. Ergo, in reality, this allocative notion is originally (when viewed alone and remote from the other allocative general equilibria of $\mathcal{E}$) a social choice theoretic one.

Remark 5. The upper extreme in the in-reference multiple inclusive ordering has already been identified as that convex neighbourhood of $I^I B_+$ which pulls close to its premises all the rest of the general equilibrium allocations in $\mathcal{E}$. *De facto*, broadly speaking, the Walrasian allocative equilibria, which are placed in middle of the previous ordering by containment relationship, are the landmark ones in $\mathcal{E}$. This is not only because allocative competitive general equilibria are supported by prices, or because stronger Nash-Walras allocative general equilibria are contained in $\mathcal{W}(\mathcal{E})$, but also because they are viable under the loosest (rational) behavioural assumptions: agents are autonomous, care only about themselves and are not dependable on the other agents in any respect, apart from when located into the grand coalition of $\mathcal{E}$. Once $\mathcal{W}(\mathcal{E})$, the consensual point of origin in neoclassicism, is targeted, the above stated ordering by containment expression ultimately equips $\mathcal{E}$ with the celebrated and imperative first welfare theorem. On the other lower extreme of this serial inclusion, $\mathcal{V}(\mathcal{E})$ contains also allocations that can be always supported by prices, so $\mathcal{V}(\mathcal{E})$ can naturally enlarge towards $\mathcal{W}(\mathcal{E})$, so as to approximately get centralised afterwards, by having approached arbitrarily close to $\mathcal{A}(\mathcal{E})$. However, $\mathcal{C}(\mathcal{E})$ does not conform to the price supportability idiosyncrasy, unless it is decentralised, recedes further away from $\mathcal{A}(\mathcal{E})$, that is to say, gets shrunk and returns back to $\mathcal{W}(\mathcal{E})$ (fulfilling, in this manner, the Edgeworth's conjecture). Ideally, $\mathcal{V}(\mathcal{E}) = \mathcal{W}(\mathcal{E}) = \mathcal{C}(\mathcal{E}) \subseteq \mathcal{A}(\mathcal{E})$. Super ideally, $\mathcal{V}(\mathcal{E}) = \mathcal{W}(\mathcal{E}) = \mathcal{C}(\mathcal{E}) = \mathcal{A}(\mathcal{E})$. The last equality means folding backwards $\mathcal{A}(\mathcal{E})$, a situation that is understood as the decentralisation of $\mathcal{A}(\mathcal{E})$, via its price supportability. By concentrating onto the focal $\mathcal{W}(\mathcal{E})$ once more, this prospect of attaining the last equation between families of allocations ultimately delivers the very attractive second welfare theorem for $\mathcal{E}$.

Note 1. It is suasive at this point to familiarise ourselves more intimately with the competitive general equilibria of $\mathcal{E}$. To start delving into this matter of prominent interest, if agents' preferences are strongly monotone, then their budget sets reduce to budget hyper-lines of $X_i = [\mathbf{0}, \mathbf{w}] \subset B_+$, $i \in I$. If this is the case, in lieu of the Marshallian demand functions, which originate from utility maximisation, Hicksian demand functions, that emanate from expenses or expenditure minimisation, are also amenable. Then, a dual definition for a (priced) competitive allocation (that is included into a competitive general equilibrium) is also straightforwardly presentable. Particularly in this dual backdrop, rational preferences secure for each agent the possession of a pack of well-shaped indifference (in utility procurement) hyper-surfaces within her consumption set, by which mathematical economic tools some degree of pair-wise substitutability (or complementarity) between all the economy's commodities, and inside each agent's utility function(s), is captured. On $X_i \subset B_+$, a well shaped indifference surface that falls into the ownership of an agent $i \in I$ is connected, convex to the origin, contains uncountable infinitely many elements and satisfies a trend for outward permutation. Jevons, Edgeworth, Pareto and Slutsky are the legendary figures who coined and engineered the specific ideas with their (historic and timeless) ingenious treatises. Indifference curves were eventually pioneered and popularised in a simple mathematical context by Allen and Hicks, 1934. In this graphically oriented behavioural version of $\mathcal{E}$, an indifference surface that is found to be into the property of some agent $i \in I$ is the geometrical locus that collects all the points (i.e., commodity combinations) of $X_i = [\mathbf{0}, \mathbf{w}] \subset B_+$ that provide equal or constant (positive) utility, i.e., some fixed level of utility, to this agent. To vividly plot such a level surface, just give exogenously an alternative (positive) value to some well defined functional format $u_i(x_i)$, $x_i \in [\mathbf{0}, \mathbf{w}]$, of some $u_i \in \mathcal{U}_i$, of some agent $i \in I$, and then take and gather together in a graph (or a set) all the possible vectors x_i of $[\mathbf{0}, \mathbf{w}]$ that salsify the resulting equational algebraic formula. The set $\mathbb{U}_i = \{u_i(x_i) : x_i \in X_i\}$ is (the gauged values of) a $u_i \in \mathcal{U}_i$ of an agent $i \in I$. Any utility level $u_i(\bullet) \in \mathbb{U}_i$ is associated with an indifference surface for the agent $i \in I$. The only exogenous utility level inside $\mathbb{U}_i$ is the $u_i(\omega_i)$, which is therefore corresponded to an exogenous level surface of agent $i \in I$. The beacon economic idea that is nested into this simple mathematical process is that all points (or consumption choices) at some indifference surface of some agent are equally preferred by the agent. This idea acts as a catalyst in general equilibrium theory. It becomes the concept that aligns, levels, bridges and syntheses the neoclassical economics with the powerful new Paretian theory[16]. According to the former, it eventually becomes meaningful only to ask which option is quantitively better than the other, while it is meaningless to ask how much better it is, so the evaluation or quantification of the extant utilitarian differences between any two selections for this agent do not matter. By extrapolating this principle in a setting with multiple agents who form a society, disparities between the agents' preferences are not important, so it is fruitless to (necessarily and strictly) contrast or interrelate (so as to aggregate for example) the personal utility functions (equivalently, the subjective preferences) of agents. This is why the vintage notion of the Paretian collective utilitarian optimality of agents does not require direct correlations and comparisons among (and consequently aggregation of) the agents' utilities. On the contrary, it finds only the discrepancies between the alternative utilities of any agent that are gained under alternatively examined re-allotments of the commodities' quantities, and across all the agents, inside $\mathcal{E}$.

Main Results

This section initially retrieves:

1. a weak equi-preference among all the allocative general equilibria of the previous section, with Pareto optimal initial endowments and any type of convexity of preferences, that is, without convexity of preferences restrictions and
2. a strong equivalence among all the allocative general equilibria of the previous section, with Pareto optimal initial endowments and strict convexity of preferences, that is, with concrete preferential restrictions with regards to convexity.

[16]The old Paretian (relative utilitarian) welfare does not even require explicit measurement of utility, and speculates that it is enough for the utility levels of an individual to be scaled according to some Likert-like scale, from worst, to worse, to good, to better and to best. Utility is a feeling, a sense or, more generally, a phycological state, so it cannot be measured and quantified in absolute quantities. If, in the aftermath, utility is calculated in real numbers, its unit of measurement is 'utils', and certainly not 'money'.

In Note 2, the penultimate deed of this section, a stricter equi-preference and a stronger equivalence among the allocative general equilibria of the previous section are smoothly deduced.

Theorem 1. If $\omega \in \mathcal{A}(\mathcal{E})$, then for any individually rational allocation $x \in \mathcal{IR}(\mathcal{E})$ it holds that $x_i \sim_i \omega_i$, $i \in I$ (i.e., the two vectors belong in the same indifference surface of the agent $i \in I$).

Proof. Take a $x \in \mathcal{IR}(\mathcal{E})$. For x it holds that $x_i \succsim_i \omega_i$, for all $i \in I$. If $x_i \succ_i \omega_i$ for some $i \in I$, then x Pareto improves upon ω through this i, which is a contradiction since $\omega \in \mathcal{A}(\mathcal{E})$. This then implies that $x_i \sim_i \omega_i$, for every $i \in I$.

Theorem 2. If ω is Pareto optimal and $\precsim_i$, $i \in I$, are strictly convex, then $\mathcal{A}(\mathcal{E}) = \{\omega\}$.

Proof. Assume that there exists another (feasible) allocation $x \in \mathcal{A}(\mathcal{E})$ such that $x \neq \omega \in \mathcal{A}(\mathcal{E})$. $x \neq \omega$ implies that there exists an agent $i \in I$ such that $x_i \neq \omega_i$, $x_i \in [\mathbf{0}, \mathbf{w}]$, $\omega_i \in (\mathbf{0}, \mathbf{w})$. For $a \in (0,1)$, take a linear combination of x and ω, $z = [ax + (1-a)\omega] \in \mathcal{A}(\mathcal{E})$, where $z \in \mathcal{A}(\mathcal{E})$ because $\mathcal{A}(\mathcal{E})$ is convex. z is a feasible allocation ($\mathbf{z} = \mathbf{w}$), since for $a \in (0,1)$:

$$\sum_{i \in I} z_i = a \sum_{i \in I} x_i + (1-a) \sum_{i \in I} \omega_i = \sum_{i \in I} \omega_i, \text{ given that } \sum_{i \in I} x_i = \sum_{i \in I} \omega_i.$$

Because $x \in \mathcal{A}(\mathcal{E})$, x is an individually rational allocation (see in the Claim of the previous section), which means that $x_i \succsim_i \omega_i$, for all $i \in I$. Pick and fix the agent $i \in I$ with $x_i \neq \omega_i$. Then we can have two cases:
1. If $x_i \succ_i \omega_i$, then, by means of the proof of Theorem 1, this option is not allowable and only $x_i \sim_i \omega_i$ is so; besides, from strict convexity of $\succsim_i$, it is then implied that $ax_i + (1-a)\omega_i \succ_i \omega_i \iff z_i \succ_i \omega_i$, which is a contradiction since this means that z Pareto improves on ω via this agent i.
2. If $x_i \sim_i \omega_i$, then again from strict convexity of $\succsim_i$ it is implied that $ax_i + (1-a)\omega_i \succ_i \omega_i \sim_i x_i \iff z_i \succ_i \omega_i$, which is the same contradiction as before.

Corollary. Theorem 1 declares that any individually rational allocation of $\mathcal{E}$ is equally preferable with the allocation $\omega \in \mathcal{A}(\mathcal{E})$, thence, is weakly isomorphic to it. Consequently, all Pareto optimal, Core, Walras and Shapley value allocations of $\mathcal{E}$, which are individually rational, round up to (i.e., collide closely into) a unique allocation, the $\omega \in \mathcal{A}(\mathcal{E})$. From Theorem 2 it is implied that ω is the unique (feasible) $x \in \mathcal{A}(\mathcal{E})$. This, in turn, implies that $\mathcal{V}(\mathcal{E}) = \mathcal{W}(\mathcal{E}) = \mathcal{C}(\mathcal{E}) = \mathcal{A}(\mathcal{E})$, i.e., that all allocations precisely meet.

Note 2. The singleton class $\mathcal{A}(\mathcal{E}) = \{\omega\}$ is indeed convex. In general, this class can be empty (which never is), a singleton set or an uncountably infinite set, given its convexity. Pareto optimality of ω can easily come apart with the mobility of agents that is associated with the formation of their coalitions. If ω is c-Pareto optimal (i.e., a core allocation), thus, Pareto optimal as well, and agents' preferences are strictly convex, then, if it is once more explicitly reported that $\mathcal{C}(\mathcal{E})$ is convex in I^{B_+}, it can be proved that $\mathcal{C}(\mathcal{E}) = \{\omega\}$, hence, that $\mathcal{V}(\mathcal{E}) = \mathcal{W}(\mathcal{E}) = \mathcal{C}(\mathcal{E}) \subseteq \mathcal{A}(\mathcal{E})$, which is a less rich, but more stable, Shapley-Walras-Core allocative equivalence. The proof would simply reprint all the steps and arguments of the proof of Theorem 2, with the difference that a c-feasible (thus, feasible as well) $x \in \mathcal{C}(\mathcal{E})$ would have to be summoned, while this x would then originally be c-individually rational, hence, eventually individually rational as well. By flipping the same coin onto its other side now, there is also in $\mathcal{E}$ a stricter, than that (but in the same motif) of Theorem 1, idea of synonymity or parity among the allocative general equilibria in reference. In a rational allocative general equilibrium of $\mathcal{E}$, when all the agents rivalrily exhibit exactly the same behaviour, all the Pareto optimal individual allocations of an agent $i \in I$ are eventually situated into the highest indifference surface of that agent, subject to the economy's allocative feasibility constraint, namely, subject to the (partial) order-boundedness condition $x_i \leq^* \mathbf{w}$; they are located, in other words, up-onto the most upwardly based - inside the rectangle $X_i = [\mathbf{0}, \mathbf{w}]$ - indifference surface of agent i; recall that the order interval $[\mathbf{0}, \mathbf{w}]$ is a Dedekind complete lattice set in side B_+. This in turn implies that the class $\mathcal{A}(\mathcal{E})$ is generated by the cross-product of those particular indifference surfaces of all agents. This specific collection and conglomeration of the agents' indifference surfaces contains all the core, the competitive and the value allocations of $\mathcal{E}$, which are Pareto optimal. The inclusion of the class $\mathcal{W}(\mathcal{E})$ in this family of indifference surfaces implies simultaneously the familiar tangency condition of each one of these classified level surfaces with the personal budget hyper-line of the agent who owns this indifference surface. If $\omega \in \mathcal{A}(\mathcal{E})$, then this allocation is also included into the formerly specified family of indifference surfaces, hence, co-exists with and is equally preferred to any Edgeworth-core, Walras-competitive and Shapley-value (Pareto optimal, thus, individually rational) allocation of $\mathcal{E}$.

Postscript. Regressively thinking, irrelevantly to the existence (or non existence) of deeper qualities for the agents' initial endowments and/or preferences, if $|I| = 2$ (and the Edgeworth's Box illustration of $\mathcal{E}$ follows through) the set of all the individually efficient and the set of all the c-individually efficient allocations acquire membership into the longer of the two aforementioned equalities or equivalences. If $\mathcal{E}$ is a Robinson Crusoe economy (i.e., $|I| = 1$), then the previous equality-equivalence is trivial.

Extensions. Any progressive reproduction of $\mathcal{E}$, that stays loyal to $\mathcal{E}$'s analytical trail, does not cancel out or neutralise this paper's deliverables. For example, it is conspicuous that all the analysis of the paper remains valid if the cardinality of I changes to the one of either $\mathbb{N}$ or $[0, 1] \subset \mathbb{R}$, or to any discrete-continuum mixture with agents' types and replica economies. The same is also state-wisely (i.e., state by state) true if Knightian (1921) uncertainty (or ambiguity), randomness and Radner's (1968, 1982) private (incomplete) informational structure are inserted into $\mathcal{E}$'s complete markets, all of them captured and represented by a set of states of nature of the world $\mathcal{S}$, which can be an up to uncountably infinite state space. Further and beyond, for the sake of the argument, nothing in the analysis is nullified when, in its state contingent visualisation with agents' differential informational partitions of $\mathcal{S}$, $\mathcal{E}$ is put in non-Bayesian decision theoretic tracks. To verify these statements, one may simply duplicate the analysis and mimic all its arguments in an economic platform with wide population and ambiguity. This all-over expanded economy is coarser, with bigger calibre, length, area or diameter, thence, more explanatory.

Reflections

The conventional general equilibrium theory is developed *ceteris paribus*, namely, by having discarded or partialled out the effect of the quality of the agents' initial endowment. This paper proposes, predicts and prescribes a unification of the general equilibrium architecture via the qualification of the initial endowment allocation. Along with its salient quantitative role, which is primarily marked by the fact that any general equilibrium allocation is just a feasible redistribution of the initial endowments allocation[17], the specific allocation plays a central qualitative role in $\mathcal{E}$.

More specifically, the extrapolative admissions that were made in the ending part of the previous section open the road for the following universal result: in any conceivable convex (or convexifiable) economy, with concave (or concavifiable) utility functions, that adequately resembles to $\mathcal{E}$, and which could be viable under any additional (within the original spirit) configurations or singularities, if the initially endowed wealth, status, power, capabilities, prospects, opportunities, property induced rights (and so forth) of the inhabitants of this economy are socially efficient, then all the Pareto optimal general equilibrium allocations that will be generated will coincide with the agents' initial assignments allocation.

This position, in turn, clouds the neoclassical *modus operandi* and casts reasonable doubts to it, whenever the decision theoretic idea that averages are always better or less precarious than - ergo, are preferred over - the extremes is prevalent. Indeed, it raises credible concerns whether it is any worthwhile or truly valuable to get into the trouble to trade, rather than just re-construct with casual negotiations and bargainings, which inevitably lead to mutual agreements, their initial holdings into Pareto optimal ones, and then quickly and safely opt to keep them, instead of stepping into the cumbersome trading procedure, in order to generate a socially efficient general equilibrium solution, by exhausting their toil and wasting their resources. It puts, in other words, the fundamental Theory of Value in jeopardy. To a great extent, therefore, the efficaciousness of general equilibrium theory is obscured by fuzziness and dubiousness, if not entirely squashed. Insofar as the economic organism of the convex sort is made immune to this pathogenicity, the whole establishment of Economics runs the danger of being obliterated.

[17] And, secondarily, by the fact that the magnitude of the initial endowment is a means of exercising market power.

To rectify this predicament, provided that the agents' preferences have to conform to rationality, and consequently the rational preferential axioms cannot be tampered with, it needs to be made certain that the initial endowments are not Pareto optimal. This is a sufficient condition for engaging into trade, instead of indulging in autarky, that is on a par with the Ricardian philosophy, but from the consumption side of a neoclassical economy.

Thereafter, one may simply create a copy for $\mathcal{E}$ which is not vacant with respect to production and still attain the same argument for (non) trade, but this time mirrored onto the economy's production side. In this economy, veritably, an agent's initial endowment would be reflected up-onto her dividend or portion from the economy's aggregate product(ion) or cake, that is, onto up-the economy's per capita (or per head) output.

But then again, when the baked pie is cut and the shares of its recipients must not be Pareto optimal so that a new pie can be for sure cooked and shared out again, the free-markets allocative mechanism has either failed (according to the impetuous pessimists) or has simply hit the bull's eye (according to the careful optimists).

There is no puzzle, riddle, dilemma or debate associated with this circumstance. To disambiguate this seemingly baffling situation, one needs to realise that the second interpretation, that one of an original *laissez faire* cross-populational socially efficient distribution of the production, is actually the only persuasive. According to it, the Smith's invisible hand, the Edgeworth's fictitious re-contracting drill and, perhaps most convincingly of all, the Walrasian imaginary auctioneer's tâtonnement procedure have been absolutely successful.

Either of them has precisely put the economy onto an optimal balancing position and path from the very beginning. It has effortlessly guaranteed a stationary convergent steady state (positioning or placement) for an economy that is trivially endowed with some optimal rule of consumption versus capital accumulation. In this economic picture, of course, $\mathcal{E}$'s commodities have all wrapped up into the economy's composite product, which gets then simplistically (and entirely) split up between durables and non-durables.

Put another way, if agents have (all of them) an identical utility function, given that they are all of them originally bequested with the same initial capital stock, then if this utility function is strictly concave[18] and strictly convex preferences are implied from it, the (massive in the literature) complete-markets neoclassical growth expedition, which is tailored to (and entrenched into) the first-best socially efficient competitive general equilibrium, is rendered fruitless.

[18]For instance, a Cobb-Douglas (1928) function, or a Dixit-Stiglitz (1977) function, which both, when they are linearly homogenous, represent homothetic preferences as well.

References

Allen R., G., D., and Hicks J., R., 1934, 'A Reconsideration of the Theory of Value. Part I and Part II', Economica, 1(1): 52-76 and 1(2): 196-219.

Aliprantis, C., D, and Brown, D., J., 1983, 'Equilibria in Markets with a Riesz space of commodities', Journal of Mathematical Economics, 11: 189-207.

Anderson, R., M., 1978, 'An Elementary Core Equivalence Theorem', Econometrica, 46: 1483-1487.

Arrow, K., 1951 (1963, 2012), 'Social choice and individual values', New Haven, Yale University Press.

Arrow, K., J., and Debreu, G., 1954, 'Existence of an Equilibrium for a Competitive Economy', Econometrica, 22(3): 265–290.

Arrow, K., J., and Hurwicz, L., 1958, 'On the Stability of the Competitive Equilibrium I', Econometrica, 26: 522-552.

Arrow, K., J., Block, H., D., and Hurwicz, L., 1959, 'On the Stability of the Competitive Equilibrium II', Econometrica, 27: 82-109.

Arrow, K., J., and Hahn, F., H., 1971, 'General Competitive Analysis', Holden-Day, Inc., San Francisco.

Aumann, R., J., 1964, 'Markets with a Continuum of Traders', Econometrica, 32: 39-50.

Aumann, R., and Shapley, L., 1974, 'Values of Non-Atomic Games', Princeton University Press, Princeton, N.J.

Aumann, R., 1975, 'Values of markets with a continuum of traders', Econometrica, 43: 611-646

Beloso, C., H., and Estevez, T., M., 1995, 'On the Existence of Continuous Preference Orderings without Utility Representations', Journal of Mathematical Economics, 24(4): 305-309.

Bewley, T., 1972, 'Existence of Equilibria in Economies with Infinitely many Commodities', Journal of Economic Theory, 4: 541-540.

Bewley, T, 1974, 'Edgeworth's conjecture', Econometrica, 41: 415-452.

Bowen, R., 1968, 'A New Proof of Theorem in Utility Theory', International Economic Review, 9: 374.

Champsaur, P., 1975, 'Cooperation vs Competition', Journal of Economic Theory, 11: 394-417.

Chipman, J., S., Hurwicz, L., Richter, M., and Sonnenschein, H., F., (eds), 1971, 'Preferences, Utility and Demand', Harcourt Brace, Jovanovich: new York.

Coase, R., H., 1960, 'The Problem of Social Cost', Journal of Law and Economics, 3: 1-44.

Cobb, C., W., and Douglas, P., H., 1928, 'A Theory of Production', American Economic Review, 18 (Supplement): 139-165.

Debreu, G., 1954, 'Representation of a Preference Ordering by a Numerical Function', in Thrall, R., M., Coombs, C., H., and Davis, R., L., (eds.), Decision Processes, Ch. XI: 159-165, Wiley, New York.

Debreu, G., 1956, 'Market equilibrium', Proceedings of the National Academy of Sciences of USA, 52: 876–878.

Debreu, G., 1959, 'Theory of Value. An Axiomatic Analysis of Economic Equilibrium', Wiley, New York.

Debreu, G., and Scarf, H., E., 1963, 'A Limit Theorem on the Core of an Economy', International Economic Review, 3: 235-246.

Debreu, G., 1964, 'Continuity Properties of Paretian Utility', International Economic Review, 5(3): 285-293.

Debreu, G., 1970, 'Economies with a Finite Set of Equilibria', Econometrica, 38: 387-392.

Debreu, G., 1972, 'Smooth Preferences', Econometrica, 40: 603-615.

Dixit, A., K., and Stiglitz, J., E., 1977, 'Monopolistic Competition and Optimum Product Diversity', The American economic review, 67(3): 297-308.

Dworkin, R., 1981a, 'What is Equality? Part 1: Equality of Welfare', Philosophy and Public Affairs, 10: 185-245.

Dworkin, R., 1981b, 'What is Equality? Part 2: Equality of Resources', Philosophy and Public Affairs, 10: 283-345.

Edgeworth, F., Y., 1881,'Mathematical Psychics: An Essay on the Application of Mathematics to the Moral Sciences', Kegan Paul, London.

Emmons, D., and Scafuri, A., J., 1985, 'Value Allocations- An Exposition', in Advances in Equilibrium Theory, Aliprantis C.D. et al. (eds). Berlin, Heidelberg, New York: Springer.

Florenzano, M., 1983, 'On the Existence of Equilibria in Economies with an Infinite Dimensional Commodity Space', Journal of Mathematical Economics, 12(3): 207-219.

Gale, D., 1955, ‘The law of supply and demand’, Mathematica Scandinavica, 3: 155–169.

Gale, D., and Mas-Colell, A., 1975, ‘An equilibrium existence theorem for a general model without ordered preferences’, Journal of Mathematical Economics 2: 9–15.

Gale, D., and Mas-Colell, A., 1979, ‘Corrections to an equilibrium existence theorem for a general model without ordered preferences’, Journal of Mathematical Economics, 6: 297–298.

Grodal, B., 1975, ‘The rate of convergence of the core for a purely competitive sequence of economies’, Journal of Mathematical Economics, 2: 171-186.

Hart, S., 1977, ‘Values of Non-differentiable Markets with a Continuum of Traders’, Journal of Mathematical Economics, 4(2): 103-116.

Harsanyi, J., C., 1967, ‘Games with Incomplete Information Played by Bayesian Players. Part 1 (of 3). The Basic Model’, Management Science, 14(3): 159–182.

Hicks, J., R., 1939 (1946), ‘Value and Capital: An Inquiry into Some Fundamental Principles of Economic Theory’, Oxford, Clarendon Press.

Hildenbrand, W, 1974, ‘Core and Equilibria of a Large Economy’, Princeton University Press, Princeton, N.J.

Hildenbrand, W, and Kirman, A., P., 1973, ‘Size Removes Inequity’, The Review of Economic Studies, 30: 305-314.

Hurwicz, L., 1972, ‘On Informationally Decentralized Systems’, in Decision and Organization, T. McGuire and R. Radner (ed.), Amsterdam, North-Holland.

McKenzie, L., W., 1954, ‘On Equilibrium in Graham’s Model of World Trade and Other Competitive Systems’, Econometrica, 22(2): 147–161.

Knight, F., H., 1921, ‘Risk, Uncertainty, and Profit’ Boston, MA: Hart, Schaffner Marx; Houghton Mifflin Company.

Kreps, D., M., 1988, ‘Notes on the Theory of Choice’, Westview Press, Boulder.

Lancaster, K., J., 1966, ‘A New Approach to Consumer Theory’, The Journal of Political Economy, 74(2): 132-57.

Mas-Colell, A., 1974, ‘An equilibrium existence theorem without complete or transitive preferences’, Journal of Mathematical Economics, 1: 237–246.

Mas-Colell, A., 1977, ‘Indivisible Commodities and General Equilibrium Theory’, Journal of Economic Theory, 16: 443-456.

Mas-Colell, 1977, ‘Competitive and Value Allocations of Large Exchange Economies’, Journal of Economic Theory, 14(2): 419-438.

McKenzie, L., W., 1959, ‘On the Existence of General Equilibrium for a Competitive Market’, Econometrica, 27: 54-71.

McKenzie, L., W., 1981, ‘The Classical Theorem on Existence of Competitive Equilibrium’, Econometrica, 49: 819-841.

Nash, J., 1950, ‘Equilibrium Points in n-Person Games’, Proceedings of the National Academy of Science, 36: 48-49.

Nash, J., F., 1951, ‘Noncooperative Games’, Annals of Mathematics, 54: 289-295.

Negishi, T., 1960, Welfare economics and existence of an equilibrium for a competitive economy’, Metroeconomica, 12, 92–97.

Negishi, T., 1962, ‘The Stability of a Competitive Economy: A Survey Article’, Econometrica, 30: 635-669.

Nikaido, H., 1956, ‘On the classical multilateral exchange problem’, Metroeconomica, 8: 135–145.

Pareto, V., 1971, ‘Manual of Political Economy’, Translated by Ann S. Schwier: English translation of the 1927 French translated edition of the original 1906 Italian text ‘Manuale di economia politica con una introduzione alla scienza sociale’, Augustus M. Kelley, New York.

Podczeck, K., and Yannelis, N., C., 2024, ‘Existence of Walrasian Equilibria with Discontinuous, Non-Ordered, Interdependent Preferences, Without Free Disposal, and with an Infinite-Dimensional Commodity Space’, Economic Theory, 78(2):1-13.

Postlewaite, A., 1979, ‘Manipulation via Endowments’, Review of Economic Studies, 46:255-262.

Radner, R., 1968, ‘Competitive Equilibrium under Uncertainty’, Econometrica, 36: 31-58.

Radner, R., 1982, ‘Equilibrium under Uncertainty”, in K.J. Arrow, M.D. Intriligator(eds.), Handbook of Mathematical Economics, vol. II, Amsterdam: North-Holland.

Samuelson, P., A., 1938, ‘A note on the pure theory of consumers’ behaviour’, Economica, 5 (17): 61–71.

Samuelson, P., A., 1948, ‘Consumption theory in terms of revealed preference’, Economica, 15 (60): 243–253.

Sandine, G., 2024, ‘An Extreme Value Type Theorem for Convex Functions on Reflexive Banach Spaces’, PSU working papers.

Scafuri, A., J,. and Yannelis, N., C., 1984, ‘Non-Symmetric Cardinal Value Allocations’, Econometrica, 52: 1365-1368.

Scarf, H., E., 1962, ‘An Analysis of Markets with a Large Number of Participants’, in Recent Advances in Game Theory, Philadelphia, PA, The Ivy Curtain Press.

Scarf, H., E., 1967, ‘The Core of a N-Person Game’, Econometrica, 35: 50-69.

Scarf, H., E., 1971, ‘On the Existence of a Cooperative Solution for a General Class of N-person Games’, Journal of Economic Theory, 35: 169-181.

Shafer, W., J., 1980, ‘On the Existence and Interpretation of Value Allocations’, Econometrica, 48: 467-476.

Shapley, L., A., 1953, ‘A Value of N-person Games’, in Contributions to the Theory of Games, Princeton: Princeton University Press, 307-317.

Shapley, L., and Shapiro, N., 1960 ‘Values of Large Gains I: A Limit Theorem’, Rand Corporation Memorandum, 26-48.

Shapley, L., A., 1969, ‘Utility Comparisons and the Theory of Games’, in La Decision, ed. by G. T. Guilbaud. Paris: Edition du CNRS.

Shapley, L., and Shubik, M., 1969, ‘Pure Competition, Coalitional Power, and Fair Division’, International Economic Review, 10: 337-362.

Shubik, M., 1959, ‘Edgeworth market games’, Contributions to the Theory of Games, IV, Princeton University Press, Princeton, N.J.

Stigler, G.,J., 1966, ‘The Theory of Price’, 3rd ed., New York, Macmillan.

Vind, K, 1964, ‘Edgeworth Allocations in an Exchange Economy with Many Traders’, International Economic Review, 5: 165-177.

Vind, K., 1965, ‘A Theorem on the Core of an Economy’, Review of Economic Studies, 5: 165-177.

Vind, K., 1973, ‘A Third Remark on the Core of an Atomless Economy’, Econometrica, 40: 585-586.

Walras, L., 2003, ‘Elements of Pure Economics (or, The Theory of Social Wealth)’, Translated by William Jaffé: English translation of a republication of the original 1874 French text ‘Éléments d’Économie Politique Pure (ou Théorie de la richesse sociale)’, Routledge, New York.

Wold, H., 1943, ‘A Synthesis of Pure Demand Analysis, I, II and III’, Scandinavisk Aktuarietidskrift, 26: 85-118, 220-263, 69-120.

Yannelis, N., C., 1985, ‘On a Market Equilibrium Theorem with an Infinite Number of Commodities’, Journal of Mathematical Analysis and Applications, 108: 595-599.

Department of Economics
Athens University of Economics and Business

List of Recent Working Papers

2022

01-22 Is Ireland the most intangible intensive economy in Europe? A growth accounting perspective, Ilias Kostarakos, KieranMcQuinn and Petros Varthalitis
02-22 Common bank supervision and profitability convergence in the EU, Ioanna Avgeri, Yiannis Dendramis and Helen Louri
03-22 Missing Values in Panel Data Unit Root Tests, Yiannis Karavias, Elias Tzavalis and Haotian Zhang
04-22 Ordering Arbitrage Portfolios and Finding Arbitrage Opportunities, Stelios Arvanitis and Thierry Post
05-22 Concentration Inequalities for Kernel Density Estimators under Uniform Mixing, Stelios Arvanitis
06-22 Public Sector Corruption and the Valuation of Systemically Important Banks, Georgios Bertsatos, Spyros Pagratis, Plutarchos Sakellaris
07-22 Finance or Demand: What drives the Responses of Young and Small Firms to Financial Crises? Stelios Giannoulakis and Plutarchos Sakellaris
08-22 Production function estimation controlling for endogenous productivity disruptions, Plutarchos Sakellaris and Dimitris Zaverdas
09-22 A panel bounds testing procedure, Georgios Bertsatos, Plutarchos Sakellaris, Mike G. Tsionas
10-22 Social policy gone bad educationally: Unintended peer effects from transferred students, Christos Genakos and Eleni Kyrkopoulou
11-22 Inconsistency for the Gaussian QMLE in GARCH-type models with infinite variance, Stelios Arvanitis and Alexandros Louka
12-22 Time to question the wisdom of active monetary policies, George C. Bitros
13-22 Investors' Behavior in Cryptocurrency Market, Stelios Arvanitis, Nikolas Topaloglou and Georgios Tsomidis
14-22 On the asking price for selling Chelsea FC, Georgios Bertsatos and Gerassimos Sapountzoglou
15-22 Hysteresis, Financial Frictions and Monetary Policy, Konstantinos Giakas
16-22 Delay in Childbearing and the Evolution of Fertility Rates, Evangelos Dioikitopoulos and Dimitrios Varvarigos
17-22 Human capital threshold effects in economic development: A panel data approach with endogenous threshold, Dimitris Christopoulos, Dimitris Smyrnakis and Elias Tzavalis
18-22 Distributional aspects of rent seeking activities in a Real Business Cycle model, Tryfonas Christou, Apostolis Philippopoulos and Vanghelis Vassilatos

2023

- **01-23 Real interest rate and monetary policy in the post Bretton Woods United States, George C. Bitros and Mara Vidali**
- **02-23 Debt targets and fiscal consolidation in a two-country HANK model: the case of Euro Area, Xiaoshan Chen, Spyridon Lazarakis and Petros Varthalitis**
- **03-23 Central bank digital currencies: Foundational issues and prospects looking forward, George C. Bitros and Anastasios G. Malliaris**
- **04-23 The State and the Economy of Modern Greece. Key Drivers from 1821 to the Present, George Alogoskoufis**
- **05-23 Sparse spanning portfolios and under-diversification with second-order stochastic dominance, Stelios Arvanitis, Olivier Scaillet, Nikolas Topaloglou**
- **06-23 What makes for survival? Key characteristics of Greek incubated early-stage startup(per)s during the Crisis: a multivariate and machine learning approach, Ioannis Besis, Ioanna Sapfo Pepelasis and Spiros Paraskevas**
- **07-23 The Twin Deficits, Monetary Instability and Debt Crises in the History of Modern Greece, George Alogoskoufis**
- **08-23 Dealing with endogenous regressors using copulas; on the problem of near multicollinearity, Dimitris Christopoulos, Dimitris Smyrnakis and Elias Tzavalis**
- **09-23 A machine learning approach to construct quarterly data on intangible investment for Eurozone, Angelos Alexopoulos and Petros Varthalitis**
- **10-23 Asymmetries in Post-War Monetary Arrangements in Europe: From Bretton Woods to the Euro Area, George Alogoskoufis, Konstantinos Gravas and Laurent Jacque**
- **11-23 Unanticipated Inflation, Unemployment Persistence and the New Keynesian Phillips Curve, George Alogoskoufis and Stelios Giannoulakis**
- **12-23 Threshold Endogeneity in Threshold VARs: An Application to Monetary State Dependence, Dimitris Christopoulos, Peter McAdam and Elias Tzavalis**
- **13-23 A DSGE Model for the European Unemployment Persistence, Konstantinos Giakas**
- **14-23 Binary public decisions with a status quo: undominated mechanisms without coercion, Efthymios Athanasiou and Giacomo Valletta**
- **15-23 Does Agents' learning explain deviations in the Euro Area between the Core and the Periphery? George Economides, Konstantinos Mavrigiannakis and Vanghelis Vassilatos**
- **16-23 Mild Explocivity, Persistent Homology and Cryptocurrencies' Bubbles: An Empirical Exercise, Stelios Arvanitis and Michalis Detsis**
- **17-23 A network and machine learning approach to detect Value Added Tax fraud, Angelos Alexopoulos, Petros Dellaportas, Stanley Gyoshev, Christos Kotsogiannis, Sofia C. Olhede, Trifon Pavkov**
- **18-23 Time Varying Three Pass Regression Filter, Yiannis Dendramis, George Kapetanios, Massimiliano Marcellino**
- **19-23 From debt arithmetic to fiscal sustainability and fiscal rules: Taking stock, George Economides, Natasha Miouli and Apostolis Philippopoulos**
- **20-23 Stochastic Arbitrage Opportunities: Set Estimation and Statistical Testing, Stelios Arvanitis and Thierry Post**
- **21-23 Behavioral Personae, Stochastic Dominance, and the Cryptocurrency Market, Stelios Arvanitis, Nikolas Topaloglou, and Georgios Tsomidis**
- **22-23 Block Empirical Likelihood Inference for Stochastic Bounding: Large Deviations Asymptotics Under *m*-Dependence, Stelios Arvanitis and Nikolas Topaloglou**

23-23 A Consolidation of the Neoclassical Macroeconomic Competitive General Equilibrium Theory via Keynesianism (Part 1 and Part 2), Angelos Angelopoulos
24-23 Limit Theory for Martingale Transforms with Heavy-Tailed Noise, Stelios Arvanitis and Alexandros Louka

2024

01-24 Market Timing & Predictive Complexity, Stelios Arvanitis, Foteini Kyriazi, Dimitrios Thomakos
02-24 Multi-Objective Frequentistic Model Averaging with an Application to Economic Growth, Stelios Arvanitis, Mehmet Pinar, Thanasis Stengos, Nikolas Topaloglou
03-24 State dependent fiscal multipliers in a Small Open Economy, Xiaoshan Chen, Jilei Huang, Petros Varthalitis
04-24 Public debt consolidation: Aggregate and distributional implications in a small open economy of the Euro Area, Eleftherios-Theodoros Roumpanis
05-24 Intangible investment during the Global Financial Crisis in the EU, Vassiliki Dimakopoulou, Stelios Sakkas and Petros Varthalitis
06-24 Time will tell! Towards the construction of instantaneous indicators of different agent-types, Iordanis Kalaitzoglou, Stelios Arvanitis
07-24 Norm Constrained Empirical Portfolio Optimization with Stochastic Dominance: Robust Optimization Non-Asymptotics, Stelios Arvanitis
08-24 Asymptotics of a QLR-type test for optimal predictive ability, Stelios Arvanitis
09-24 Strongly Equitable General Equilibrium Allocations, Angelos Angelopoulos
10-24 The Greek macroeconomy: A note on the current situation and future outlook, Apostolis Philippopoulos
11-24 Evolution of Greek Tax System, A Survey of Legislated Tax Changes from 1974 to 2018, Panagiotis Asimakopoulos
12-24 Macroeconomic Impact of Tax Changes, The case of Greece from 1974 to 2018, Panagiotis Asimakopoulos
13-24 `Pareto, Edgeworth, Walras, Shapley' Equivalence in a Small Economy, Angelos Angelopoulos

Department of Economics
Athens University of Economics and Business

The Department is the oldest Department of Economics in Greece with a pioneering role in organising postgraduate studies in Economics since 1978. Its priority has always been to bring together highly qualified academics and top quality students. Faculty members specialize in a wide range of topics in economics, with teaching and research experience in world-class universities and publications in top academic journals.

The Department constantly strives to maintain its high level of research and teaching standards. It covers a wide range of economic studies in micro-and macroeconomic analysis, banking and finance, public and monetary economics, international and rural economics, labour economics, industrial organization and strategy, economics of the environment and natural resources, economic history and relevant quantitative tools of mathematics, statistics and econometrics.

Its undergraduate program attracts high quality students who, after successful completion of their studies, have excellent prospects for employment in the private and public sector, including areas such as business, banking, finance and advisory services. Also, graduates of the program have solid foundations in economics and related tools and are regularly admitted to top graduate programs internationally. Three specializations are offered:1. Economic Theory and Policy, 2. Business Economics and Finance and 3. International and European Economics. The postgraduate programs of the Department (M.Sc and Ph.D) are highly regarded and attract a large number of quality candidates every year.

For more information:

https://www.dept.aueb.gr/en/econ/