Department of Economics

Athens University of Economics and Business

WORKING PAPER no. 09-2024

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September 2024

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Strongly Equitable General Equilibrium Allocations

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Abstract

The literature massively recognises equitable (fair and impartial) divisions as cake-cutting solutions in which none recipient envies the share of any other person. There is, however, a stronger idea of equitability of distributions, that precedes the previous one. An allocation is equitable if and only if none individual is jealous of the standard of living, not simply the allocative quantity, that is conveyed by an allocation. The class of individually rational allocations fulfils equitability of this sense. Hence, the literature's most distinguishable allocative general equilibria do so.

Key Words: General Equilibrium, Allocations, Enviousness, Equitability.

Classification: D5.

Acknowledgements: The author wishes to acknowledge fruitful private conversations with Emma Moreno-García and Carlos Hervés-Beloso. The author is indebted to those two persons, both for their contribution to the generation of this paper, and for the overall academic support that they have generously offered to him.

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Preliminaries

In the simplest conceptually neoclassical general equilibrium theory that is sustainable by a pure exchange (or trade into markets) consumption economy with rational agents that do not cooperate, nor condition their behaviour into other agents' actions, transfers (or side-payments) of consumption bundles among the agents are solidly conceivable, if not necessary, when justifying the normative stability of general equilibrium outcomes. To this end, endogenous general equilibrium allocations, which are alternative - and ultimately both unitary and socially optimal - feasible redistributions of the agents' initial endowments allocation, may be freely re-allocated between the agents' population. The same, therefore, can happen for the agents' initial endowments allocation before even agents engage into trade. Albeit, unbendably in the standard general equilibrium theory, during these redistributions of allocated quantities of commodities, the agents' preferential comparisons and rankings over the commodities' baskets are preserved. The exogenously stated heterogenous utility functions of the agents remain invariant. This a strong assumption, without doubt.

To deliver the arguments of this note upon refinements and weakenings of the aforementioned stringent ties, let the following simplistic finite $m-markets \times n-agents$ economy:

$$\mathcal{E} = \{\mathbb{R}^m_+; \precsim_i, \omega_i : i \in I = \{1, 2, , , , , , n\}\}.$$

There are $m \in \mathbb{N}$ commodities in $\mathcal{E}$. All agents have the same consumption set, which is $\mathbb{R}^m_+$ for simplicity. This condition allows for the inclusion of public goods in $\mathcal{E}$, jointly with the consideration in it of the standard private goods, enables agents to exhibit unbounded rationality, while bans $\mathcal{E}$ from admitting indivisible goods. $\omega_i \in \mathbb{R}^m_+ \setminus \{\mathbf{0}\}$ is the exogenous initial endowment of agent $i \in I$. $\precsim_i$, which is agent's $i \in I$ subjective (or personalised) preferences, obeys to certain objective ordering criteria for the consumption bundles of $\mathbb{R}^m_+$: totality, reflexivity, transitivity, continuity and monotonicity (convexity of preferences is the bonus axiom that can be summoned upon request). Otherwise, $\precsim_i$, $i \in I$, are not rational and cannot be globally satiated. Equivalently, the ordinally equivalent utility functions that represent them (see in Debreu, 1954, 1959, 1964) do not attain a global maximum. The previous arrangements imply that a utility function $u_i : \mathbb{R}^m_+ \to \mathbb{R}_+$ arises for each agent $i \in I$ in the foreground, which function is continuous on its domain with respect to the standard topology of $\mathbb{R}^m$, and increasing on each one of its arguments separately (hence, quasi-concave on its domain). This function is unique up to any monotone transformation.

The following definition of equitability (in the weak sense) is due to Foley (1967), Schmeilder-Yaari (1971) and Varian (1974) (among others).

Definition 1. Let $x \in \mathbb{R}_{+}^{m \times n}$ be a feasible allocation of $\mathcal{E}$ (iff $\sum_{i \in I} x_i = \sum_{i \in I} \omega_i$; strong monotonicity of preferences is required for the strict equality). Then: x is equitable (just and neutral) if and only if x is an envy-free or envy-less allocation of $\mathcal{E}$ if and only if there do not exist $i \neq j \in I$ such that i envies the allocative quantity x_j of j if and only if there do not exist $i \neq j \in I$ such that $u_i(x_j) > u_i(x_i)$.

Remark 1. Observe that this definition allows for exchanges of allocative quantities among the agents, since agent $i \in I$ can think of obtaining the individual allocation x_j of agent $i \in I$. Such private re-sharings of commodities across the agents are permittable and realisable only because agents have the same consumption set.

Remark 2. It is a widely known result in the literature that a (priced) Walrasian or competitive (feasible) allocation of $\mathcal{E}$, the feasibility condition of which clears the markets of $\mathcal{E}$ with the supply equals demand condition, is equitable in the sense of Definition 1. For the existence of this state-of-the-art general equilibrium concept, the allocations of which are innately both Pareto optimal, i.e., socially efficient, and individually rational, i.e., personally efficient, see in Arrow and Debreu (1954) and McKenzie (1954).

Strong Equitability

How do the exogenously stated differential preferences of agents in $\mathcal{E}$ are formed? Under which latent protocols? Why are not they just homogenous? General equilibrium theory remains silent on this issue.

One appealing explanation of this procedure is that agents' preferences are dependable on their initial endowments. If, for example, $\leq$ is a partial order on $\mathbb{R}^m$, then agents may be entering in $\mathcal{E}$ with explicit disparities, as rich versus poor: $\omega_i \leq \omega_j$ implies that i is poorer than j, or that j is wealthier than i. In that case, agents with greater (smaller, respectively) wealth, rights or power are likely (if not certainly) to have more (less, respectively) sophisticated needs to satisfy. Accordingly, agents are naturally (from their origins) bound to form dissimilar preferences, so each asymmetric u_i that represents $\precsim_i$ of the trader $i \in I$, and is idiosyncratically selected for trade from consumer i, is conditional up-onto ω_i of i, a situation which is stated as $u_i^{\omega_i}$ for this agent i.

The idea that naturally ensues is that an agent may be envying the potentials, opportunities or prospects of another agent, thus, this agent's initial endowment. So

what if agents can be seen as swapping their initial endowments so as to test the robustness of the non-enviousness condition in $\mathcal{E}$? To wit, if ω denotes the initial endowments allocation of $\mathcal{E}$, what if re-shuffles of this allocation, ω', take place? If this is the case, an agent would be then, by default, allowed to change her preferences (equivalently, her utility function) contingently upon the initial endowment she gets assigned with each time, without becoming worse off in terms of utilitarian welfare.

The four definitions that follow contextualise these conceptualisations.

Definition 2. The feasible allocation x of $\mathcal{E}$ is called individually equitable iff before the feasible allocation x, for every agent $i \in I$, there does not exist ω_j, $j \neq i$, such that $u_i^{\omega_j}(x_i) > u_i^{\omega_i}(x_i)$.

Definition 3. The feasible allocation x of $\mathcal{E}$ is called individually efficiently-equitable iff before the feasible allocation x and any feasible re-allocation x' of x, for every agent $i \in I$, there does not exist ω_j, $j \neq i$, such that $u_i^{\omega_j}(x_i') > u_i^{\omega_i}(x_i)$.

Definition 4. The feasible allocation x of $\mathcal{E}$ is called socially equitable iff before the feasible allocation x, there does not exist an ω' of ω such that $u_i^{\omega_{j\neq i}}(x_i) \geq u_i^{\omega_i}(x_i)$, for all $i \in I$, whilst specifically $u_i^{\omega_{j\neq i}}(x_i) > u_i^{\omega_i}(x_i)$, for at least one $i \in I$.

Definition 5. The feasible allocation x of $\mathcal{E}$ is called socially efficiently-equitable iff before the feasible allocation x and any feasible re-allocation x' of x, there does not exist an ω' of ω such that $u_i^{\omega_{j\neq i}}(x_i') \geq u_i^{\omega_i}(x_i)$, for all $i \in I$, whilst specifically $u_i^{\omega_{j\neq i}}(x_i') > u_i^{\omega_i}(x_i)$, for at least one $i \in I$.

Remark 3. In Definition 2, more strongly than in the story of Definition1, the status and not just the consumption of j are not envied by i for the specific allocation x. In Definition 3, double more strongly than the scenario of Definition 1, the initial position and not just the sequel consumption of j are efficiently not envied by i for any redistribution x' of x. Observe that individual (efficient) equitability of an allocation implies the social (efficient) equitability of this allocation.

The Theorem infra that closes this section bridges individual rationality (or efficiency) with individual equitability, providing in this way sideways, by using an analytical shortcut, the normatively required individual efficient-equitability of allocations (of Definition 3). Social efficient-equitability of allocations then follows through, using the implication in Remark 3.

Theorem. If ω satisfies individual efficient-equitability, then every (feasible) individually rational allocation of $\mathcal{E}$ is individually equitable.

Proof. Take a (feasible) individually rational allocation $x \in \mathbb{R}_{+}^{m \times n}$. For x it holds that $u_i^{\omega_i}(x_i) \geq u_i^{\omega_i}(\omega_i)$, for all $i \in I$. Suppose that x is not individually equitable. Then for some agent $i \in I$ there exists an ω_j, $j \neq i$, such that $u_i^{\omega_j}(x_i) > u_i^{\omega_i}(x_i)$, from which inequality (when jointly taken together with the previous one) it is implied that $u_i^{\omega_j}(x_i) > u_i^{\omega_i}(\omega_i)$. This is a contradiction by virtue of the condition that ω (which is a feasible allocation by default) is individually efficient-equitable as of Definition 3, which means that $u_i^{\omega_i}(\omega_i) \geq u_i^{\omega_j}(\omega_i')$, for every ω_j, $j \neq i$, and for every feasible redistribution ω' of the allocation ω, where ω_i' replaces some individual allocation x_i that comes from a feasible allocation x.

Remark 4. If agents do not trade at all and keep their initial endowments, or if they do trade and still end up with their initial endowments, then the condition that ω is individually efficiently-equitable, generally, reasonably and forcefully holds.

Epilogue

This short paper fosters the idea that it is more sagacious to think that it is not the inequalities in consumption that agents may be jealous of, but probably, more primitively and more generally, it is the discrepancies in their statuses and lifestyles. If such polarisations are not envy-full by the individuals, then a criterion for equitability (fairness and impartiality) in the pie-cutting process is established, even if the personally allocated shares are not egalitarian; which should not be the case either ways, unless all agents in $\mathcal{E}$ are identical, so that then the equal treatment property of $\mathcal{E}$'s general equilibrium allocations should be conscripted. Individually efficient allocations are equitable in this essay's strong sense. The pre trade level of the non-zero vector $\omega_i \in \mathbb{R}_{+}^{m}$ of an agent $i \in I$ affects the level of the quantity $x_i \in \mathbb{R}_{+}^{m}$ that this agent gets allocated with post-trade. Thence, the equitability notion that is introduced in this entry is stronger than the one that already and affluently exists in the cake-cutting literature.

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