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Social Welfare Functions; a proposal for a two-dimensional form

Διπλωματική Εργασία

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Ο Ορέστης Βαπτιστής Κουζούπης βεβαιώνω ότι το έργο που εκπονήθηκε και παρουσιάζεται στην υποβαλλόμενη διπλωματική εργασία είναι αποκλειστικά ατομικά δικό μου. Όποιες πληροφορίες και υλικό που περιέχονται έχουν αντληθεί από άλλες πηγές, έχουν καταλλήλως αναφερθεί στην παρούσα διπλωματική εργασία. Επιπλέον τελώ εν γνώσει ότι σε περίπτωση διαπίστωσης ότι δεν συντρέχουν όσα βεβαιώνονται από μέρους μου η εργασία μου θα μηδενιστεί.

Introduction

The subject of this paper is social welfare functions. More specifically, their definitions, the many proposed forms, the philosophies behind each proposal, the differences between them and their shortfalls, as well as the axioms they have to adhere to, and the many opinions on all aforementioned subjects. Alternative methods for ranking policy options, such as Cost Benefit Analysis, will also be explained and discussed. After that, there will be a proposal from myself for a new social welfare function with two metrics. Moreover, I will be explaining the reasons that led me to the form, how it works, both in certainty and uncertainty of outcomes, and the way it compares alternative policies, either through a graph or through estimating the percentage differences of the two metrics between two states. Finally, there will be a numeric example in order to compare the two most well-known social welfare functions, the utilitarian and the leximin, with my proposed two-dimensional social welfare function, using the utilities of two individuals in a hypothetical proposal of two income tax rates.

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1. **Social Welfare Functions**

1.1. What are they?

As stated in the introduction, the main subject of this paper will be social welfare functions. However, to begin, one must define the term. Unfortunately, there is no clear definition. Adler (Matthew D. Adler, 2019) says that a social welfare function is a "policy-assessment tool" and that "it functions to produce a ranking of policies". Kakwani and Son (Nanak Kakwani, Hyun Hwa Son, 2016) argue that social welfare functions "provide a rule in aggregating different utilities across individuals in society". Of course, they failed to mention what's the case with social welfare functions whose form is not a sum, for example the Leximin/Rawlsian, which is mentioned below. Based on these definitions, which tread on basically, but not exactly, the same lines, and my understanding of the matter at hand, I've come to the conclusion that social welfare functions are algebraic formulas that estimate the impact a policy has (or might have, if uncertainty of outcome is included) on the total utility of a population, thus becoming a helpful tool for a decision-maker in their efforts to rank comparable policies and choose the best option. This personal definition will be the basis of everything discussed below, and, partly, the reason for my proposal.

1.2. Their forms

Historically, there have been many proposals as to the form of a social welfare function, based on different philosophies. Adler in his book "Measuring Social Welfare: An introduction" (2019) highlights five forms as the leading ones. Namely, the utilitarian, the continuous-prioritarian, the leximin, the rank-weighted and the sufficientint.

The utilitarian social welfare function is the most well-known and more widely used one in theoretical discussions, and it is defined as the sum of the utilities of the individuals in the population. Of course, a higher sum for a particular policy P relative to every other policy means that that is the best policy out of all available options.

The continuous-prioritarian social welfare function is a family of nonlinear social welfare functions, which attempt to counter some of the shortfalls of the utilitarian form, the ones they view as most important, specifically the non-inclusion of the distribution of the total utility within the population. Adler gives the sum of the square roots of the utilities of the individuals in the population as an adequate example of the family, and this will be the example used when there is any reference to the continuous-prioritarian below. As a ranking tool, it, also, favours policies that have a higher sum than the other options.

The leximin social welfare function is the minimum utility from the individual utilities in the population, based on the philosophy that "a chain is as strong as its weakest link". Logically, a higher minimum is better.

The rank-weighted social welfare function is exactly as advertised, a sum of individual utilities multiplied by a weight. This form gives the decision-maker the opportunity to prioritise the utilities of certain individuals within the population, probably ones most affected by the policies, and diminish the effect of outliers, which skew results in other forms. Again, when ranking, a higher sum is preferable.

Lastly, there is the sufficientist social welfare function, which sets a welfare threshold, arbitrarily, above which the function is the utilitarian and below which it is the continuous-prioritarian, meaning that utilities below the threshold are taken into account using the continuous-prioritarian and utilities above are taken into account using the utilitarian. As was the case with all the other social welfare functions, a higher score is best.

Kakwani and Son (Nanak Kakwani, Hyun Hwa Son, 2016) mention 8 social welfare functions in their 2016 paper. The Benthamite, which is another name for the utilitarian, and the Bergson-Samuelson, which is an individualistic social welfare function, both of which they characterise as "general forms", thus did not really expand on them in their paper. They did, however, provide the equations for Atkinson's money metric social welfare function: $W=\int u(x)*f(x)dx$, the Gini social welfare function: $W=2*\int x*[1-F(x)]*f(x)dx$, Sen's social welfare function: $W=2*\int x*[1-F(x)]*f(x)dx$, Kakwani's social welfare function: $G=(2/\mu)*\int x*[F(x)-(\frac{1}{2})]*f(x)dx$, the Rawlsian, which is another name for the aforementioned Leximin, and, finally, the combination of Kakwani and Sen's social welfare functions: $Wg=\mu*(1-G)$.

During the 1970s, Harsanyi (John C. Harsanyi, 1975 and 1977) and Sen (Amartya Sen, 1977) argued through lectures, papers and rejoinders, on the subject of nonlinear social welfare functions. Harsanyi believed that the only social welfare function permissible by Bayesian decision theory is the average of individual utilities, allowing room only for the inclusion of weights. On the other side of the argument, Sen, whose social welfare function is a couple of lines above, argued for and favoured nonlinear social welfare functions. The extent of their argument can be found in Harsanyi's 1975 paper and 1977 rejoinder and Sen's 1977 paper. The first paper was a response to one of Sen's lectures.

At this point, after this brief account of social welfare function history, I would like to point out that every aforementioned form, whether linear or nonlinear, is a single-equation social welfare function. In other words, they are all one-dimensional social welfare functions, as I like to refer to them.

1.3. Their shortfalls

Each aforementioned form has certain shortfalls, which every future form attempts to address. The utilitarian social welfare function is one that has been under the most scrutiny, due to its simple form and the fact it was one of the first forms ever proposed, for example, Adler writes that "a key shortfall of the utilitarian SWF… is that it takes no account of the distribution of well-being. ….., it might be thought that the distribution of well-being itself is something we should care about." (Matthew D. Adler, 2019). Rawls in his attempt to address the non-inclusion of distribution created the leximin social welfare function, which, however, has some shortfalls of its own. Many criticise it for giving too much weight on the opinion of a potential outlier and others have said that "the leximin SWF is absolutist. Any gap-diminishing leaky transfer from a better-off to a worse-off individual is ethically recommended, regardless of how close to zero the fractional gain to the worse-off individual." Furthermore, the sufficientist social welfare function was created because, as Adler says, "Crisp criticizes prioritarians for giving greater ethical weight to well-being changes affecting worse-off individuals, as compared to the weight accorded changes affecting the better off, even if all the individuals involved are quite well off…. "the Beverly Hills case", in which the prioritarian prefers to give fine wine to a small group of rich individuals as opposed to giving fine wine to a larger group of super-rich individuals, where the individual welfare benefit of fine wine is the same for all the individuals in both groups".

As for the functions in Kakwani and Son's paper (Nanak Kakwani, Hyun Hwa Son, 2016), most of them are mainly functions of income, not utility or welfare. They are also pretty much different versions of the same idea.

Apart from these theoretical shortfalls, some social welfare functions also struggle with adhering to many axioms. "[The sufficientist SWF] doesn't satisfy the Pigou-Dalton axiom", "Rank-weighted SWFs violate Separability", "The utilitarian SWF satisfies neither [Pigou-Dalton and minimal Pigou-Dalton].", "The leximin and sufficientist SWFs do not [satisfy the Continuity axiom]."

Moreover, the main shortfall I find in all of them is that they strip away the decision-makers power in making an informed choice. Their one-dimensional form restricts the outlook they give on the policies into whatever the creator of the social welfare function thought was most important, whereas social welfare functions should, in my opinion, give as much important information to the decision-maker as possible about the policies, creating a rounded perspective, so they can then make, a close to perfectly, informed choice. Furthermore, the current forms, in actuality, are the ones that make the decisions, by assigning one score, thus transforming the decision-maker into an intermediary, without any power, between the philosophy of the creator of the social welfare function and the policies, which is problematic since decision-makers at the policy level were elected by the public, in democracies, and are expected to be the ones deciding. The first point is why one could not characterise a decision made using these forms as an informed one and the second point is why one could not characterise it as a decision.

1.4. Axioms

It has been widely accepted that social welfare functions must adhere to certain axioms, which act as broad rules that ensure the validity of the rankings and subsequent choices made using the functions, by testing the form of each function in regards to being able to highlight key characteristics of policies. There is discourse behind the weight each axiom has, or rather should have, when comparing or evaluating social welfare functions. Adler, in his aforementioned book (Matthew D. Adler, 2019) has categorised them thus:

Fundamental:

1. Pareto Indifference, which is the rule that equal vectors are equally good,

2. Strong Pareto, which says that if the only difference between two vectors is that one person is better off, then the one with that person is the better vector,

3. Anonymity, basically that the order of the numbers in each vector should not be a factor when comparing them,

4. Fundamental Principle of Invariance, if a vector is greater than another vector, then the addition of the same constant in both vectors should not change the comparison. Adler gives the example w<w* ⇔ w+2<w*+2 (Matthew D. Adler, 2019, page 97),

Additional:

1. Pigou-Dalton, a transfer of utility from an individual with a high utility to an individual with a low utility would result in a better vector. Adler gives the example 7,15,17,6 is better than 7,12,20,6 (Matthew D. Adler, 2019, page 99),

2. Minimal Pigou-Dalton, that the opposite transfer is never better,

3. Separability, that unaffected individuals don’t affect the ranking of vectors,

4. Continuity, that the function permits the existence of a region between x and x*, with x greater than x*, where other vectors are also greater than x*,

Uncertainty:

1. Expected Value Ethical Decision-Making, is the rule that one should follow the rankings made using the function when deciding which policy option to choose,

2. Dominance, basically if P is greater than P* in every state of nature then P is better than P*,

3. Ex ante Pareto Indifference, if the expected value of P is equal to the expected value of P* then P is equal to P*,

4. Ex ante Strong Pareto, if the expected value of P is greater or equal to the expected value of P* for everyone and the expected value of P is greater than the expected value of P* for at least one individual, then P is greater than P*,

Arrow's Conditions:

1. Universal Domain, everything is possible,

2. Independence of Irrelevant Alternatives, if x is equal to y then the rank of x should be equal to the rank of y,

3. Weak Pareto, if x is greater than y then the rank of x should be greater than the rank of y,

4. Nondictatorship, is the rule that every person's opinion is equal, and that the opinion of one specific individual should not dictate the ranking of options.

Harsanyi, on the other hand, believed that the only axioms that mattered were the Bayesian Rationality axioms in individual behaviour and moral choices between alternative social policies.

Of course, it should be noted that Adler is writing in the 21st century, while Harsanyi in the late 20th, so there has been substantial progress in the field between the statements of the latter and the former's opinions. Harsanyi was actually one of the first to lay the groundwork on the field, which means that his opinion should be noted and respected. However it should also be made apparent that the field is not in its infancy anymore, so more complicated analyses like Adler's hold more weight, even if Harsanyi as a scientist should be more revered. Furthermore, it should also be mentioned that Harsanyi had an absolutist idea about the form of the social welfare function, as is apparent in his back and forth with Sen, so any opinions he had about axioms are, probably, based on the form he sees as being the only correct one.

1.5. Use of SWFs

There is only theoretical, not practical, use of social welfare functions, since there is no way to calculate utility, which is the variable used in all of them (sometimes coupled with income). Moreover, even if one could estimate utility, one would have to assume that every individual has the same utility function, which is a heroic assumption, to put it mildly. Their actual practical use is to highlight the characteristics of policies/projects that should hold the most weight when decisions are made. In other words, to influence the perspective from which policies/projects are viewed. Social welfare functions in all of their forms are equations with theoretical backgrounds; the backgrounds are what's more important, i.e. the reasons behind the form of each equation, because once one understands the theory behind each form, it is easy to realise that the equations were an inevitability, a logical outcome; not the beginning, but the end of the thought-process.

2. <u>Other Methods to Rank Policies</u>

Due to use of social welfare functions only theoretically, many have come up with different methods of ranking policies and projects, which attempt to circumvent the issues that restrain social welfare functions from being applicable, whilst trying to approach the subject in a way that gives clear and accurate results.

Brun and Hadorn (Georg Brun, Gertrude Hirsch Hadorn, 2007) offer three different methods, namely Measured Merits, Ordered Values and Permissible Preference Orderings. Measured Merits was proposed by Chang, and it makes use of one covering value of the policies in order to rank them. Ordered Values was created by Griffin, and it is founded on basic value-judgments. Permissible Preference Orderings was Rabinowicz's idea where the deciding factors are the evaluative relations between value-bearers.

Urquhart and Eastman (N. Scott Urquhart, Clyde E. Eastman, 1977) put forth a pairwise ranking, more commonly known as Condorcet, when dealing with a partially informed population. Ties, in their example, were attributed to indecisiveness. In it a representative group from the population is presented with policy options, two each time, from which they choose the one they prefer. After this process, the data is collected, and each option is ranked based on the number of times it was chosen over others. It's also a great way to identify the median option, which is one that political scientists and politicians would like to know, due to the theory of the median voter.

Plaut (Pnina O. Plaut, 1998) proposes a three-coefficient social welfare function to estimate the effects of travel pollution, with consumption and travel being positive coefficients and the pollutant being a negative. However, I have not included it in the forms of social welfare functions, because it makes use of the utilitarian approach with more complex individual utilities. I should highlight that most others assume that the utilities of the individuals have already factored those coefficients, so it is not completely necessary to highlight them. However, it would make sense to do so, if these coefficients are given weights not by the individuals, but from the decision-maker, which would require more in-depth knowledge of every individual’s utility, something practically impossible, since we are not able to have even surface level knowledge of it, but theoretically an interesting concept.

Taeihagh, Givoni and Beñares-Alcántara (Araz Taeihagh, Moshe Givoni, René Beñares-Alcántara, 2013) use Multicriteria Decision Analysis tools along with network analysis. Steele, Carmel, Cross and Wilcox, also preferred Multicriteria Decision Analysis, but warned of potential “misuses” (Katie Steele, Yohay Carmel, Jean Cross, Chris Wilcox, 2008)

Hardaker, Richardson, Lien and Schumann (J. Brian Hardaker, James W. Richardson, Gudbrand Lien, Keith D. Schumann, 2004) prefer stochastic efficiency with respect to a function.

Apart from the methods discussed above, that were used by the respective researchers for their subjects, there is also the very well-known method of Cost-Benefit Analysis and Social Cost-Benefit Analysis. Cost-Benefit Analysis works by estimating the net present value of a policy or project and calculating if the NPV is positive or negative. If it is negative the policy/project is rejected entirely; if it is positive then the policy/project is put in consideration with other comparable ones that also have positive NPVs, because a positive NPV means that there will be a return on the investment taking place. After all NPVs have been estimated, the best option is considered to be the policy/project with the highest one. Cost-Benefit Analysis is usually a tool for companies, not policy-makers, due to its noninclusion of social costs and benefits, which is why Social Cost-Benefit Analysis was created. The main difference between the two is that the latter takes into account both economic and non-economic costs and benefits (the aforementioned social ones), while the former takes into account only economic costs and benefits. Moreover, Social Cost Benefit Analysis uses a social discount rate, which can also fluctuate through time when the policies are long-term ones, which is more likely the case with policies, rather than projects. Furthermore, regarding Social Cost-Benefit Analysis, environmental economics has put forth many methods with which one can estimate the non-economic costs and benefits of a policy. Namely, Hedonic and Travel Cost Analysis, Contingent Valuation Method, Choice Experiment Method, and Random Utility Model.

Hedonic Analysis is based on estimating a regression with the characteristics of a market good, for example the real estate market to evaluate the price of clean air. Travel Cost Analysis is the same thing but with travel packages, so that the value of preserving natural parks is estimated.

Contingent Valuation Method uses questionnaires on the predispositions, and certain valuation scenarios, where respondents assign their own price based on the characteristics of the scenarios, which exclude the costs.

Choice Experiment Method works the same way as Contingent Valuation, with the only difference being that within the scenarios are included prices or costs, and respondents rank their preferences based not only on the characteristics of each scenario, but on the preassigned cost as well.

Random Utility Model is a rank-ordered logit model that estimates the marginal willingness to pay based on respondents' rankings of different choice cards with slightly altered characteristics, such as environmental status, financial impact, quality of services (e.g. water supply) and price.

The selection of the Model used is based on the existence of a market that would allow Hedonic Analysis and the funding of the research, since questionnaires can be an expensive process, depending on the method used to get responses. Unfortunately, methods with higher accurate response rates are also the most expensive.

Concerning the aforementioned Social Discount Rate, there are three ways of estimating it. Social Rate of Time Preference, Social Opportunity Cost and the Weighted Average Method.

Social Rate of Time Preference uses the Ramsey Formula to estimate the rate at which society discounts future benefits in the present year. The Formula is: $pt= \delta + n*gt$.

Social Opportunity Cost claims that the discount rate is equal to the rate of return required by the private sector. The required rate of return (RRR) is estimated with CAPM (Capital Asset Pricing Model). Basically, RRR= Riskless Rate + Risk premium, the latter is found with CAPM.

The Weighted Average Method estimates, as the name suggests, a weighted average of Social Rate of Time Preference and Social Opportunity

Cost to create the Social Discount Rate. (Tom Tietenberg, Lynne Lewis, 2009)

For long-term projects, a Socially Efficient Discount Rate is used, more likely one that declines with time. (Tom Tietenberg, Lynne Lewis, 2009)

3. My proposal

3.1. The two-dimensional Social Welfare Function (two equations): mean and standard deviation

The former metric calculates average impact on society by the application of a policy, the latter estimates the (in)equality in the distribution of the aforementioned impact within the affected population.

3.2. The algebraic form of the function

$Wf = (Ua + Ub)/2$ (or ($\sum_{i=1}^{N} Ui$)/n, generally speaking) and $s.d. = Sum\ of\ square\ root\ of\ (Ui - Wf) squared\ divided\ by\ n, i = 1, \ldots, n.$.

3.3. How it works

Example with 2 scenarios: A and B. If the Wf of scenario A is higher than the Wf of scenario B, a society should choose (or rather prefer) scenario A. If the s.d. of scenario A is higher than the s.d. of scenario B, a society should prefer scenario B. If both conditions are true at the same time, then it is up to the society to decide what they value more. In my opinion, the way this deadlock can be solved is by comparing the percentage differences in the two metrics. For example, if (W1f- W2f) /W2f> (s.d.1-s.d.2) /s.d.2, I would advise preferring scenario 1, whereas if (W1f- W2f)/W2f < (s.d.1-s.d.2)/ s.d.2, logic dictates that scenario 2 is preferrable. Of course, there is the possibility, although slim, that (W1f- W2f)/W2f = (s.d.1-s.d.2)/ s.d.2. In that case, the choice is up to debate. It should be noted that the aforementioned are my solution to a problem that may arise, and it is not mandatory. Meaning a society can find another way to solve this puzzle. Although, I do think that my solution is, by all accounts, well thought out.

3.4. What do the metrics mean?

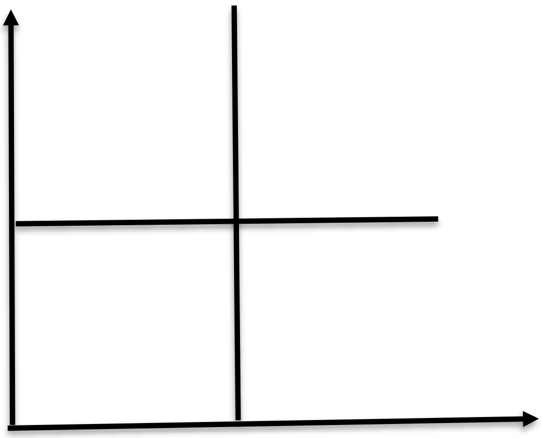
Wf calculates the mean utility that a scenario gives to society, standard deviation calculates how the utility was distributed to the people. In other words, Wf measures the increase in the average person’s welfare caused by the application of the scenario and s.d. measures the equality of the distribution of the aforementioned increase. From these, one can easily deduce that if a society values Wf more than s.d., it is more like a utilitarian society and if it values s.d. more than Wf, it is more like a prioritarian. But they are not identical to these cases, because even if they value one more than the other, they should take both into account when taking a decision, which forces a utilitarian society to act in a more prioritarian way and a prioritarian to see the valid arguments of the utilitarian, thus bringing the two, extreme, in my opinion, theories closer.

3.5. Why these metrics

In my opinion, the use of two metrics when deciding between scenarios is necessary, especially the two metrics that I have proposed, because they give the decision-maker the full picture. Whereas previously employed social welfare functions used only one of each metrics, thus constraining the decision-maker into what the creator of the function assessed was the most important characteristic of any scenario. These two metrics also allow a decision-maker to realise the true impact a scenario will have on society, by calculating the impact on the average person’s welfare and the difference in impact across the population. Regarding the average person, that was one of the reasons why I thought of this function. I thought that the two functions were concentrating on the wrong people, one was giving weight to the opinion of the one who was more affected by the policy, either positively or negatively, and the other focused more about the least affected, when, in reality, we should be concerned about the average impact of a scenario, not the impact on the outliers. That does not mean that one should not be concerned about the inequality of the impact, that is why the second metric is just as important as the first, because it gives you a clearer view on inequality, by taking everyone’s experience into account, not just the outliers.

3.6. The graph

Wf



Graph 1 **Sd**

We can use this graph to categorise scenarios. On the y axis is Wf and the x axis is s.d. The best case is for a scenario to fall somewhere at the top left corner, because that means that it has high Wf and low s.d. Worst case is bottom right, low Wf and high s.d. The choice between two scenarios, one in the top left and the other in the bottom right is obvious. What is not obvious is the choice between a scenario in the bottom left, low Wf and low s.d., and one on the top right, high Wf and high s.d. If a society chooses top right options more, it values utility of the average person over equality of distribution of utility and vice versa.

3.7 Alternative metrics

I understand that various researchers before me have proposed a social welfare function using average, however existing social welfare functions weren't paired with standard deviation, which, in my opinion, is of extreme importance and the differentiating factor of my proposal. This is why I would also like to propose alternative metrics for the function. By this I mean the different versions that Wf can have, whilst complying with the same principle. One example would be a weighted average social welfare function, where weight is assigned by an expert to the utilities of the respondents, probably based on the relevance of the scenario to them and Wf equals the sum of their utilities multiplied by their assigned weight (where w are the corresponding weights). But this version has the risk of subjectivity, because of the weights and who assigns them. One could also disregard the responses of the respondents who are purposefully negatively affected by a scenario, for example the utility of corporations is obviously going to be negative when talking about raising corporate taxes, but that is the point, so one must make the decision of counting or disregarding the opinions of the corporations. It could also be divided into two separate functions, one that takes relevancy to the subject into consideration (S (ri*Ui, r: relevancy), and one that assigns weight to people’s opinion based on other criteria, for example expertise (S(wi*Ui, w:weight)). Another alternative is one that uses behavioural economics’ reference point utility, thus making each respondents utility equal to Ui-U*i, I would suggest simplifying it by using Uwi= Ui- U*i and then the function would be equal to Wf= (Uw1+…+Uwn)/n. Moreover, one could make a hybrid out of my proposed alternatives, for example one that uses both weight and reference points, because that would give them a much more realistic result and would have taken every factor into consideration. I proposed the original function, because it is the basis for the alternatives and because I wanted to explain the metrics themselves using the simplest of examples, plus I believe that it can still be used for research purposes.

For standard deviation, one could also use variance, but I do not see any point, since standard deviation is the square root of variance and, contextually, it gives us the information needed in the correct form.

3.8 Conclusion of the proposal

To sum up, if one were to use my proposal and apply it to a real-life situation, they would begin by measuring the utility that each scenario gives each respondent. They, then, would collect the data and estimate the value of the two metrics, Wf and s.d., for every scenario using whichever variation they thought was best. Afterwards, they would compare the scenarios with my proposed method(s), either with comparing the percentage differences of the two metrics, with the graph or with both. They would then be able to make an informed decision on the favoured scenario. I believe that the example I’ve just described is one that answers most of the questions raised about the accuracy and objectivity of welfare functions and of welfare economics, in general, and it is a very reliable and robust procedure that also allows decision-makers to have freedom of choice and not be constrained by the philosophy of the person suggesting the SWF.

3.9 Uncertainty

If uncertainty of outcome is included in the calculations, then, for every scenario, one would estimate the ex ante average outcome, the standard deviation of that ex ante outcome, the ex ante standard deviation of individual utility from the ex ante average outcome and the standard deviation of the ex ante standard deviation. In other words, one would have to calculate: E(Wf), s.d(E(Wf)), E(s.d.) and s.d.(E(s.d.)). The first and the third are metrics that calculate the expected average social welfare and the expected (in)equality of distribution of said average, the second and fourth metrics calculate the risk of the expected average social welfare not materialising and the volatility of the (in)equality of distribution along the possible outcomes.

In terms of the graph method, one could create four of them when there is uncertainty about the outcome of applying each scenario: 1. E(Wf) on the y axis and E(s.d.) on the x axis, 2. E(Wf) on the y axis and s.d.(E(Wf)) on the x axis, 3. E(s.d.) on the y axis and s.d.(E(s.d.)) on the x axis and 4. s.d.(E(Wf)) on the y axis and s.d.(E(s.d.)). The first graph works in the same manner as the one with no uncertainty. The second gives the decision-maker an idea about the risk of the expected outcomes not coming into fruition. The third one creates a picture for the actual distribution of welfare amongst the possible outcomes of a scenario. The fourth one puts risk and volatility together, informing the decision-maker of the total risk of application of each scenario.

3.10 Adherence to the Axioms in Adler (Measuring Social Welfare: An Introduction, 2019)

It adheres to all of the fundamental and additional axioms (Pareto Indifference, Strong Pareto, Anonymity, Fundamental Principle of Invariance and Pigou-Dalton, Minimal Pigou-Dalton, Separability, Continuity). Pareto Indifference: equal vectors means equal averages and standard deviations, so they're equally as good. Strong Pareto: one person better off means higher average, effect on standard deviation varies on account of each hypothetical vector, however the percentage change is probably higher for the average, so the new vector is better than the old one. Anonymity: order of numbers is not a factor, since there are no weights assigned by order. Fundamental Principle of Invariance: w+2 would mean higher average, same standard deviation, so if w<w*, then w+n<w*+n, n being a number. Pigou-Dalton (7,15,17,6 is better than 7,12,20,6): same average, lower standard deviation, so based on the two-dimensional SWF it's true. If Pigou-Dalton is adhered, then minimal Pigou-Dalton is also. Separability: of course. Continuity: The existence of space between the two scenarios on the graph proves the adherence to this axiom. As for the, per Adler, Uncertainty Axioms, Expected Value Ethical Decision-Making: not up to the SWF, but on the decision-maker. Dominance: of course. Ex ante Pareto Indifference: of course. Ex ante strong Pareto: it would lead to a higher mean, so probably.

4. **Numeric Example**

The example is of a proposed tax on income. There are two options: 2 or 3%. Below are estimations of two individuals' utilities and the social welfare after the application of the tax rates. Their utility equations are:

$$u1 = ln((1-t)Y) + 2ln(tY), u2 = ln((1-t)Y) + 4ln(tY), Y = 1000, t = 2\% \ or \ 3\%$$

Their utilities with tax=2% are:

$$u1 = ln(980) + 2ln(20) = 12.88, u2 = ln(980) + 4ln(20) = 18.87, t = 2\%$$

Their utilities with tax=3% are:

$$u1 = ln(970) + 2ln(30) = 13.68, u2 = ln(970) + 4ln(30) = 20.48, t = 3\%$$

Using my proposal the social welfare for tax=2% is:

$$Wf2 = 1/2(12.88 + 18.87) = 15.875, s.d.2 = (2.995 + 2.995)/2 = 2.995$$

And for tax=3%, it is:

$$Wf3 = 1/2(13.68 + 20.48) = 17.08, s.d.3 = (3.4 + 3.4)/2 = 3.4$$

By applying the percentage difference methodology, we find out that:

$$[(Wf3 - Wf2)/Wf2]100 = 7.6\%, [(s.d.3 - s.d.2)/s.d.2]100 = 13.5\%,$$

Which means that, based on the proposal, one should prefer t=2%

Let us compare with the Utilitarian/Benthamite and Rawlsian/Leximin social welfare functions.

Using the former, the results are:

$$Wut2 = 31.75, Wut3 = 34.16$$

Meaning tax=3% is preferable to tax=2%.

Using the latter, the results are:

$$Wlex2 = 12.88, Wlex3 = 13.68$$

Meaning tax=3% is, again, preferable to tax=2%.

This example shows quite strikingly the differences between the two-dimensional social welfare function and the utilitarian and the leximin social welfare functions. Both of them preferred tax=3%, due to their inherent shortfalls (utilitarian: its one-dimensional form, leximin: focus on the outlier, not the entire population). Interestingly, the leximin preferred the option which increased the inequality in distribution of outcomes for the individual with the minimum utility. The utilitarian is known to not take into account distribution, so the results were not as surprising. The two-dimensional gave, in my opinion, a full view of the two options' outcomes and their impact on the society in question and gave its preference without discounting any aspect.

5. Conclusion

In the first part of the paper, we defined social welfare functions and their utility. After that, eleven forms of social welfare functions were presented, namely the utilitarian/Bentemite, the Rawlsian/leximin, the Continuous-prioritarian, the rank-weighted, the sufficientist (Matthew D. Adler, 2019) and the Bergson-Samuelson, Atkinson's social welfare function, the Gini social welfare function, Sen's social welfare function, Kakwani's social welfare function and the combination of Sen's and Kakwani's social welfare functions (Nanak Kakwani, Hyun Hwa Son, 2016). Then there was a reference to the debate between professors Sen and Harsanyi about the correct form of social welfare functions and the existence of nonlinear ones, with Sen being for nonlinear social welfare functions and Harsanyi being absolutely against and, also, against any form other than average (John C. Harsanyi, 1975 and 1977)(Amartya Sen, 1977). Afterward, the axioms social welfare functions must adhere to were discussed [Pareto Indifference, Strong Pareto, Anonymity, Fundamental Principle of Invariance, Pigou-Dalton, Separability, Continuity, Expected Value Ethical Decision-Making, Dominance, Ex Ante Pareto Indifference, Ex Ante Strong Pareto, Universal Domain, Independence of Irrelevant Alternatives, Weak Pareto, Nondictatorship (Matthew D. Adler, 2019), as well as the Bayesian Rationality Axioms (John C. Harsanyi, 1975)], together with the shortfalls of the aforementioned forms, such as nonadherence to certain axioms.

In the second part of the paper, alternative methods for ranking policies used by many researchers were presented [Measured Merits, Ordered Values, Permissible Preference Orderings (Georg Brun, Gertrude Hirsch Hadorn, 2007), Pairwise Ranking (N. Scott Urquhart, Clyde E. Eastman, 1997), a Social Welfare Function with three coefficients (Plaut), Multicriteria Decision Analysis (Araz Taeihagh, Moshe Givoni, René Beñares-Alcántara, 2013, Katie Steele, Yohay Carmel, Jean Cross, Chris Wilcox, 2008), Stochastic efficiency with respect to a function (J. Brian Hardaker, James W. Richardson, Gudbrand Lien, Keith D. Schumann, 2004)] as well as methods used by Environmental Economics to measure non-market value [Hedonic and Travel Cost Analysis, Contingent Valuation Method, Choice Experiment Method, Random Utility Model, Social Cost Benefit Analysis (Tom Tietenberg, Lynne Lewis, 2009)]. Normal Cost Benefit Analysis was also mentioned, along with the differences between the former and Social Cost Benefit Analysis, with special reference to the Social Discount Rate and the methods used to estimate it [Social Rate of Time Preference, Social Opportunity Cost, Weighted Average (Tietenberg, Lewis)].

The third part of the paper was my proposal for a new form of a social welfare function. More specifically a two-dimensional social welfare function with two metrics: average and standard deviation from the average. The former symbolises the average impact on the utility of an individual in the population, whilst the latter the actual equality in the distribution of said average impact amongst the individuals of the population. Alternative metrics with slight differences to the original two were also given, such as weighted average and variance. After I explained how the metrics work and how they can be utilised to compare policy options. Regarding that last subject, I proposed two methods: the graph and percentage difference. The graph is simple, on the y axis is the average and on the x axis is standard deviation, it has been divided into four quadrants, the top left one encompasses the best policy options, the bottom right the worst, whilst the two remaining ones are to be debated. The percentage difference method is one that settles that debate, by comparing the percentage difference in average and standard deviation between two policy options. If the percentage difference of the former is higher than the latter’s, then the policy in the top right quadrant should be chosen, and vice versa. I also analysed the use of the new form in situations where uncertainty of outcome needs to be taken into account, introducing a new metric which calculates the volatility in the equality of the distribution amongst the many possible outcomes. Finally, I discussed whether the new form adheres to the aforementioned axioms.

The fourth and final part of the paper was a numeric example of a choice between two income tax rates. The population was two people, their incomes were equal, but not their utility functions, with one of them gaining more utility from public goods than the other. After calculating the utility of each person in each scenario, I, firstly, estimated the results given by the two dimensional social welfare function, coming to the conclusion that the first option was preferable, based on the percentage difference method. I then did the same with the two most well-known social welfare functions, the utilitarian and the leximin, and found that both of them favoured the second policy option. An analysis on the results and the reasons for them followed these findings.

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