Optimal Monetary Policy with and without Debt*

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Abstract

We derive optimal monetary policy rules when government debt may be a constraint for the monetary authority. We focus on an environment where fiscal policy is exogenous, setting taxes according to a rule that specifies the tax rate as a function of lagged debt. In the case where taxes do not adjust sufficiently to ensure the solvency of debt, then the monetary authority is burdened by debt sustainability. Under this scenario, optimal monetary policy is a 'passive money rule', setting the interest rate to weakly respond to inflation. We characterize analytically the optimal inflation coefficients under alternative specifications of the central bank loss function. We show that the maturity structure of debt is a key variable behind optimal policy. When debt maturity is calibrated to US data, our model predicts that a simple inflation targeting rule where the inflation coefficient is $1 - \frac{1}{\text{Maturity}}$ is a good approximation of the optimal policy.

Lastly, our framework nests the case where fiscal policy adjusts taxes to satisfy the intertemporal debt constraint. In this scenario optimal monetary policy is an active policy rule. We contrast the properties of active and passive policies, using the analytical optimal policy rules derived from this framework of monetary/fiscal interactions.

Keywords: Fiscal/monetary policy interactions, Fiscal theory of the price level, Ramsey policy, Optimal monetary policy rules.

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1 Introduction

Since the 2008-9 recession and more recently due to the spending and transfer programs launched to deal with the effects of the COVID 19 recession, governments in developed economies accumulated large stocks of debt. High debt levels require bold fiscal adjustments to ensure the solvency of government budgets. However, in many cases it is questionable whether fiscal authorities will be able/willing to generate the large required surpluses to finance debt.[1]

At high debt levels, and when fiscal authorities may not be able/willing to adjust taxes sufficiently, debt may become an important constraint for monetary policy. Under such circumstances, ensuring debt solvency becomes a task burdening the monetary authority and inflation needs to be used to finance debt. How should policy be then designed to be optimal?

A sizable literature has studied optimal policy in the 3-equation New Keynesian model (see e.g. Woodford (2001b); Giannoni and Woodford (2003); Orphanides and Williams (2007); Svensson (1999, 2003); Woodford (2003b); Giannoni (2014) among numerous others), that is in the case where monetary policy is not burdened with debt substainability. These papers have used the baseline New Keynesian framework to develop practically relevant recommendations for the design of interest rate rules, the desirable response of the main instrument of monetary policy to macroeconomic conditions.

Another strand of literature has analyzed optimal policy in environments where inflation responds to debt. Using the so called *Ramsey approach* to optimal policy, Chari and Kehoe (1999); Siu (2004); Schmitt-Grohé and Uribe (2004); Faraglia, Marcet, Oikonomou, and Scott (2013); Lustig, Sleet, and Yeltekin (2008); Leeper and Zhou (2021) among others, have solved policy problems in which a Ramsey planner can simultaneously set inflation and fiscal variables (taxes) to satisfy the intertemporal solvency of debt.

From this second class of models, however, it is not easy to derive transparent conclusions concerning the conduct of monetary policy. Firstly, because bringing together monetary and fiscal policies under one authority makes it difficult to disentangle which of the implications of the models concern monetary policy and which do not.[2] Secondly, and most importantly, because in the optimal policy equilibrium in these models macroeconomic variables become functions of the current and lagged values of the Lagrange multiplier attached to the consolidated budget

[1]Even prior to the COVID crisis, the US had a debt to GDP ratio exceeding 100 percent and that was projected to rise. At the same time, on the fiscal side, there was no announced increase in tax rates to stabilize debt. In light of this, several papers have built models based on the assumption that taxes do not adjust to ensure fiscal solvency and investigating the effects on the macroeconomy (see Bianchi and Melosi (2017, 2019) and the recent strand of the literature on monetary/fiscal interactions we summarize below).

In the Euro area, similar conditions held after the 2010-11 debt crisis, the main concern being that debt has been explosive in some countries, and sustainable in others. This has led to a development of a considerable academic and policy literature on 'fiscal divergence'.

These problems are obviously more relevant today due to the effects of COVID on debt levels in OECD economies.

[2]In many of these models the planner will use the tax schedule not only to generate surpluses and finance debt, but also to 'manipulate interest rates' and hence change the real costs of financing debt (see Lustig et al., 2008; Faraglia et al., 2013; Leeper and Zhou, 2021). Taxes will therefore affect real and nominal interest rates but so does monetary policy, operating through inflation and output targets. In effect, the optimal interest rate path is jointly determined by the fiscal/monetary policies.

constraint, the object that defines the dynamics of debt. The multipliers are state variables and it is not easy to derive an interest rate rule that expresses the nominal rate solely as a function of macroeconomic conditions, without involving Lagrange multipliers.

This is evidently an important limitation of the analysis. Lagrange multipliers cannot be observed in practice, and therefore solving the Ramsey first order conditions (expressing the path of the nominal rate as a function of the multipliers) cannot provide any meaningful guidance to monetary policy.

This paper makes progress with characterizing optimal interest rate rules in an environment where the monetary authority may have to take into account debt sustainability. We utilize a standard New Keynesian model augmented with a fiscal block, the consolidated budget constraint and a fiscal policy rule specifying taxes as a function of the lagged value of debt. We solve an optimal policy problem in this linear quadratic framework assuming, as in much of the literature, that the central bank may seek to minimize inflation and output variability.

In Section 2 we describe the competitive equilibrium equations in our baseline model and setup the optimal policy program. We derive a formula expressing the inflation output trade-off under the optimal policy: The sum of inflation and output growth (the latter scaled by the relative weight attached to output stabilization in the loss function) is equated to a weighted sum of the current and lagged Lagrange multipliers attached to the consolidated budget. Were these multipliers equal to zero in the model (equivalently debt did not matter for optimal policy) then optimality would take the form of the so called *target policy criterion* in the canonical NK Keynesian model. It would then be fairly easy (given previous work by Giannoni and Woodford (2003) and others) to express optimal monetary policy as a rule in which the interest rate reacts to macroeconomic variables. However, in the presence of the Lagrange multipliers, characterizing optimal policy through such a rule does not seem an obvious property of the model.

In Sections 3 and 4 we strive to find optimal interest rate rules, by either substituting out the Lagrange multipliers, or identifying conditions under which the multipliers are zero and consequently the optimal rules we can derive from our model are those of the standard New Keynesian benchmark. We show that the model admits two types of optimal policy equilibria: In one case, when taxes adjust strongly to debt (in broadly used terminology fiscal policy is 'passive'), then the Lagrange multipliers equal 0, and the debt constraint is slack. In the second case, when taxes do not adjust to debt, (fiscal policy is 'active') then the debt constraint is binding; the multipliers are not zero and exert an influence on the optimal policy.

We then focus on the latter scenario to find an equivalent writing of the optimal interest rate rule where the right hand side variables are macroeconomic variables only, the Lagrange multipliers can be dropped. Our key finding is that such rules do exist, and they are fairly simple inflation targeting rules in which the coefficients attached to inflation can be expressed as functions of the model parameters. Moreover, the nominal rate tracks the real interest rate and (under certain conditions) a stochastic intercept term, a function of the contemporaneous shocks to the economy, also exerts an influence. The optimal rules are *passive money rules*: the nominal rate responds weakly to inflation (e.g. Leeper, 1991).

We characterize analytically these optimal rules in alternative versions of our model, in order to highlight the different channels of optimal policy. We firstly use a simplistic Fisherian setup (e.g. Cochrane (2001); Sims (2013) among others) in which the central bank only seeks to minimize the volatility of inflation. In this context, aggregate output does not exert any influence on debt (does not matter for fiscal solvency) enabling us to focus on the direct impact of inflation on public debt dynamics. We show that the optimal policy is a simple inflation targeting rule, in which the inflation coefficient equals δ, the decay factor of the coupon payments on government debt.

This result is intuitive. Under passive monetary policy, a higher inflation coefficient implies more persistence, given that inflation is a backward looking process. When the maturity of debt is short (i.e. $\delta = 0$) persistence is undesirable, since only short term price changes can contribute towards making debt sustainable when a shock occurs. With long maturity (δ close to 1) it is optimal for inflation to revert to target at the same rate as the coupon payments on government debt decay. This enables to spread efficiently the costs of inflation over time.

Our analysis then departs from this simplistic setup to consider more plausible specifications of the model, considering a central bank that desires to smooth both output and inflation and assuming that the real interest rate is endogenously dependent on output growth, as in the canonical New Keynesian model. Though we can derive explicitly the optimal interest rate rule when both of these modelling assumptions are made, we first separately consider each of them in order to continue highlighting transparently the key forces at work.

Smoothing output adds inertia to the optimal policy rule, making inflation a more persistent process for any debt maturity. Intuitively, when a smooth path of output is desired, temporary innovations to inflation (that would otherwise be optimal when debt is short term) are not warranted, as they result in output variability and increase the losses of the central bank. Our analytical formula characterizes the optimal coefficients in this case, as a function of the relative weight attached to output stabilization and of the debt maturity.

In the canonical model, where the real interest rates depend on output growth, optimal policy also takes the form of a simple inflation targeting rule. The optimal inflation coefficient is now not only a function of the average maturity of debt but also accounts for the *indirect effects* of inflation, through output, on the real bond prices, since prices matter for the intertemporal solvency of debt. We provide a simple formula showing the dependence of the optimal rule on the parameters that account for the direct and indirect channels of inflation.

Bringing these margins together we characterize analytically the optimal policy rule in the canonical model with a dual mandate objective function. The rule that we can derive is basically a fusion of the two separate scenarios, featuring an inertial response to inflation and accounting for the direct and indirect effects. Such a policy function remains optimal when we add subsidies to the model to eliminate distortions from monopolistic competition, when we assume that the central bank targets the natural rate of output, or when it pursues a simpler objective targeting a constant (steady state) output level.

Section 4 delves deeper into these properties characterizing the impulse responses of macroeconomic variables to shocks. We draw two important findings from this analysis: First, when the maturity of debt is long, the objective to smooth output lines up with the objective to spread the distortions of inflation across periods. Second, the indirect effect of inflation turns out not to matter much with long term debt, since any impact of the output path on long bond prices is compensated by an analogous impact on the real discount rates of future government surpluses.

These results lead us to the following important policy conclusion: When the maturity of debt is calibrated to the US data, a simple rule setting the inflation coefficient equal to $\delta \equiv 1 - \frac{1}{\text{Maturity}}$ provides a very good approximation of optimal policy in the canonical model and when the central bank has the dual mandate to stabilize both output and inflation. A transparent rule where the average maturity of debt is the only relevant moment, works well regardless of whether the objective function of the central bank is ad hoc (e.g. targets constant output) or is derived through a second order approximation of the household welfare function.

Section 5 summarizes the results from several extensions of the model considered in the appendix, showing the robustness of our findings to alternative modelling assumptions. A final section concludes the paper.

This paper contributes and is related to several strands of literature. First, the main contribution of our work is to characterize optimal interest rate policies when government debt matters for inflation. As discussed previously, though a considerable amount of work has been done in the standard New Keynesian framework (where debt is irrelevant for inflation, e.g. Giannoni and Woodford (2003) and previously referenced papers), to our knowledge no existing theoretical work has derived explicitly optimal interest rate rules when monetary policy needs to account for debt. As we explained, the main difficulty with this task is that Ramsey optimality conditions feature the history of Lagrange multipliers attached to the debt constraint. Our substantive finding is that commitment to rules that set the nominal interest rate as functions of inflation and target real rates are sufficient to implement optimal policy outcomes.

An alternative approach (which has been adopted by some papers in the literature) is to numerically characterize optimal interest rate rules. For example, Schmitt-Grohé and Uribe (2007) (the only paper that we are aware of doing this for the case of active fiscal policy) consider a rule in which the nominal rate responds to inflation, output and one lag of the interest rate and then numerically solve for the coefficients that maximize the welfare of the household. Interestingly, Schmitt-Grohé and Uribe (2007) find that such standard rules can approximate Ramsey outcomes when active fiscal policy entails a too strong reaction of taxes to debt, but not when taxes are weakly responding to the debt level, as we assume here.[3] They reach this finding in a model with one period (quarterly) debt.

Our analytical formulae complement this line of work. Researchers interested in finding rules

[3]To explain this better, active fiscal policy means that debt has an explosive root. This can be the case when taxes are constant (or nearly constant) or when taxes over-adjust so that the root is smaller than minus 1 (in that equilibrium debt features explosive oscillations). Our analysis concerns the former (and more standard) scenario where the rules employed by Schmitt-Grohé and Uribe (2007) do not perform as well.

In the appendix we revisit this conclusion, showing that rules similar to the ones used by Schmitt-Grohé and Uribe (2007), with optimized coefficients, underperform relative to our optimal (Ramsey) rules across all maturity structures. Differently from Schmitt-Grohé and Uribe (2007) however, who solve their model using a second order accurate solution, we employ the standard linear quadratic framework with the usual microfounded loss function.

that bring model outcomes close to optimal policy (or want to understand where simple rules of the sorts used in DGSE models may fail in doing so) will find in our solutions a useful benchmark. For instance, our result that a simple inflation targeting rule with a coefficient equal to δ can approximate the Ramsey outcome, holds for an empirically plausible debt maturity structure, but not with quarterly debt. Our formulae reveal that when debt is short term, then an optimal rule may need to respond contemporaneously to shocks, or in the presence of strong indirect effects, the inflation coefficient may be slightly negative! The methodology that we use in order to arrive to these analytic results and which is spelled out in the online appendix (applied to the several variants of the baseline model we consider), will be useful to researchers that may want to extend our findings to alternative environments (e.g. models featuring more sources of rigidities than in pricing, as we assumed here).

Our work is also related to a large literature studying the interactions between monetary and fiscal policies in macroeconomic models (e.g. Sargent, Wallace et al., 1981; Leeper, 1991; Sims, 1994; Woodford, 1994, 1995, 2001a; Cochrane, 1998, 2001; Schmitt-Grohé and Uribe, 2000; Bassetto, 2002; Eggertsson, 2008; Canzoneri, Cumby, and Diba, 2010; Del Negro and Sims, 2015; Davig and Leeper, 2007; Reis, 2016; Jarociński and Maćkowiak, 2018; Leeper and Leith, 2016; Bianchi and Ilut, 2017; Bianchi and Melosi, 2017; Davig and Leeper, 2007; Bianchi and Melosi, 2019; Leeper, Traum, and Walker, 2017; Kumhof, Nunes, and Yakadina, 2010; Bi and Kumhof, 2011; Benigno and Woodford, 2007; among many others).[4] Particularly related is the framing of the interactions in Leeper (1991), in terms of two distinct regimes: In one regime fiscal/monetary policies are active/passive and in the second regime they are passive/active (for any other configuration of policies, i.e. active/active or passive/passive, the model does not have a unique stable rational expectations equilibrium).

As discussed previously, our model admits two types of equilibria, under passive and active fiscal policies. When taxes strongly adjust to debt (passive fiscal) the Lagrange multipliers attached to the debt constraint are endogenously zero and we obtain the optimal policy of the standard New Keynesian model. The optimal interest rate rules in this scenario are active. We characterize these rules analytically for the different versions of the model we consider. Besides illuminating the contrast with the case of active fiscal policy, these derivations demonstrate that the model nests the two equilibria of active/passive policies defined in Leeper (1991).[5] Whereas in Leeper (1991) and the rest of the literature that used his influential framework, interest rate rules are ad-hoc, here they are optimal. Our framework can thus be seen as an extension of this important model to optimal monetary policy. This is a separate contribution of our paper.[6]

[4]See in particular Leeper and Leith (2016) for a very comprehensive overview of this literature focusing on the interactions between monetary and fiscal policy.

[5]Our solution gives rise to two stable equilibria, there is no room here for any lack of coordination between the two policies that could bring about multiple equilibria (as for example in the case of a passive/passive regime). This has to do with the assumed structure of the policy program. The fiscal authority 'commits' to a tax rule and this is internalized by the optimizing monetary authority. Essentially, monetary policy acts like a 'Stackelberg leader' (though this term is not fully correct since we do not model optimal fiscal policy). Given this framing, it is not surprising that optimal monetary policy leads to unique equilibria.

[6]Our framework can be used to study optimal policies in context of the various significant extensions of Leeper

Our work is also related to a considerable literature studying optimal policy with debt. A large strand of this literature has studied the properties of optimal distortionary taxes and debt issuance in real models (e.g. Lucas and Stokey (1983); Aiyagari, Marcet, Sargent, and Seppälä (2002); Angeletos (2002); Buera and Nicolini (2004); Faraglia, Marcet, Oikonomou, and Scott (2019, 2016) among many others). Lucas and Stokey (1983) were the first to tackle this problem in a complete financial market setting, assuming that debt is issued in state contingent securities. Aiyagari et al. (2002) and Marcet and Scott (2009) instead focused on the case where the optimizing government issues only non-state contingent debt, a short term bond. Faraglia et al. (2016) extended this approach to long term government bonds.

Our approach to modelling optimal policy is methodologically similar to the approach of these papers. We assume incomplete markets as Aiyagari et al. (2002), Marcet and Scott (2009), and Faraglia et al. (2016) do. Moreover, even though taxes are exogenous here and instead policy sets distortionary inflation to make debt solvent, many of the features of the optimal allocation are common with optimal taxation models. We therefore frequently refer to well known results from this literature to explain our findings.

From the second strand of this related literature studying optimal policy with debt- the papers on Ramsey monetary/fiscal policies previously referenced- the work of Leeper and Zhou (2021) is most closely related to ours. Leeper and Zhou (2021) utilize a linear quadratic framework broadly similar to the one employed here, to investigate how the optimal mix of inflation and taxes varies with the debt maturity and with the relative importance attached to smoothing inflation v.s. smoothing output fluctuations in the objective of the planner. While their paper makes considerable progress with deriving analytical results in this context, their interest is not in deriving optimal monetary policy rules. Thus, our findings are complementary.[7]

Lastly, in recent work, Chafwehé, de Beauffort, and Oikonomou (2022) use the simplistic Fisherian model we present in Section 3 to derive optimal interest rate rate rules in a model with active fiscal policy. Their paper experiments with alternative ways of modelling government bonds, most notably considering separately the case where governments can engage in debt buybacks and the case where they cannot. We use a standard setup of modelling government debt (as in e.g; Woodford, 2001a; Bianchi and Ilut, 2017; Leeper and Zhou, 2021 and others) and moreover, relative to Chafwehé et al. (2022), our derivations extend to more plausible calibrations of the New Keynesian model with debt.

(1991) considered in the literature. For example, it can be applied to study regime fluctuations, (Davig and Leeper (2007); Bianchi and Melosi (2017, 2019)), an extension we are pursuing in current work. Our analytical rules and methodology should also be applicable in the context of models with jointly optimal monetary/fiscal policies, the papers previously referenced.

[7]Our findings are also complementary to Kirsanova and Wren-Lewis (2012) who, departing from the jointly optimal problem, consider the case where fiscal variables follow feedback rules similar to the ones assumed here. Their numerical solutions suggest that under a strong reaction of spending or taxes to (short term) debt, optimal monetary policy resembles an active policy. However, their characterization of active/passive policy is based on the model solution which links the nominal rate with the state variables of their model (shocks and debt stocks). This is a very different approach than the one we adopt in this paper. Kirsanova and Wren-Lewis (2012) do not derive standard interest rate rules.

2 Theoretical Framework

We consider an optimal policy problem where a Ramsey planner (the Fed) sets the path of macroeconomic variables, interest rates, inflation and output, subject to the dynamic equations that define the competitive equilibrium. Our framework is a standard New Keynesian model, with monopolistically competitive firms operating technologies which are linear in the labour input and setting prices subject to adjustment costs as in Rotemberg (1982). The model is augmented with a fiscal block, the consolidated budget constraint and a tax rule that determines the response of taxes to the lagged value of government debt. We derive our main results under the assumption that taxes are lump sum.

Since this is a standard setup, we will describe the competitive equilibrium using the equations of the log-linear model. We leave it to the appendix to characterize the household and firm optimal behavior from the (background) non-linear model.

2.1 The model

We use the standard notation $\hat{x}$ to denote the log deviation of variable x (in the nonlinear model) from its steady state value, $\overline{x}$. The model equations are the following:

$$\hat{\pi}_t = \kappa_1 \hat{Y}_t - \kappa_2 \hat{G}_t + \beta E_t \hat{\pi}_{t+1}, \tag{1}$$

where $\kappa_1 \equiv -\frac{(1+\eta)\overline{Y}}{\theta}(\gamma_h + \sigma\frac{\overline{Y}}{\overline{C}}) > 0$, and $\kappa_2 \equiv -\frac{(1+\eta)\overline{Y}}{\theta}\sigma\frac{\overline{G}}{\overline{C}} > 0$,

$$\hat{i}_t = E_t\left(\hat{\pi}_{t+1} - \hat{\xi}_{t+1} + \hat{\xi}_t - \sigma\left[\frac{\overline{Y}}{\overline{C}}(\hat{Y}_t - \hat{Y}_{t+1}) - \frac{\overline{G}}{\overline{C}}(\hat{G}_t - \hat{G}_{t+1})\right]\right), \tag{2}$$

$$\overline{p}_\delta \overline{b}\hat{b}_{t,\delta} + \overline{p}_\delta \overline{b}\hat{p}_{t,\delta} = -\overline{s}\hat{s}_t + (1+\delta\overline{p}_\delta)\overline{b}\left(\hat{b}_{t-1,\delta} - \hat{\pi}_t\right) + \delta\overline{p}_\delta\overline{b}\hat{p}_{t,\delta}, \tag{3}$$

$$\overline{p}_\delta \hat{p}_{t,\delta} = \beta(1+\overline{p}_\delta\delta)E_t\left[-\sigma\left(\frac{\overline{Y}}{\overline{C}}(\hat{Y}_{t+1} - \hat{Y}_t) - \frac{\overline{G}}{\overline{C}}(\hat{G}_{t+1} - \hat{G}_t)\right) - \hat{\pi}_{t+1} + \hat{\xi}_{t+1} - \hat{\xi}_t\right] + \beta\delta\overline{p}_\delta E_t\hat{p}_{t+1,\delta}\Bigg]. \tag{4}$$

(1) is the Phillips curve at the heart of our model. $\hat{\pi}_t$ represents inflation and $\hat{Y}_t$ is output. $\hat{G}_t$ denotes government spending in t. Parameters $\eta < 0$ and $\theta > 0$ govern the elasticity of substitution across the differentiated (monopolistically competitive) goods produced in the economy and the degree of price stickiness, respectively.[8]

σ denotes the inverse of the intertemporal elasticity of substitution and γ_h is the inverse of the Frisch elasticity of labour supply. These objects influence the slope of the Phillips curve, κ_1, through their influence on the response of hours/output to changes in marginal costs (wages). σ influences the magnitude of coefficient κ_2 due to the income effect on labour supply.

(2) is the standard log-linear IS-Euler equation which prices a short term nominal asset. $\hat{\xi}$ is a

[8] θ is the parameter that governs the magnitude of price adjustment costs in the standard quadratic cost function of Rotemberg (1982). When θ equals zero prices are fully flexible.

standard preference shock which affects the relative valuation of current vs. future utility by the household. A drop in $\hat{\xi}$ makes the household relatively patient, willing to substitute current for future consumption.[9]

(3) is the consolidated budget constraint. The left hand side (LHS) of this equation represents the value of debt issued in period t. The leading term, $\hat{b}_{t,\delta}$, denotes the quantity of real net government bonds issued in t and held by the private sector, whereas the second term, $\hat{p}_{t,\delta}$, represents the price of the newly issued debt in deviation from steady state. When $\delta > 0$ debt is issued in a perpetuity bond that pays decaying coupons.[10] When $\delta = 0$ only short bonds are issued by the government. On the right hand side (RHS) of (3) we have the government's surplus ($\overline{s}\hat{s}_t = \overline{\tau}\hat{\tau}_t - \overline{G}\hat{G}_t$, where τ denotes taxes) and the real value of debt that was issued in $t-1$ (remaining terms).

Finally, equation (4) defines the recursive formula that determines the price of debt in period t. Iterating forward this equation and substituting the equilibrium price in (3) and rearranging, it is possible to write the consolidated constraint as:

$$\begin{aligned}\frac{\beta\overline{b}}{1-\beta\delta}\hat{b}_{t,\delta} + \overline{b}\sum_{j=1}^{\infty}\beta^j\delta^{j-1}\left[E_t\left(-\sigma\Big(\frac{\overline{Y}}{\overline{C}}\hat{Y}_{t+j} - \frac{\overline{G}}{\overline{C}}\hat{G}_{t+j}\Big) - \sum_{l=1}^{j}\hat{\pi}_{t+l} + \hat{\xi}_{t+j}\right)\right] \\ = -\overline{s}\hat{S}_t - \overline{b}\sigma\left(\frac{\overline{Y}}{\overline{C}}\hat{Y}_t - \frac{\overline{G}}{\overline{C}}\hat{G}_t\right) + \overline{b}\hat{\xi}_t \\ + \frac{\overline{b}}{1-\beta\delta}(\hat{b}_{t-1,\delta} - \hat{\pi}_t) + \delta\overline{b}\sum_{j=1}^{\infty}\beta^j\delta^{j-1}E_t\left(-\sigma\Big(\frac{\overline{Y}}{\overline{C}}\hat{Y}_{t+j} - \frac{\overline{G}}{\overline{C}}\hat{G}_{t+j}\Big) - \sum_{l=1}^{j}\hat{\pi}_{t+l} + \hat{\xi}_{t+j}\right)\end{aligned} \tag{5}$$

where $\hat{S}_t$ denotes the surplus scaled by marginal utility in deviation from the steady state and

$$\overline{s}\hat{S}_t \equiv \left[-\overline{G}\left(\hat{G}_t\Big(1+\sigma\frac{\overline{G}}{\overline{C}}\Big) - \sigma\frac{\overline{Y}}{\overline{C}}\hat{Y}_t + \hat{\xi}_t\right) + \overline{\tau}\left(\hat{\tau}_t - \sigma\Big(\frac{\overline{Y}}{\overline{C}}\hat{Y}_t - \frac{\overline{G}}{\overline{C}}\hat{G}_t\Big) + \hat{\xi}_t\right)\right]$$

2.1.1 Tax rule

Fiscal policy is assumed to follow a standard rule that links the level of taxes to the face value of debt outstanding:

$$\hat{\tau}_t = \phi_{\tau,b}\hat{b}_{t-1,\delta} \tag{6}$$

[9] We focus on these two shocks as firstly fiscal shocks are clearly important in the context of a paper on fiscal inflation, and secondly, changes in real interest rates driven by discount factor shocks have been shown an important source of fluctuations for government budgets. For example, de Lannoy, Bhandari, Evans, Golosov, and Sargent (2022) argue that such shocks are an important source of risk that debt management should ward off to ensure the intertemporal solvency of US government debt. Here the focus is on the inflation consequences of these shocks. See also Bianchi and Melosi (2017, 2019) for in depth analyses of the monetary/fiscal interactions in the context of DSGE models with recessions being induced by discount factor shocks.

Finally, it should also be noted that the formulae that we will derive below will continue being relevant when we assume additional sources of shocks, e.g. shocks to government transfers, or cost-push shocks. We discuss this further below.

[10] In this case we assume that short term debt is in zero net supply.

Coefficient $\phi_{\tau,b}$ is a crucial object. Below we will separately consider cases where taxes adjust strongly to debt, so that debt becomes solvent through fiscal policy and cases where taxes do not strongly adjust and the solvency of debt will be (endogenously) ensured by monetary policy. In standard terminology, fiscal policy is passive in the former scenario and active in the latter (Leeper, 1991).

To derive our results analytically, we will assume that when fiscal policy is 'active', then $\phi_{\tau,b} = 0$. This assumption is also made (for example) by Bianchi and Ilut (2017) and Bianchi and Melosi (2017). For passive fiscal policy, we will set $\phi_{\tau,b} > \widetilde{\phi_\tau}$, where $\widetilde{\phi_\tau}$ is a threshold value defined below as a function of model parameters.[11]

2.2 Optimal Policy

Policy Objective. The central bank sets the inflation and output sequences to maximize the following objective function:

$$-\frac{1}{2}\sum_{t=0}^{\infty}\beta^t E_0\left\{\hat{\pi}_t^2 + \lambda_Y \widetilde{Y}_t^2\right\} \tag{7}$$

for $\lambda_Y \geq 0$ and where $\widetilde{Y}_t$ defines the output target.

A couple of lines are needed to motivate this choice. First, notice that (7) is a standard dual mandate objective function when the central bank assigns a positive weight λ_Y to output stabilization. It is well known (see e.g. Woodford (2003a), Ch. 6), that λ_Y can be derived endogenously as a function of the structural parameters of the model through a second order approximation of the household utility function. However, our derivations below will establish formulas applicable to any $\lambda_Y \geq 0$, including $\lambda_Y = 0$ (in which case the optimal policy focuses on inflation stabilization only). This broader approach will offer analytical convenience and help us to separately highlight the channels of optimal policy so that our (more complex) formulae under the quadratic welfare approximation are more easily interpretable.

Furthermore, notice that in (7) the central bank seeks to stabilize a measure of the output gap defined by $\widetilde{Y}_t$. We will consider two separate cases: $\widetilde{Y}_t = \hat{Y}_t$ (the objective is to stabilize output relative to its steady state level) and $\widetilde{Y}_t = \hat{Y}_t - \hat{Y}_t^n$ (stabilize output relative to the *natural level.*)[12] Assuming a steady state output target is quite common in the literature (e.g. Giannoni and Woodford, 2003 and Galí, 2015) and moreover it is plausible since in practice central banks are called to achieve such simple objectives (McKay and Wolf, 2022).[13] On the other hand, targeting the natural output is consistent with the micro-founded approach to optimal policy. We will show

[11]Our results do not depend on the exact specification of (6). Since taxes are lump sum, the precise timing of taxes is irrelevant. Therefore, we could (for instance) assume a more persistent tax response, adding a first order autoregressive component to (6). Only the distinction between active/passive fiscal policies is relevant for our results.

[12]We use the standard definition of the natural output, $\hat{Y}_t^n = \frac{\kappa_2}{\kappa_1}\hat{G}_t = \frac{\overline{G}}{\overline{Y}+\frac{\gamma_h}{\sigma}\overline{C}}\hat{G}_t$. See for example Woodford (2003a).

[13]Some of the well known results in the literature have been derived assuming steady state output targets and for this reason this is a case that is worth considering. For example, in the appendix we extend our results to the

that our formulae can easily accommodate both of these scenarios.[14]

Optimality. Maximization of (7) is subject to the dynamic equations (1) and (5) and given the tax rule (6).[15] As it is standard, we solve for optimal policies with a Lagrangian. Letting $\psi_{\pi,t}$ be the multiplier attached to the Phillips curve constraint, and $\psi_{gov,t}$ the analogous multiplier attached to the consolidated budget, the first order conditions for the optimum are given by:[16]

$$-\hat{\pi}_t + \Delta\psi_{\pi,t} + \frac{\bar{b}}{1-\beta\delta}\sum_{l=0}^{\infty}\delta^l \Delta\psi_{gov,t-l} = 0 \tag{8}$$

$$-\lambda_Y \widetilde{Y}_t - \psi_{\pi,t}\kappa_1 + \sigma\frac{\overline{Y}}{\overline{C}}\bar{b}\sum_{l=0}^{\infty}\delta^l \Delta\psi_{gov,t-l} + \sigma\frac{\overline{Y}}{\overline{C}}(\overline{G}-\overline{\tau})\psi_{gov,t} = 0 \tag{9}$$

$$\frac{\bar{b}}{1-\beta\delta}\Big(\psi_{gov,t} - E_t\psi_{gov,t+1}\Big) + \phi_{\tau,b}\overline{\tau}E_t\psi_{gov,t+1} = 0 \tag{10}$$

(8) is the FONC with respect to $\hat{\pi}_t$; (9), (10) are first order conditions with respect to $\widetilde{Y}_t$ and $\hat{b}_{t,\delta}$ respectively.

2.2.1 Interpreting the first order conditions

To inspect these optimality conditions, let us consider first $\psi_{gov,t} = 0$ for all t. Notice that this corresponds to the case where the consolidated budget constraint exerts no influence on optimal policy and it will later be shown an endogenous outcome of the model under passive fiscal policy. Under this assumption, combining equations (8) and (9) to substitute out the multiplier ψ_π, we get:

$$\hat{\pi}_t + \frac{\lambda_Y}{\kappa_1}\Delta\widetilde{Y}_t = 0 \tag{11}$$

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objective function used by Giannoni and Woodford, 2003,

$$-\frac{1}{2}\sum_{t=0}^{\infty}\beta^t E_0\Big\{\hat{\pi}_t^2 + \lambda_Y \hat{Y}_t^2 + \lambda_i \hat{i}_t^2\Big\}$$

when the policy objective targets steady state output, featuring also interest rate smoothing. We derive supplementary results for this scenario.

[14]The welfare based criterion requires to be derived under the assumption that the fiscal authority subsidizes factor inputs to eliminate the inefficiency emanating from imperfect competition. We have not (yet) introduced subsidies to the model, however, this will only be a simple extension of our derivations.

[15]Given optimal policies we can use (4) to solve for $\hat{p}_{t,\delta}$. In other words, we do not have to keep track of the bond price in the optimal policy program. Analogously, (2) is slack, and given a path of the optimal policy variables we can find the sequence $\{\hat{i}_t\}$ to satisfy this constraint.

[16]Note that we solve for optimal policy from a *timeless perspective*. We thus do not consider (for example) the usual initial allocation problem whereby the planner may inflate away public debt at the beginning of the horizon. As is well known, solving for optimal policies from a timeless perspective, requires to introduce additional constraints on the initial allocation (e.g Giannoni and Woodford, 2003), or the program can be stated in terms of an objective function that accounts explicitly for the lagged Lagrange multipliers at the beginning of the planning horizon (e.g Faraglia et al. (2016)). To avoid introducing explicitly all these elements we do not state the Lagrangian here.

Equation (11) designates the trade-off between inflation and output growth under the optimal policy. It is the same optimality condition as the one we would derive from the standard 3-equation New Keynesian model without the debt constraint (e.g. Woodford (2003a)). As is well known, this optimal plan is inertial (the FONC features the lag of the Lagrange multiplier $\psi_{\pi,t-1}$) due to the presence of forward expectations in the Phillips curve (Woodford (2003a); Giannoni and Woodford (2003)). Committing to a higher inflation rate in t changes $\widetilde{Y}_t$ by $\frac{1}{\kappa_1}$ and $\widetilde{Y}_{t-1}$ by $-\frac{\beta}{\kappa_1}$. Given discounting in the planner's objective, we obtain (11).

Consider now $\psi_{gov,t} \neq 0$. Rearranging the FONC we get :

$$\hat{\pi}_t + \frac{\lambda_Y}{\kappa_1}\Delta\widetilde{Y}_t = \overbrace{\frac{\overline{b}}{1-\beta\delta}\sum_{l=0}^{\infty}\delta^l\Delta\psi_{gov,t-l} + \sigma\frac{\overline{Y}}{\overline{C}}\overline{b}\sum_{l=0}^{\infty}\delta^l\left(\Delta\psi_{gov,t-l} - \Delta\psi_{gov,t-l-1}\right) + \sigma\frac{\overline{Y}}{\overline{C}}(\overline{G}-\overline{\tau})\Delta\psi_{gov,t}}^{:=\mathcal{D}_t} \tag{12}$$

Thus, the optimal policy will not equate $\hat{\pi}_t + \frac{\lambda_Y}{\kappa_1}\Delta\widetilde{Y}_t$ to 0, but to a weighted sum of the current and lagged values of the multiplier on the consolidated budget. To save notation we label this sum $\mathcal{D}_t$.

What do these terms capture? Shocks to government spending or to preferences will impact the value of debt and the deficit. When the debt constraint becomes important for the optimal allocation, inflation and output adjust to satisfy the constraint and ensure the solvency of debt. The terms on the RHS of (12) essentially capture changes in $\hat{\pi}_t$ and $\widetilde{Y}_t$ driven by shocks being filtered through the consolidated budget constraint.

To further clarify this, consider the intertemporal budget constraint that can be obtained by iterating forward on (5):

$$E_t\sum_{j=0}^{\infty}\beta^j\overline{s}\hat{S}_{t+j} = \frac{\overline{b}}{1-\beta\delta}\hat{b}_{t-1,\delta} + \overline{b}\sum_{j=0}^{\infty}\beta^j\delta^j E_t\left[-\sigma\left(\frac{\overline{Y}}{\overline{C}}\hat{Y}_{t+j} - \frac{\overline{G}}{\overline{C}}\hat{G}_{t+j}\right) - \sum_{l=0}^{j}\hat{\pi}_{t+l} + \hat{\xi}_{t+j}\right] \tag{13}$$

The intertemporal constraint (13) links the present value of the government's surplus (LHS) to the real value of debt outstanding in t (RHS). Note also that (13) is equivalent to (5) in terms of the optimal policy.[17] Consider the impact of a shock which lowers the LHS of (13) relative to the RHS. This may, for example, occur following a shock which increases spending. In response to such a shock, the constraint tightens and the value of the multiplier ψ_{gov} increases. To satisfy the constraint, the monetary authority may need to engineer a drop in the real payout of debt (the RHS of (13)) either through increasing inflation and/or adjusting output when $\sigma > 0$. Moreover, since optimal policy features full commitment, it is feasible to adjust both the current and future values of these variables. The lagged values of the multipliers in the date t optimality condition capture the promises made by the planner to change inflation and output in t in response to shocks which have occurred in the past.

[17] See for example Aiyagari et al. (2002).

The multiplier under active fiscal policy. Assume that fiscal policy is active and so $\phi_{\tau,b} = 0$. Then (10) becomes:

$$\psi_{gov,t} = E_t \psi_{gov,t+1}$$

The multiplier evolves according to a random walk.

This result is standard in models of optimal policy under *incomplete markets*. Aiyagari et al. (2002); Faraglia et al. (2016) examine models where a Ramsey planner optimally sets the path of distortionary taxes to finance spending shocks. In Schmitt-Grohé and Uribe (2004); Lustig et al. (2008); Faraglia et al. (2013); Leeper and Zhou (2021) the planner can simultaneously set distortionary taxes and inflation to satisfy the intertemporal budget constraint. The multiplier on the debt constraint measures the burden of the distortions and it follows a random walk, as the planner aims to evenly spread the costs over time.

The same principle applies here. Although we have assumed that taxes are lump sum and exogenous to the planning program, inflation is distortionary due to the Phillips curve and the objective (7). The random walk property implies that the optimal policy aims to evenly distribute the burden of the distortions caused by inflation across periods.

3 Optimal Monetary Policy and Interactions with Fiscal Policy

The previous paragraph showed that optimality in the Ramsey program takes the form of condition (12) expressing the inflation output trade-off as a function of the sum of current and lagged Lagrange multipliers. In the case where $\psi_{gov,t} = \mathcal{D}_t = 0$ (the debt constraint is not relevant for optimal policy) then (11), the standard optimality of the 3 equation New Keynesian model, applied.

When $\hat{\pi}_t + \frac{\lambda_Y}{\kappa_1}\Delta\widetilde{Y}_t = 0$ optimal monetary policy can be represented by an interest rate rule setting $\hat{i}_t$ as a function of macroeconomic variables. One can simply use the Euler equation to back out the appropriate policy rule satisfying $\hat{\pi}_t + \frac{\lambda_Y}{\kappa_1}\Delta\widetilde{Y}_t = 0$ for all t. (See for example Giannoni and Woodford (2003) and our derivations below).

However, when the debt constraint affects optimal policy, such that $\psi_{gov,t}, \mathcal{D}_t \neq 0$, finding a rule that relates interest rates to macroeconomic variables is not straightforward. Condition (12) (together with the Euler equation) suggests that the sequence of optimal interest rates depends on the current and lagged multipliers. Such a representation of interest rate policy is of limited practical relevance.

We show in this section that optimal monetary policy can be summarized by standard rules that express the nominal rate as a function of macroeconomic variables, both when $\mathcal{D}_t = 0$ and when $\mathcal{D}_t \neq 0$. The two scenarios will be relevant because the model admits two types of equilibria: When fiscal policy is passive and debt sustainability is ensured by taxes, we will have $\mathcal{D}_t = 0$.

When fiscal policy is active then we will have $\mathcal{D}_t \neq 0$.

Furthermore, in the case where $\mathcal{D}_t \neq 0$, the optimal interest rate rule will be a passive money rule (e.g. Leeper, 1991); and when fiscal policy is passive, the optimal policy will be an active rule. We will present our results as two separate equilibria, one in which the monetary/fiscal regime is active/passive and one in which it is passive/active. This framing follows Leeper (1991).

We provide analytical formulae for these optimal rules for several calibrations of the model. We start with a very simple setup: a Fisherian economy in which the central bank aims at stabilizing only inflation, $\lambda_Y = \sigma = 0$. Under this calibration the real interest rate is exogenous (driven by shocks $\hat{\xi}_t$) and, since income effects on labour supply are absent, we also have $\kappa_2 = 0$.[18] This setup offers considerable analytical convenience and it will help us build intuition for the optimal policy in more realistic calibrations of the model, which we will consider next. Analogous Fisherian models can be found in Aiyagari et al. (2002); Cochrane (2001); Davig and Leeper (2007); Cochrane (2018); Sims (2013); Faraglia et al. (2016); Bouakez, Oikonomou, and Priftis (2018); Bianchi and Melosi (2019).

3.1 A Fisherian model with an inflation stabilization objective.

3.1.1 Fiscal Policy

We begin by characterizing the equilibrium multiplier ψ_{gov} under passive and active fiscal policies. To simplify the algebra, let us also assume $\delta = 0$ and that shocks to preferences and spending are i.i.d.[19] Then, inflation will evolve according to:

$$\hat{\pi}_t = \bar{b}\Delta\psi_{gov,t} \tag{14}$$

Using this expression to substitute inflation out from the consolidated budget and using the Phillips curve to substitute output, together with the tax rule (6), we get:

$$\hat{b}_{t,\delta} - \bar{b}E_t\Delta\psi_{gov,t+1} + \frac{1}{\beta\bar{b}}\left((\bar{s}-\bar{b})\hat{\xi}_t - \overline{G}\hat{G}_t\right) = \frac{1}{\beta}\left[1 - \frac{\overline{\tau}\phi_{\tau,b}}{\bar{b}}\right]\hat{b}_{t-1,\delta} - \bar{b}\Delta\psi_{gov,t} \tag{15}$$

Equation (15) together with (10) form the system of equations that needs to be resolved to find the optimal allocation.

Active Fiscal Policy. Consider first $\phi_{\tau,b} = 0$. Since from (10) $\psi_{gov,t}$ is a random walk we can write (15) as:

$$\hat{b}_{t,\delta} + \frac{1}{\beta}\widetilde{\chi}_t = \frac{1}{\beta}\hat{b}_{t-1,\delta} - \bar{b}\Delta\psi_{gov,t}$$

[18] Thus, a spending shock under the Fisherian setup does not impact the Euler equation or the Phillips curve, it only appears in the government budget constraint. It is essentially equivalent to pure (lump sum) transfer shock.

[19] At the end of this subsection we generalize the results to $\delta \geq 0$. Moreover, we will derive most of our analytical results in Sections 3 and 4 assuming i.i.d shocks for simplicity, but in Section 5 we will show that it is straightforward to extend our analytical formulae to the case of persistent shocks.

where $\widetilde{\chi}_t \equiv \frac{1}{\overline{b}}\left((\overline{s}-\overline{b})\hat{\xi}_t - \overline{G}\hat{G}_t\right)$. This difference equation has an unstable root, $\frac{1}{\beta}$. It can be solved forward to give:

$$\hat{b}_{t-1,\delta} = \sum_{j\geq 0} \beta^j E_t\left[\widetilde{\chi}_{t+j} + \overline{b}\Delta\psi_{gov,t+j}\right] = \widetilde{\chi}_t + \overline{b}\Delta\psi_{gov,t} \tag{16}$$

where the second equality makes use of the assumption that shocks are i.i.d and the random walk property of the multiplier. Clearly, (16) can be consistent with the random walk if and only if $\hat{b}_{t,\delta} = 0$ for all t. In equilibrium we then have:

$$\Delta\psi_{gov,t} = -\frac{1}{\overline{b}}\widetilde{\chi}_t \qquad \text{and} \qquad \hat{\pi}_t = -\widetilde{\chi}_t.$$

Passive Fiscal Policy. Now assume $\phi_{\tau,b}$ is positive and its value exceeds some threshold $\widetilde{\phi}_\tau > 0$. $\psi_{gov,t}$ evolves according to:

$$\psi_{gov,t} = \left(1 - \frac{\overline{\tau}\phi_{\tau,b}}{\overline{b}}\right)E_t\psi_{gov,t+1}$$

and solving forward we get $\psi_{gov,t} = 0$ for all t[20].

Inflation will be zero at all times. Using these results and the consolidated budget (15) we can show that debt is a stable process if the following condition is met:

$$\phi_{\tau,b} > (1-\beta)\frac{\overline{b}}{\overline{\tau}} \equiv \widetilde{\phi}_\tau \tag{17}$$

The model admits two types of equilibria: In one case, when fiscal policy is active, we have that $\psi_{gov,t} \neq 0$ and inflation responds to shocks that hit the consolidated budget constraint. In the second scenario, fiscal policy adjusts taxes to debt so that condition (17) holds; then, taxes ensure the intertemporal solvency of debt and $\psi_{gov,t} = 0$. Since the debt constraint is slack, it is not necessary to use inflation to adjust the real value of debt.

The above derivations can be easily extended to the case where $\delta > 0$. The algebra is a bit cumbersome and so we simply highlight the main result with the following Proposition:

Proposition 1. *If $\phi_{\tau,b}$ satisfies*

$$\phi_{\tau,b} > \frac{(1-\beta)}{(1-\beta\delta)}\frac{\overline{b}}{\overline{\tau}} \equiv \widetilde{\phi}_\tau$$

then $\psi_{gov,t} = 0$. If $\phi_{\tau,b} = 0$ then $\psi_{gov,t} \neq 0$.

[20]We will focus on cases where $\left(1 - \frac{\overline{\tau}\phi_{\tau,b}}{\overline{b}}\right) > 0$. Scenarios in which taxes respond too strongly to debt and violate this condition are implausible since they imply that debt has a negative root and oscillates through time.

Finally, note that Proposition 1 does not only hold under the calibration $\lambda_Y = \sigma = 0$ considered in this paragraph. It will continue to hold when these parameters are considered positive in subsequent sections.

3.1.2 Optimal Monetary Policy Rules

We now turn to the optimal monetary policy rules that can implement the Ramsey outcome. We let $\delta \geq 0$. We will verify that the following simple inflation targeting rule can implement the allocation under the optimal policy:

$$\hat{i}_t = \hat{\xi}_t + \phi_\pi \hat{\pi}_t \tag{18}$$

Consider first the case where fiscal policy is passive, $\phi_{\tau,b} \geq \widetilde{\phi}_\tau$. As we showed previously, in this equilibrium, optimal policy sets $\hat{\pi}_t = 0$. From the Phillips curve we find $\hat{Y}_t = 0$. It is easy to show that the interest rate rule (18) can implement this outcome. Combining (18) with the Euler equation we get the following difference equation in inflation:

$$\hat{\pi}_t \phi_\pi = E_t \hat{\pi}_{t+1}$$

Standard results yield $\hat{\pi}_t = 0$ if and only if $\phi_\pi > 1$.

Consider now the case of active fiscal policy. In equilibrium, inflation and the interest rate are given by

$$\hat{\pi}_t = \frac{\bar{b}}{1-\beta\delta}\sum_{l=0}^{\infty}\delta^l \Delta\psi_{gov,t-l} \qquad \hat{i}_t = \hat{\xi}_t + \frac{\bar{b}\delta}{1-\beta\delta}\sum_{l=0}^{\infty}\delta^l \Delta\psi_{gov,t-l}$$

A rule of the form (18) can implement this outcome if the following holds:

$$\phi_\pi \frac{\bar{b}}{1-\beta\delta}\sum_{l=0}^{\infty}\delta^l \Delta\psi_{gov,t-l} = \delta \frac{\bar{b}}{1-\beta\delta}\sum_{l=0}^{\infty}\delta^l \Delta\psi_{gov,t-l}$$

Hence, $\phi_\pi = \delta \in [0, 1]$.

We highlight these results with the following Proposition:

Proposition 2. *Assume $\lambda_Y = \sigma = 0$. The optimal policy is a rule of the form (18). In the case where fiscal policy is passive, the optimal monetary policy sets $\phi_\pi > 1$. In the case where fiscal policy is active, the optimal inflation coefficient is $\phi_\pi = \delta \in [0, 1]$. Optimal monetary policy is then 'passive'.*

Several comments are in order. First, note that Proposition 2 clearly reveals the analogy between our framework and the equilibria with active/passive monetary/fiscal policies defined in Leeper (1991). Whereas in Leeper (1991), and in the rest of the literature that used this influential framework, monetary policy is modeled using ad hoc interest rate rules, here interest rates are optimal in the sense that commitment to rule (18) can implement the optimal allocation when the parameter values are as stated in Proposition 2. Moreover, simple inflation targeting rules as in (18) are commonly used in the literature. The analytical derivations we provide here show the conditions under which (18) is optimal in terms of the objective of the planner and the fundamental model parameters.

Interpretation. According to Proposition 2, in the case of passive fiscal policy, the optimal allocation can be implemented with any rule that satisfies the Taylor principle ($\phi_\pi > 1$). In contrast, in the active fiscal scenario, optimal monetary policy constrains the parameter ϕ_π to equal δ, the rate at which the coupon payments on debt decay. The average debt maturity ($\frac{1}{1-\delta}$) becomes a key variable determining the reaction to inflation. A higher maturity of debt entails a stronger reaction.

Why is this the desired response to inflation? To illuminate the forces at work, let us assume that the economy is hit by a shock in t assuming also (for simplicity) that no shock hits the economy thereafter. When monetary policy follows (18) and $\phi_\pi < 1$, then inflation becomes a backward looking process. We can therefore solve the path of inflation as $\hat{\pi}_{t+j} = \phi_\pi^j \hat{\pi}_t,\ j = 1, 2, ...$ up to the initial condition $\hat{\pi}_t$. In turn, $\hat{\pi}_t$ can be found to satisfy the intertemporal budget constraint given the shock. Inflation will jump initially, and revert to steady state at rate ϕ_π.

Now consider the optimal policy plan. The path of inflation is: $\hat{\pi}_{t+j} = \frac{\bar{b}}{1-\beta\delta}\delta^j \Delta\psi_{gov,t}$. The shock will induce a change in the value of the multiplier in t and thereafter ψ_{gov} will remain constant; under the optimal policy, inflation will jump in t (to again satisfy the intertemporal constraint) and converge back towards the steady state at rate δ.

It is quite evident that setting $\phi_\pi = \delta$ produces the exact same responses in the two cases (the initial jumps would also be equal to satisfy effectively the same intertemporal budget). Intuitively, if ϕ_π exceeded δ then inflation persistence would be too high under the interest rate rule and inflation would be far from 0 even when the coupon payments are close to 0. High future inflation would however not contribute significantly towards making debt sustainable and a policy setting $\phi_\pi > \delta$ couldn't be optimal.

Analogously, if ϕ_π is lower than δ, then inflation will be more frontloaded than under the optimal policy. To satisfy the intertemporal budget, inflation will then need to be higher in period t, since it decays faster towards 0. But this path doesn't fully exploit the maturity structure of debt; it is possible to reduce inflation in t by targeting a smoother and more persistent path of inflation, relying on a bigger adjustment of the real payout of future coupon payments to the shock. This will enable to spread the burden of fiscal inflation more efficiently and reduce the losses of the central bank.

Finally, note that under both active and passive fiscal policies, the optimal interest rate rule tracks the real interest rate which, in this simple Fisherian model, is equal to $\hat{\xi}_t$. This feature of optimal monetary policy enables to eliminate the demand shock from the Euler equation. When fiscal policy is passive, it leads to the zero inflation/divine coincidence outcome we previously showed. In the case of active fiscal policy inflation is not zero, since the preference shock is also filtered through the government budget constraint. Real interest rate tracking will be a feature of all versions of the model we will consider subsequently.[21]

3.2 Alternative Settings

The simple model of the previous subsection highlighted an important property of optimal monetary policy in the active fiscal regime: inflation persistence governed by the coefficient ϕ_π, is desirable with long term debt. The condition $\phi_\pi = \delta$ sets the persistence of inflation equal to the persistence of the payment profile of the long bond.

This mechanism continues being relevant in the case where λ_Y, σ are not constrained to equal zero, to which we will now turn. However, introducing an output smoothing objective ($\lambda_Y > 0$) or setting $\sigma > 0$, to make real interest rates endogenously vary with aggregate output (as in the canonical New Keynesian model) introduces additional channels that impact optimal inflation and the interest rates. When $\lambda_Y > 0$ making inflation an i.i.d process is not warranted even when debt is short term, since temporary jumps in inflation result in excess output volatility, increasing the losses of the central bank. As we shall see, output smoothing leads to optimal inflation persistence. Furthermore, whilst in a Fisherian model output fluctuations are irrelevant for the debt solvency, when $\sigma > 0$, changes in output growth result in fluctuations in bond prices that affect the solvency of the intertemporal budget. This channel will also impact the optimal monetary policy.

We now derive the optimal interest rate rules for the cases of output smoothing and endogenous real interest rates, extending our previous analytical results. In order to highlight transparently what each of these features brings to optimal policy, we study them in isolation, starting from $\sigma = 0$, $\lambda_Y > 0$ (Fisherian model with output smoothing) and then considering $\sigma > 0$ $\lambda_Y = 0$ (canonical New Keynesian model with a pure inflation smoothing objective).

In this subsection we will show our formulae for the interest rate rules. Then, in the first part

[21] **Multiplicity:** It should be noted that the optimal interest rate rule in this simplistic version of the model is not unique in the sense that other specifications of the interest rate reaction function can deliver the same outcome as (18). For example, consider a rule of the form

$$\hat{i}_t = \hat{\xi}_t + \phi_\pi \hat{\pi}_t + \phi_Y \hat{Y}_t \tag{19}$$

where the nominal rate responds to both output and inflation. We can show that (19) can implement the optimal policy outcome under active fiscal policy provided that $\phi_\pi + \frac{\phi_Y}{\kappa_1}(1-\beta\delta) = \delta$. When this condition holds, then inflation will again decay at rate δ following a shock to the economy (the analogous condition to get the passive fiscal policy equilibrium under rule (19) is the familiar $\phi_\pi + \frac{\phi_Y}{\kappa_1}(1-\beta) > 1$).

Though (19) is an optimal interest rate rule in the Fisherian model of this paragraph, we prefer to work with the simple rule in (18) for two main reasons: First, because it is less appealing to assume that monetary policy targets output in a model where output fluctuations do not contribute towards making debt sustainable (i.e. when $\sigma = 0$ see below) and do not enter into the central bank's objective function; second, rule (18) leads to a more transparent policy recommendation, setting the inflation coefficient equal to δ, relative to (19) where the analogous condition involves estimating additional parameters β and κ_1.

Finally, note that multiplicity may be specific to the assumptions in this model (where output is a slack variable). In the less restrictive versions of the model that we will consider next, it will not be easy to test explicitly whether multiplicity arises under active fiscal policy. The optimal policy rules that we will derive will be simple, transparent and readily interpretable functions of inflation.

of Section 4 we will use our analytical results to clarify the forces behind optimal policy through studying the dynamic adjustment of the economy to shocks. Finally, at the end Section 4 we will be in place to show the optimal interest rate policy when $\lambda_Y, \sigma > 0$, which will basically combine the margins of the two separate cases we consider here.

3.2.1 Case 1: $\lambda_Y > 0$, $\sigma = 0$.

Assume that the planner's objective is to smooth both inflation and the output gap, focusing on the case where $\lambda_Y > 0$ but $\sigma = 0$. Natural output is constant, and thus $\widetilde{Y}_t = \hat{Y}_t$. Combining (8) and (9) we can show that inflation satisfies the following condition:

$$-\hat{\pi}_t - \frac{\lambda_Y}{\kappa_1}\Delta\hat{Y}_t + \frac{\overline{b}}{1-\beta\delta}\sum_{k=0}^{\infty}\delta^k\Delta\psi_{gov,t-l} = 0 \tag{20}$$

Moreover, using the Phillips curve to substitute out aggregate output and after rearranging we obtain the following :

$$E_t\hat{\pi}_{t+1} - (1 + \frac{1}{\beta} + \widetilde{\kappa})\hat{\pi}_t + \frac{1}{\beta}\hat{\pi}_{t-1} = -\zeta_t \tag{21}$$

where $\widetilde{\kappa} = \frac{\kappa_1^2}{\lambda_Y\beta}$ and $\zeta_t \neq 0$ (=0) when fiscal policy is active (passive).[22] Inflation thus follows a second order difference equation.

Consider the equilibrium where fiscal policy is passive. Equation (21) can be written as:

$$E_t\hat{\pi}_{t+1} = (\widetilde{\lambda}_1 + \widetilde{\lambda}_2)\hat{\pi}_t - \widetilde{\lambda}_1\widetilde{\lambda}_2\hat{\pi}_{t-1}$$

where $\widetilde{\lambda}_{1,2}$ are the roots of the characteristic polynomial in (21):

$$\widetilde{\lambda}_{1,2} = \frac{1}{2}\left((1 + \frac{1}{\beta} + \widetilde{\kappa}) \pm \sqrt{(1 + \frac{1}{\beta} + \widetilde{\kappa})^2 - \frac{4}{\beta}}\right)$$

Since one of the roots is unstable (say $\widetilde{\lambda}_2$) and the other root is stable, the interest rate rule that implements the optimal allocation

$$\hat{i}_t = \hat{\xi}_t + (\widetilde{\lambda}_1 + \widetilde{\lambda}_2)\hat{\pi}_t - \widetilde{\lambda}_1\widetilde{\lambda}_2\hat{\pi}_{t-1} \tag{22}$$

defines an active monetary policy.

Turning to the active fiscal scenario, we can show that the optimal interest rate rule is:

$$\hat{i}_t = \hat{\xi}_t + (\widetilde{\lambda}_1 + \delta)\hat{\pi}_t - \widetilde{\lambda}_1\delta\hat{\pi}_{t-1} - \delta\frac{\widetilde{o}}{\widetilde{\lambda}_2 - 1}\Delta\psi_{gov,t} \tag{23}$$

[22] For brevity we define the forcing term ζ_t in the appendix.

where $\widetilde{o} > 0$ is defined in the appendix for brevity.

Proposition 3 summarizes these results:

Proposition 3. *Assume $\lambda_Y > 0$ and $\sigma = 0$. The Ramsey optimal interest rate rule is given by (22) when fiscal policy is passive and $\psi_{gov,t} = 0$. It is given by (23) when fiscal policy is active and $\psi_{gov,t} \neq 0$.*

Proof: See appendix.

The result in Proposition 3 and in particular the active fiscal scenario deserves a brief comment. Note first that the systematic response of the nominal rate to inflation in (23) indeed defines a passive monetary policy. Since $\widetilde{\lambda}_1 + \delta - \widetilde{\lambda}_1\delta < 1$ an x per cent rise in inflation will lead to a less than x per cent increase in the nominal rate. Thus, this case also conforms with the principle that when fiscal policy is active, optimal monetary policy can be expressed as a passive money rule.

Moreover, as we will explain in detail in Section 4, the fact that the rule is inertial (the nominal rate responds to both the current and lagged inflation rates) implies that the optimal path of inflation is more persistent in this model where the planner desires to smooth output fluctuations. To show this in the simplest case possible, let $\delta = 0$. Then (23) becomes $\hat{i}_t = \hat{\xi}_t + \widetilde{\lambda}_1\hat{\pi}_t$ and so, following any shock to the consolidated budget constraint, inflation will jump and revert to 0 at rate $\widetilde{\lambda}_1$. In contrast, as we previously saw, setting $\lambda_Y = 0$ and with short term debt, the optimal inflation was an i.i.d process.

Furthermore, notice that in the case where $\delta > 0$ optimal policy, besides responding to inflation and the real rate, also responds to the term $-\delta\frac{\widetilde{o}}{\widetilde{\lambda}_2-1}\Delta\psi_{gov,t}$, a function of the Lagrange multiplier. We did not (yet) derive a rule expressing the nominal rate as a function of macroeconomic variables only.

This term is a *stochastic intercept*. It introduces a temporary innovation to the interest rate rule whenever a shock hits the economy leading to $\Delta\psi_{gov,t} \neq 0$. For example, assume that spending increases in t and so $\Delta\psi_{gov,t} > 0$. Then, according to (23), optimal policy will keep the nominal rate slightly lower in t than the value implied by the systematic component of the interest rate rule. This effect concerns only period t since $\Delta\psi_{gov,t}$ is an i.i.d. variable.

It turns out that, in equilibrium, $\Delta\psi_{gov,t}$ can be written as a function of the two shocks (see Section 4 for the analytical expression). Thus, we could replace the stochastic intercept with the shocks, or even some linear combination of macroeconomic variables can substitute out $\Delta\psi_{gov,t}$. We choose not to expand on this here. In Section 4, when we will evaluate the model, we will explore in detail the stochastic intercept term and investigate its significance for optimal policy. We will then show that over plausible calibrations of the model this term exerts only a small influence on optimal policy.

3.2.2 Case 2: $\sigma > 0$, $\lambda_Y = 0$

Consider now the case where $\sigma > 0$. For illustrative purposes and to simplify the formulae we will derive in this subsection, let us first assume that only preference shocks can hit the economy, setting $\overline{G} = 0$.

Combining the FONC of the planner's program we get:

$$\hat{\pi}_t = \frac{\bar{b}}{1-\beta\delta}\sum_{l=0}^{\infty}\delta^l \Delta\psi_{gov,t-l} + \frac{\sigma}{\kappa_1}\bar{b}\sum_{l=0}^{\infty}\delta^l\Big(\Delta\psi_{gov,t-l} - \Delta\psi_{gov,t-l-1}\Big) - \frac{\sigma\overline{\tau}}{\kappa_1}\Delta\psi_{gov,t} \tag{24}$$

which expresses inflation as a function of the multipliers.

It is evident that optimal inflation is zero in the equilibrium where $\psi_{gov,t} = 0$ for all t. A simple inflation targeting rule as in (18) can implement this outcome insofar as $\phi_\pi > 1$.

Consider now the case where $\psi_{gov,t} \neq 0$. Using the Phillips curve to eliminate aggregate output we can write the Euler equation as:

$$\hat{i}_t = \frac{\sigma}{\kappa_1}E_t\Big(\hat{\pi}_{t+1} - \beta\hat{\pi}_{t+2} - \hat{\pi}_t + \beta\hat{\pi}_{t+1}\Big) + E_t\hat{\pi}_{t+1} + \hat{\xi}_t$$

and using (24) it is simple to show that:

$$E_t\hat{\pi}_{t+1} = \delta\hat{\pi}_t - \frac{\sigma}{\kappa_1}(\bar{b} - \delta\overline{\tau})\Delta\psi_{gov,t} \quad \text{and} \quad E_t\hat{\pi}_{t+2} = \delta E_t\hat{\pi}_{t+1}$$

With appropriate substitutions (see appendix) we obtain the following expression for the interest rate rule:

$$\hat{i}_t = \hat{\xi}_t + \Big(\delta + \frac{\sigma}{\kappa_1}(1-\delta)(\delta\beta - 1)\Big)\hat{\pi}_t - \widetilde{\omega}\Delta\psi_{gov,t} \tag{25}$$

where $\widetilde{\omega} > 0$ is derived in the appendix and $\Big(\delta + \frac{\sigma}{\kappa_1}(1-\delta)(\delta\beta - 1)\Big) < 1$ so that (25) defines a passive monetary policy.

The nominal rate thus again follows a simple rule which tracks the real rate $\hat{\xi}_t$, and responds to inflation. The optimal coefficient ϕ_π is a function of debt maturity, but now also parameters κ_1 (the slope of the Phillips curve) and σ (the inverse of the IES) influence the value of the coefficient. We will later explain in detail how these parameters are relevant, but basically they pertain to the effects that changes in output have on real bond prices and on the intertemporal debt constraint, when $\sigma > 0$. These effects are internalized by optimal policy through the term $\frac{\sigma}{\kappa_1}(1-\delta)(\delta\beta - 1)$.

Notice further that, as in the previous paragraph, the optimal rule features a stochastic intercept, the final term on the RHS of (25). The loading on this term is negative and therefore when $\Delta\psi_{gov,t} > 0$ optimal policy will keep the nominal interest rate lower in t than what is implied by the systematic component of the rule. We again leave it to Section 4 to explain this term.

We next state the optimal policy rule in the case where fiscal shocks can hit the economy and

$\overline{G} > 0$.

Proposition 4. *Assume $\sigma > 0$, $\lambda_Y = 0$ and $\overline{G} > 0$. The optimal monetary policy rule under active fiscal policy is:*

$$\hat{i}_t = \widetilde{r}_t + \left(\delta + \frac{\overline{Y}}{\overline{C}}\frac{\sigma}{\kappa_1}(1-\delta)(\delta\beta - 1)\right)\hat{\pi}_t - \underbrace{f\Delta\psi_{gov,t}}_{\text{Stochastic Intercept}} \tag{26}$$

where $\widetilde{r}_t \equiv \hat{\xi}_t + \frac{\gamma_h \overline{G}}{\overline{Y} + \frac{\gamma_h}{\sigma}\overline{C}}\hat{G}_t \equiv \widetilde{r}^n_t$

Proof: See appendix.[23]

For brevity, we give the formula for coefficient f (which depends on the output gap measure $\widetilde{Y}$) in the appendix. (26) is similar to (25), the main difference is that the real interest rate $\widetilde{r}_t$ that needs to be tracked is now also a function of the spending shock. According to Proposition 4, $\widetilde{r}_t$ is equal to the *natural rate of interest,* $\widetilde{r}^n_t$ (see Woodford, 2003a).

4 Going deeper into optimal policy

We now build on the analytical results of the previous section to go deeper into the mechanics of optimal policy, unraveling the various channels under the different scenarios we considered. To do so, we complement our derivations with new formulae characterizing the transmission of the shocks to the macroeconomy, focusing in particular on the dynamic response of inflation.

Throughout this section we focus on the case of active fiscal policy. The passive fiscal model, besides having been very well investigated in the literature, will result in zero inflation. Thus, it is simple for the reader to compare the properties of the equilibrium under passive and active fiscal policies.

The key properties of optimal policy that we focus on in this section are the following: First, an explicit output smoothing objective is a source of inflation persistence in the model. Even when the maturity of debt is short, the planner still finds optimal to let inflation deviate from target for a while, following a shock to the economy. With long term debt however, the incentive to make inflation deviations persistent in order to smooth output fluctuations lines up with the incentive to reduce real debt after a shock. Second, as discussed previously, in the canonical New Keynesian model, inflation fluctuations do not only impinge a *direct impact* on the real payout of government debt (by changing the price level they change the real value of a promised stream of payments), but also *indirect effects,* by altering the path of output they affect the real bond prices and the intertemporal debt constraint. We examine closely this margin using our analytical

[23] The rule in the passive fiscal scenario is of the form:

$$\hat{i}_t = \widetilde{r}_t + \phi_\pi \hat{\pi}_t$$

where again $\phi_\pi > 1$. Since this is easy to verify, we left it outside Proposition 4, but state it here for completeness.

formulae and show that indirect effects become weaker as the maturity of debt increases. With a long enough maturity of debt, optimal monetary policy needs to be only concerned about the direct effects of inflation.

Bringing these findings together, we derive a key conclusion of this paper: for an average maturity of debt calibrated to the US data, the optimal monetary policy in the canonical model with both inflation and output smoothing objectives can be approximated by a simple inflation targeting rule where the optimal inflation coefficient is δ.

4.1 Impulse Responses

4.1.1 Output smoothing ($\lambda_Y \geq 0$)

Consider first the Fisherian model with the dual mandate objective, $\lambda_Y \geq 0$. We characterize analytically the impulse response functions to shocks $\{\hat{G}_t, \hat{\xi}_t\}$ at date t. In the appendix we prove the following dynamic response of inflation to the shocks:

$$\hat{\pi}_t = \frac{\widetilde{\lambda}_2}{\widetilde{\lambda}_2 - 1}\widetilde{o}\Delta\psi_{gov,t}$$
$$\hat{\pi}_{t+j} = \widetilde{o}\frac{(\delta^{j+1} - \widetilde{\lambda}_1^{j+1})}{\delta - \widetilde{\lambda}_1}\Delta\psi_{gov,t} + \widetilde{o}\frac{\widetilde{\lambda}_1^j}{\widetilde{\lambda}_2 - 1}\Delta\psi_{gov,t}, \qquad j \geq 1 \tag{27}$$

where $0 < \widetilde{\lambda}_1 < 1$,$\widetilde{\lambda}_2 > 1$ and $\widetilde{o}$ were defined previously.[24] Moreover,

$$\Delta\psi_{gov,t} = \widetilde{\psi}(\overline{G}\hat{G}_t + (\bar{b} - \bar{s})\hat{\xi}_t) \tag{28}$$

where $\widetilde{\psi} > 0$.

A positive demand (spending) shock. According to (28) a positive spending shock in t will yield $\Delta\psi_{gov,t} > 0$. This is not surprising; the increase in spending reduces the present value of government revenues and the consolidated budget constraint tightens. The planner needs to increase inflation to make debt solvent.

(27) shows that this will have a persistent effect on inflation. Persistence derives from two sources: First, from the maturity of debt, with a higher coefficient δ implying more persistence; and second, it derives from the objective to smooth output through time (coefficient $\widetilde{\lambda}_1$ is increasing in λ_Y).

The rationale behind the first channel was stated previously. Assume again that $\lambda_Y = 0$. We can then show that $\widetilde{\lambda}_1 = 0$, $\widetilde{\lambda}_2 \to \infty$ and $\widetilde{o} = \frac{\bar{b}}{1-\beta\delta}$. Then, the above formulae tell us that inflation will display persistence equal to δ.

In addition to this channel, output stabilization also contributes to persistence. Assume that

[24] This formula assumes the most plausible scenario $\widetilde{\lambda}_1 \neq \delta$.

$\delta = 0$ and so debt is only short term. We then have:

$$\hat{\pi}_{t+j} = \widetilde{o}\frac{\widetilde{\lambda}_2}{\widetilde{\lambda}_2 - 1}\widetilde{\lambda}_1^j \Delta\psi_{gov,t}, \qquad j \geq 0$$

and inflation decays at rate $\widetilde{\lambda}_1$. We can further show that $\frac{d\widetilde{\lambda}_1}{d\lambda_Y} > 0$ and in the limit, when $\lambda_Y \to \infty$, $\widetilde{\lambda}_1 \to 1$. Even in the presence of short term debt, the deviations of inflation from target can be very persistent, depending on the desire to smooth output fluctuations. The intuition for this property is simple: Making inflation respond only in period t (the optimal policy under $\delta = 0$ and no output smoothing) will entail a large contemporaneous response of output. When a smooth path of output is desired, inflation needs to adjust gradually to the shock.

Let us now investigate how the output smoothing objective affects the *magnitude of the response* of inflation to the shock. A stark result is that when $\delta = 0$, coefficient λ_Y has no bearing on the level of inflation in t.[25] In contrast, when $\delta > 0$, then a stronger incentive to smooth output implies a smaller initial response of inflation to the shock. To understand these properties notice first that, in this Fisherian model, the path of output does not matter at all for fiscal sustainability. Since real bond prices and the intertemporal surplus are not functions of output, it is only inflation that can adjust to make debt sustainable. When $\delta = 0$ all of the burden of the adjustment falls on period t inflation. Thus, more inflation persistence when $\lambda_Y > 0$ will not change the level of inflation in t required to satisfy the intertemporal constraint. In contrast, with long term debt, a more persistent response of inflation will imply a larger fall in the real payout of debt following the spending shock, and the increase in inflation in t required to satisfy the intertemporal budget will be smaller.

It may thus seem that when the maturity of debt is long an explicit output smoothing objective will complement the inflation smoothing motive of the planner. However, this is really not so. Once again, output has nothing to do with satisfying the intertemporal debt constraint, and inflation persistence driven by output stabilization may not be desirable in terms of the inflation smoothing objective.

The term $\widetilde{o}\frac{(\delta^{j+1} - \widetilde{\lambda}_1^{j+1})}{\delta - \widetilde{\lambda}_1}\Delta\psi_{gov,t}$ in (27) reveals this property. It is essentially a correction for the persistence of the inflation process, relative to the second term in (27), which represents persistence driven purely by output smoothing. Whenever $\delta < \widetilde{\lambda}_1$ then persistence deriving from smoothing output is too high and the first term will frontload inflation to match more closely the payment profile of debt. In contrast, if $\delta > \widetilde{\lambda}_1$ then the first term in (27) will add persistence, the planner targets a flatter path of inflation than what is dictated by the objective to smooth output.

[25] To see this, combine (27) and (28), evaluated at $\delta = 0$. We get

$$\hat{\pi}_t = \frac{\overline{G}\hat{G}_t + (\overline{b} - \overline{s})\hat{\xi}_t}{\overline{b}}$$

which is independent of $\widetilde{\lambda}_1$ and $\widetilde{\lambda}_2$ and hence also independent of λ_Y.

Graphical impulse response analysis. We complement the above formulae with plots showing the impulse responses of macroeconomic variables to the spending shock under different calibrations of λ_Y and δ. Table 1 reports the assumed numerical values of the model's parameters and the notes of the table briefly explain our calibration targets.

Table 1: **Calibration**

Parameter	Value	Label
β	0.995	Discount factor
λ_Y	{0, 0.12, 0.5}	Loss function - weight on output
θ	17.5	Price Stickiness
η	-6.88	Elasticity of Demand
σ	{0, 1}	Inverse of IES
γ_h	1	Inverse of Frisch Elasticity
$\bar{b}$	0.132	SS bond quantity
$\bar{\tau}$	0.11	SS tax Rate
$\overline{Y}$	1	SS output
$\overline{G}$	0.1	SS gov. spending

Notes: The table reports the values of model parameters assumed in the numerical examples in Section 4 of the paper. β denotes the discount factor chosen to target a steady state (annual) real interest rate of 2 percent. Parameter η is calibrated to target markups of 17 percent in steady state. θ governs the price adjustment cost and is calibrated as in Schmitt-Grohé and Uribe (2004). The steady state level of debt is assumed equal to 60 percent of GDP (at annual horizon), and the level of public spending is 10 percent of aggregate output which is normalized to unity in steady state. The value of the tax rate is such that the steady state government budget constraint holds. We further assumed that inflation at steady state is 0.

These parameter values are held constant throughout the numerical experiments of Section 4. Parameters λ_Y and σ (the relative weights on output and interest rates stabilization and the inverse of the IES respectively) vary across experiments. We set $\sigma = 1$ as our baseline in the canonical New Keynesian model. Moreover, when we assume an ad hoc loss function we let $\lambda_Y \in \{0, 0.5\}$. For the microfounded loss function, using the formula $\lambda_Y = \frac{\sigma\frac{\overline{Y}}{\overline{C}}+\gamma_h}{\theta}$, we get $\lambda_Y \approx 0.12$.

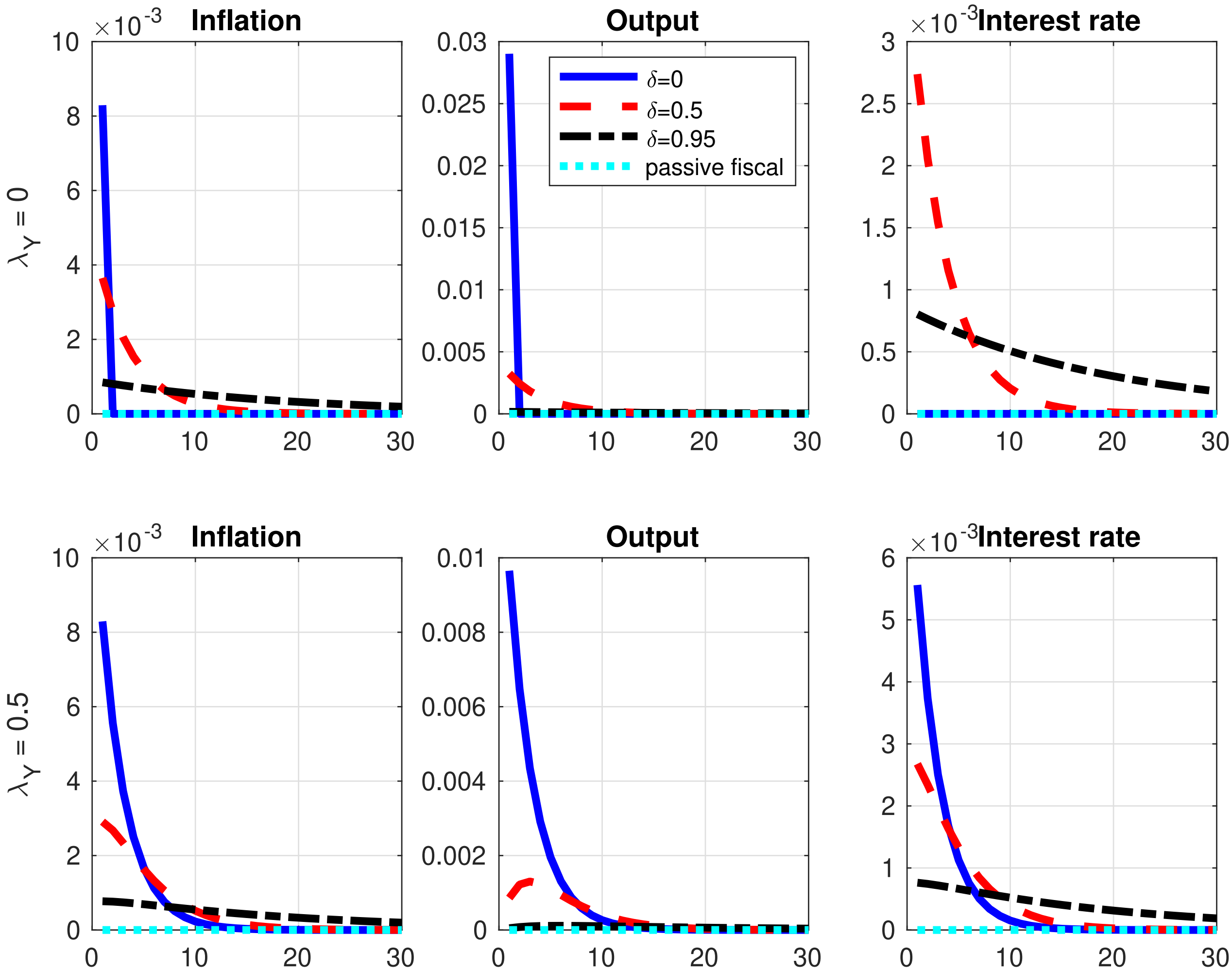
The top panel of Figure 1 shows the responses to the spending shock when $\lambda_Y = 0$. From left to right we plot the response of inflation, output and the nominal interest rate. The blue, red and black lines represent the case of active fiscal policy under $\delta = 0, 0.5, 0.95$ respectively.[26] The bottom panel of the Figure plots the same responses when $\lambda_Y = 0.5$.

The graphs are qualitatively consistent with our analytical results. Consider the behavior of the interest rate shown in the top panels. When debt is short term and the planner only cares about smoothing inflation, the nominal rate is kept constant after the spending shock. With long term debt, optimal policy increases the interest rate with inflation, the optimal response is equal to δ.

Introducing the objective to stabilize output exerts an influence on the path of macroeconomic variables but only when debt is short or medium term. Under long term debt (i.e. $\delta = 0.95$) setting λ_Y to 0 or to 0.5 makes only a small difference to the path of inflation and the nominal

[26]For comparison, the cyan line shows the passive fiscal outcome.

Figure 1: **Impulse response functions, G shock**



Notes: The figure displays the impulse responses of inflation, output, and the nominal interest rate following a government spending shock, in the case where $\sigma = 0$. Top panels assume $\lambda_Y = 0$, while in the bottom panels we set $\lambda_Y = 0.5$. In each plot, the solid blue line depicts impulse responses in the case where government debt is short term ($\delta = 0$); the dashed red lines and dash-dotted black lines plot the responses of variables when $\delta = 0.5$ and $\delta = 0.95$, respectively. The dotted cyan line considers the case where fiscal policy is passive.

rate.

This property is worth highlighting. With long debt, inflation rises (almost) permanently to absorb the shock, and the path of output is guaranteed to be smooth regardless of λ_Y. The incentive to make inflation persistent in order to smooth output fluctuations lines up with the incentive to spread inflation efficiently following a shock.[27]

Preference shock. From the formula in (28) it is clear that the dynamic adjustment of the model variables to a preference shock is (qualitatively) the same to the adjustment to the spending shock. For brevity, we plot the impulse responses and discuss in detail the channels via which preference shocks impact inflation in the online appendix (Section C.4).

4.1.2 The canonical New Keynesian model ($\sigma > 0$)

We now turn to the case where $\sigma > 0$. The following formula characterizes the path of inflation under active fiscal policy and $\lambda_Y = 0$:

$$\hat{\pi}_{t+j} = \left\{ \begin{array}{ll} \left[\frac{\bar{b}}{1-\beta\delta} + \frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}(\bar{b}-\bar{s})\right]\Delta\psi_{gov,t} & j=0 \\ \\ \left[\frac{\bar{b}}{1-\beta\delta}\delta^j - \frac{\sigma}{\kappa_1}(1-\delta)\delta^{j-1}\frac{\overline{Y}}{\overline{C}}\bar{b}\right]\Delta\psi_{gov,t} & j \geq 1 \end{array} \right\} \tag{29}$$

where

$$\Delta\psi_{gov,t} = \widetilde{\epsilon}\left[(\overline{G} + (\bar{b}-\bar{s})\sigma\frac{\overline{G}}{\overline{C}})\hat{G}_t + (\bar{b}-\bar{s})\hat{\xi}_t\right] \tag{30}$$

and $\widetilde{\epsilon} > 0$.[28]

Let us explain this formula starting from the impact effect of a shock on inflation. In (29) there are two main channels driving period t inflation: The term $\frac{\bar{b}}{1-\beta\delta}$ measures the *direct effect* on the real payout of all outstanding debt. An increase in inflation in t lowers the real value of the entire stream of payments, $\bar{b}, \bar{b}\beta\delta, \bar{b}(\beta\delta)^2, \ldots$

[27]Note that in terms of the equation (28), this means that if δ is close to 1 and $\delta > \widetilde{\lambda}_1$, then the dynamics of inflation are effectively the same, regardless of λ_Y. For $\delta = 0.95$ it will take a very high λ_Y for output smoothing to exert an influence.

We set $\lambda_Y = 0.5$ for our output smoothing scenario, which is at the high end of the values typically assumed in the literature. (The value assumed by Giannoni and Woodford (2003) is an order of magnitude smaller, and around what one would typically get from a second order approximation of household welfare). For comparison purposes, it would perhaps be useful to investigate whether our calibration of λ_Y may be too low relative to the value implied by the welfare based loss function in this Fisherian model. A second order approximation of household welfare would give us the following objective function:

$$-\frac{1}{2}E(\hat{\pi}_t^2 + \frac{\gamma_h}{\theta}\hat{Y}_t^2)$$

(see online appendix in Chafwehé et al. (2022)). With $\gamma_h = 1$ and $\theta = 17.5$ (Table 1) we get $\lambda_Y = 0.057$. Thus, $\lambda_Y = 0.5$ is already high, relative to the weight implied by a second order approximation of the welfare function.

[28]The expression for $\widetilde{\epsilon}$ is cumbersome and for the sake of the exposition is left to the appendix.

The second term represents the *indirect effect* of inflation, through output, on the intertemporal budget constraint. Since we now assume $\sigma > 0$ the path of output affects the constraint through two channels: First, through impacting real bond prices it impacts the real value of debt (this is measured by the term $\overline{b}\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}$); second, output also affects the value of the government's surplus (the term $\overline{s}\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}$).

Next, consider inflation after t. Again, we have the two distinct channels: The direct effect, $\frac{\overline{b}}{1-\beta\delta}\delta^j$ (the magnitude decays at the coupon rate since $\hat{\pi}_{t+j}$ affects the real value of payments to be made in $t+j$ and thereafter) and the indirect effect, $-\frac{\sigma}{\kappa_1}(1-\delta)\delta^{j-1}\frac{\overline{Y}}{\overline{C}}\overline{b}$.

Let us focus on the new indirect output channel in t and after t and note that what is particularly striking here is that whereas in t the output term contributes positively to the variability of inflation (i.e. it holds that $\overline{b} > \overline{s}$), after t, the sign switches and the effect becomes negative.

To understand this, consider again the intertemporal budget constraint (13). Assume $\delta = 0$ and for simplicity let us also assume that only spending shocks can hit the economy. Then, (13) can be written as:

$$-\overline{G}\hat{G}_t + \overline{s}\frac{\overline{G}}{\overline{C}}\sigma\hat{G}_t - \overline{s}\frac{\overline{Y}}{\overline{C}}\sigma\hat{Y}_t + E_t\sum_{j=1}^{\infty}\beta^j\overline{s}\hat{S}_{t+j} = \overline{b}\hat{b}_{t-1,\delta} - \overline{b}\left[\sigma\left(\frac{\overline{Y}}{\overline{C}}\hat{Y}_t - \frac{\overline{G}}{\overline{C}}\hat{G}_t\right) + \hat{\pi}_t\right] \tag{31}$$

With short term debt, the RHS of the constraint decreases in period t output. The LHS also decreases in output, but the effect on the RHS dominates. Thus, following a positive spending shock, the planner will increase output to satisfy the constraint. The dependence of the importance of this channel on parameter κ_1 follows from the Phillips curve.

Moreover, to understand why inflation will continue responding to the shock in $t+1$ and turn negative, isolate the term

$$E_t\sum_{j=1}^{\infty}\beta^j\overline{s}\hat{S}_{t+j} = -E_t\sum_{j=1}^{\infty}\beta^j\overline{s}\sigma\frac{\overline{Y}}{\overline{C}}\hat{Y}_{t+j} = -\overline{s}\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\beta E_t\hat{\pi}_{t+1} \tag{32}$$

where the second equality makes use of the Phillips curve.[29] Thus, committing to negative inflation in $t+1$ contributes to the satisfaction of (31).

In this model with short term debt, the path of output affects the intertemporal budget constraint through its influence on the path of real interest rates and the present value of (constant) surpluses. Lowering the real discount rates accomplishes to increase the present value, and to 'relax' the debt constraint after the spending shock.[30] Letting inflation (and hence output) turn negative in $t+1$ is the optimal way to accomplish this.

[29] Notice that with constant taxes we have $\overline{s}\hat{S}_{t+j} = -\overline{s}\sigma\frac{\overline{Y}}{\overline{C}}\hat{Y}_{t+j} - (\overline{G} - \overline{s}\sigma\frac{\overline{G}}{\overline{C}})\hat{G}_{t+j} + \overline{s}\hat{\xi}_{t+j}$. Then dropping the shocks (as these will cancel out due to the i.i.d processes) we get $E_t\sum_{j=1}^{\infty}\beta^j\overline{s}\hat{S}_{t+j} = -E_t\sum_{j=1}^{\infty}\beta^j\overline{s}\sigma\frac{\overline{Y}}{\overline{C}}\hat{Y}_{t+j}$.

[30] The effect is analogous to what Leeper and Zhou (2021) label a 'discount factor' impact of policy. Whereas in their model this channel concerns both inflation and tax policies, here it is a pure inflation effect.

Notice further that another way of interpreting the negative inflation in $t+1$ is the following: When the planner is not concerned about output stabilization, she will optimally shift as much of the burden of the adjustment as possible to date t output. A higher output level is possible if $\hat{\pi}_{t+1} < 0$ (from the Phillips curve) and since the distortions stemming from inflation are convex, it is optimal to tolerate a negative inflation rate in $t+1$, in exchange for a smaller inflation adjustment to the shock in t. This margin will become less significant when $\lambda_Y > 0$, as we will confirm later on.

Let us now turn to the case of long debt to explain why these considerations become progressively less important when $\delta > 0$. In this case, distorting output may increase the value of the surpluses, however, it will also increase the price of the long term debt. There is thus less of a gain of making inflation turn negative in $t+1$ in terms of fiscal solvency and in the limit, when long bonds are consols, this margin becomes absolutely irrelevant. Simple inspection of the intertemporal budget in (13) is sufficient to see this property. It is also confirmed in (29); the term $\frac{\sigma}{\kappa_1}(1-\delta)\delta^{j-1}\frac{\overline{Y}}{\overline{C}}\overline{b}$ becomes smaller in magnitude as δ increases.[31]

The direct and indirect channels of inflation we highlighted in this paragraph are clearly present in the optimal interest rate rule we derived for this model, in Proposition 4. The optimal inflation coefficient was derived equal to $\delta + \frac{\sigma}{\kappa_1}(1-\delta)(\delta\beta - 1)$ which echoes the principle that the optimal policy is mindful not only of the direct impact of inflation on the real debt, but also of the indirect output impact. Moreover, our analytical formula clearly shows that with long term debt the indirect channel is less significant. For δ sufficiently close to 1, $\frac{\sigma}{\kappa_1}(1-\delta)(\delta\beta - 1)$ is approximately 0, and the optimal policy can be approximated by a simpler rule setting the inflation coefficient equal to δ. This is an important property to which we will return in the final paragraph of this section.

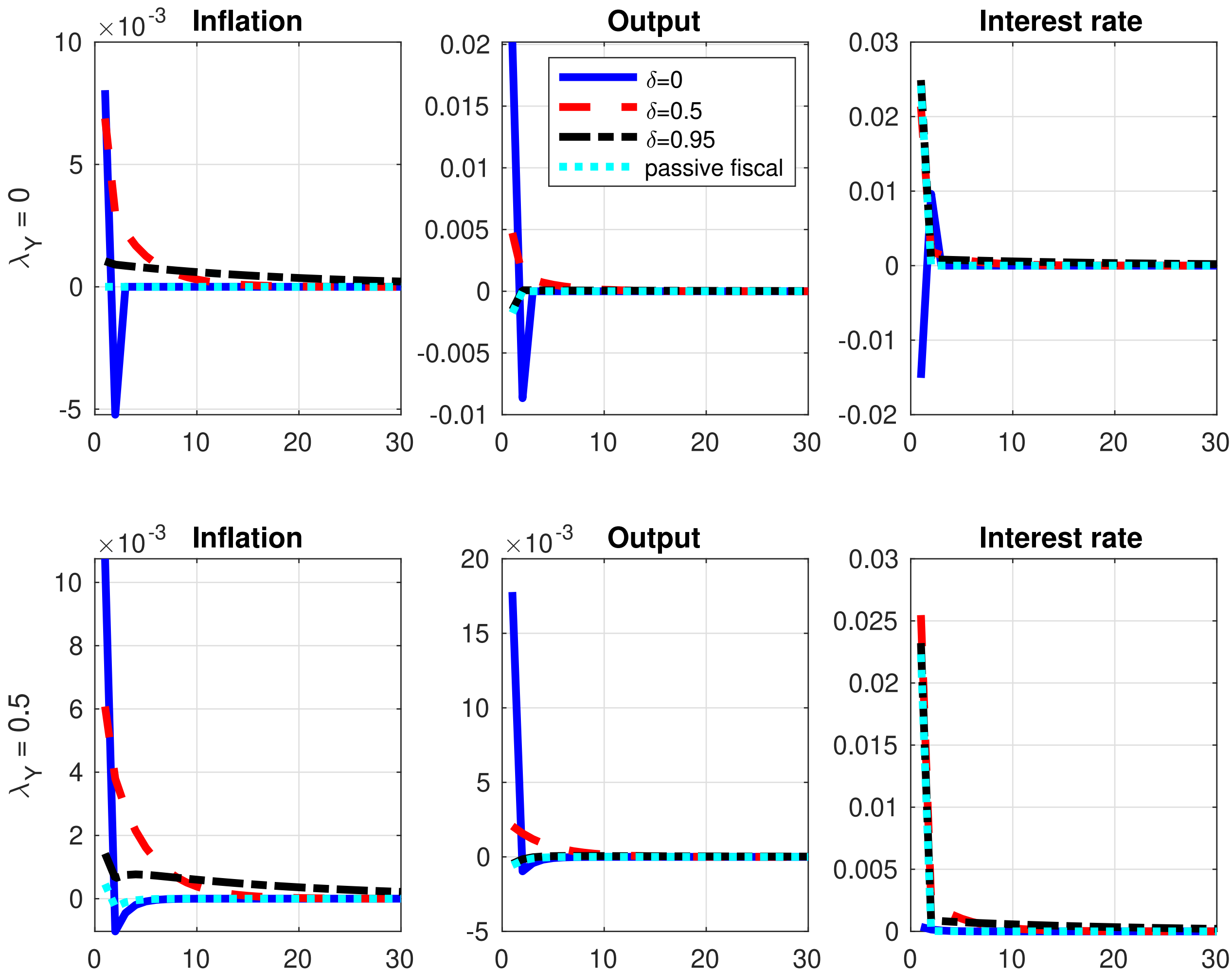
Graphs. Figure 2 plots the usual IRFS for the spending shock (see appendix for the analogous plots with preference shocks). Focus on the top panel in Figure 2 to continue studying the effect of the spending shock when $\lambda_Y = 0$. As can be seen from the Figure, when debt is short term, output and inflation increase contemporaneously with the shock and subsequently drop in $t+1$. The nominal interest rate drops initially and turns positive in $t+1$. This is due to monetary policy operating through the indirect output channel, when $\delta = 0$.

With long maturity debt ($\delta = 0.95$) optimal policy targets a smooth path of inflation and output. The nominal rate rises on impact (since it is optimal to track $\widetilde{r}_t$) and subsequently reacts to inflation only.

Finally, these properties continue being relevant in the case $\lambda_Y > 0$ studied in the bottom panels of the Figure. As expected, the indirect output channel now becomes less important. However, it remains an active margin of policy when debt is short term.

[31] The no discounting result under consols is well known to the literature (see for example Lucas and Stokey (1983); Leeper and Zhou (2021)). In the optimal fiscal policy setting of Lucas and Stokey (1983) the optimizing government will not want to distort output intertemporally when long bonds are consols, and the optimal allocation becomes time consistent.

Figure 2: **Impulse response functions with $\sigma > 0$ (G shock)**



Notes: The figure displays the impulse responses of inflation, output, and the nominal interest rate following a government spending shock, in the case where $\sigma = 1$. Top panels assume $\lambda_Y = 0$, while in the bottom panels we set $\lambda_Y = 0.5$. In each plot, the solid blue line depicts impulse responses in the case where government debt is short term ($\delta = 0$); the dashed red lines and dash-dotted black lines plot the responses of variables when $\delta = 0.5$ and $\delta = 0.95$, respectively. The dotted cyan line considers the case where fiscal policy is passive.

4.2 Optimal Policy under the Dual Mandate Objective

We have now analyzed the key forces behind optimal monetary policy in the two versions of the model. As we have seen, assuming that the monetary authority desires to smooth inflation and output introduces inertia in the optimal policy, $\hat{i}_t$ responds to both current and lagged inflation. Moreover, in the canonical model with $\sigma > 0$ where optimal monetary policy set the interest rate as a function of current inflation only, the optimal inflation coefficient reflected both the direct effect of inflation on real debt and the indirect effect through output on the debt constraint. Finally, our results in Section 3 demonstrated that in both of these cases optimal interest rate rules featured stochastic intercepts, terms involving the multiplier $\Delta\psi_{gov,t}$.

We can now derive the optimal interest rate rule in the canonical New Keynesian model with a dual mandate objective, $\sigma, \lambda_Y > 0$. The appendix proves the following Proposition:

Proposition 5: *Assume that fiscal policy is active and $\sigma, \lambda_Y > 0$. The optimal interest rate rule is:*

$$\hat{i}_t = \widetilde{r}_t + \left\{\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{\overline{C}}}\left[\left(1-\left(\widetilde{\lambda}_1+\delta\right)\right)\left(\left(\widetilde{\lambda}_1+\delta\right)\beta-1\right)+\beta\delta\widetilde{\lambda}_1\right]+\left(\widetilde{\lambda}_1+\delta\right)\right\}\hat{\pi}_t -$$
$$-\delta\widetilde{\lambda}_1\left\{\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{\overline{C}}}\left[(1+\beta)-\beta\left(\widetilde{\lambda}_1+\delta\right)\right]+1\right\}\hat{\pi}_{t-1}+\underbrace{\widetilde{f}\Delta\psi_{gov,t}}_{\text{Stochastic intercept}} \tag{33}$$

where $\widetilde{r}_t = \widetilde{r}^n_t$ under the natural output target and $\widetilde{r}_t = \widetilde{r}^n_t + \widetilde{v}_t$ under the steady state output target.

The appendix delivers the expression for the term $\widetilde{v}_t$ affecting the real rate that monetary policy needs to track under the constant output target. This term is not zero because government spending shocks shift the Phillips curve in the presence of income effects on labour supply. These impacts are accounted for by natural output. However, with the constant output target, they are not. When $\lambda_Y > 0$ this matters for optimal policy.[32] Moreover, parameter $\widetilde{f}$ also depends on the output target. For brevity the derivations are relegated to the appendix.

The reader will note in formula (33) the key elements of the optimal interest rate rules of Propositions 3 and 4. The parameter $\widetilde{\lambda}_1$ in current and lagged inflation serves the output smoothing objective (Proposition 3) and the bracketed terms multiplied by $\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{\overline{C}}}$ represent the indirect output effects (Proposition 4). The indirect effects are now dependent also on parameter $\widetilde{\lambda}_1$, since as we saw previously, output smoothing alters the relative importance of this margin. We conclude that the dual mandate optimal policy is basically a fusion of the rules of Propositions 3 and 4.

4.2.1 The microfounded loss function

The optimality of (33) holds when we assume a microfounded objective function for the planner. The only difference in this case is the addition of a subsidy to the model that ensures the efficiency of the steady state. This introduces a new term to the consolidated budget, dependent on the

[32] In Proposition 4 we had $\widetilde{v}_t = 0$ when $\lambda_Y = 0$. When the planner focuses on stabilizing inflation, shocks to the Phillips curve are fully absorbed by output and do not affect the optimal interest rate rule.

current output, which the optimal policy will internalize. Consequently, the stochastic intercept $\widetilde{f}\Delta\psi_{gov,t}$ in Proposition 5 will now also reflect this margin. Otherwise, the optimal rule remains the same as in Proposition 5.

4.3 Optimal Simple Rules

We now turn to a more detailed examination of the elements of the optimal interest rate rules derived in Propositions 3 to 5. Specifically, we aim to verify the significance of the stochastic intercept terms for optimal policy. As previously discussed, these terms introduce temporal variation in interest rates, incorporating a non-systematic element into the policy functions. Our goal is to determine whether these non-systematic components are a crucial feature of optimal Ramsey rules across different maturity structures of debt. In addition, while Propositions 3 to 5 provided an analytical characterization of the optimal systematic reaction to inflation—where the coefficients are simple functions of model parameters—we will now delve deeper into these expressions to investigate whether the indirect effects of inflation play a significant role in shaping policy.

Our main finding in this paragraph is that under a plausible calibration of the maturity structure (for example $\delta = 0.95$ implying a 5 year average maturity of debt, a value that aligns with the US data) not accounting for stochastic intercepts and indirect output effects or for lagged inflation does not change dramatically the macroeconomic outcomes. We therefore find that the rule $\hat{i}_t = \widetilde{r}_t + \delta\hat{\pi}_t$ approximates very well the Ramsey policy. Commitment to a rule that tracks the real interest rate and sets the inflation coefficient equal to $1 - \frac{1}{\text{Maturity}}$ (where Maturity denotes the average maturity of government debt) is sufficient to bring the economy very close to the Ramsey outcome. We thus show that our model gives rise to a very simple and practical policy rule for an empirically relevant maturity structure.

4.3.1 The (un)importance of stochastic intercepts.

To show transparently our results we first revisit Cases 1 and 2 of Section 3.[33] This enables us to identify whether stochastic intercepts may matter in the context of the output smoothing objective or in the context of targeting real discount rates and bond prices through monetary policy.

Let us first consider the model of subsection 3.2.1 and in particular equation (23) characterizing optimal policy when $\lambda_Y > 0$. Using also the analytical formula for $\Delta\psi_{gov,t}$ in equation (28) we can write:

$$-\delta\frac{\widetilde{o}}{\widetilde{\lambda}_2 - 1}\Delta\psi_{gov,t} = -\delta\frac{\widetilde{o}}{\widetilde{\lambda}_2 - 1}\widetilde{\psi}\overline{G}\hat{G}_t$$

Consider a positive fiscal shock. In response to this shock optimal policy will keep the nominal rate slightly lower in t than the value implied by $(\widetilde{\lambda}_1 + \delta)\hat{\pi}_t - \widetilde{\lambda}_1\delta\hat{\pi}_{t-1}$. This temporal variation in the interest rate aims at having a more gradual/smoother path of output.

[33] We again focus only on the case of spending shocks, however, our results also apply to preference shock induced fluctuations.

Figure 3 shows the responses. The solid blue lines are the Ramsey IRFS, the dashed red lines are the analogous objects drawn from a model in which monetary policy sets interest rates as in (23) but without the stochastic intercept. The top panel corresponds to short term debt. The middle and bottom panels set $\delta = 0.5$ and $\delta = 0.95$ respectively. Unsurprisingly, the stochastic intercept has no bearing on the outcomes in the case of short debt.[34] However, even when we assume long term debt the differences between the Ramsey outcome and the model without the intercept are not significant. We conclude that the stochastic intercept is not an important feature of policy to accomplish the output smoothing objective.

Next, consider the model of subsection 3.2.2 (setting $\sigma > 0$) focusing on equation (26), and noting that we can again express the stochastic intercept $-f\Delta\psi_{gov,t}$ as a function of the spending shock (e.g. equation (30)). This term will effectively keep the nominal rate lower in t following a positive innovation to government spending. The effect of this channel is revealed in Figure 4 which plots the IRFS under the optimal policy and in a model where monetary policy is set according to

$$\hat{i}_t = \widetilde{r}_t + \left(\delta + \frac{\overline{Y}}{\overline{\overline{C}}}\frac{\sigma}{\kappa_1}(1-\delta)(\delta\beta - 1)\right)\hat{\pi}_t$$

The top, middle and bottom panels vary the maturity of debt δ.

When debt is short or medium term, the stochastic intercept does affect optimal policy. Most notably, in the case where $\delta = 0$ the interest rate rule without the intercept predicts that output and inflation increase when the shock hits, but they will not fall in the next period as is the case under the optimal policy. This leads to a smaller increase in output in t which needs to be compensated by a larger reaction of inflation contemporaneously to the shock, to satisfy the intertemporal budget constraint. Thus, the stochastic intercept enables to reduce the variability of inflation by relying on a stronger response of output to make debt solvent. It is at the heart of the indirect output channel.

Finally, as is evident from the bottom panel of the Figure, the stochastic intercept is not at all important when we set $\delta = 0.95$.

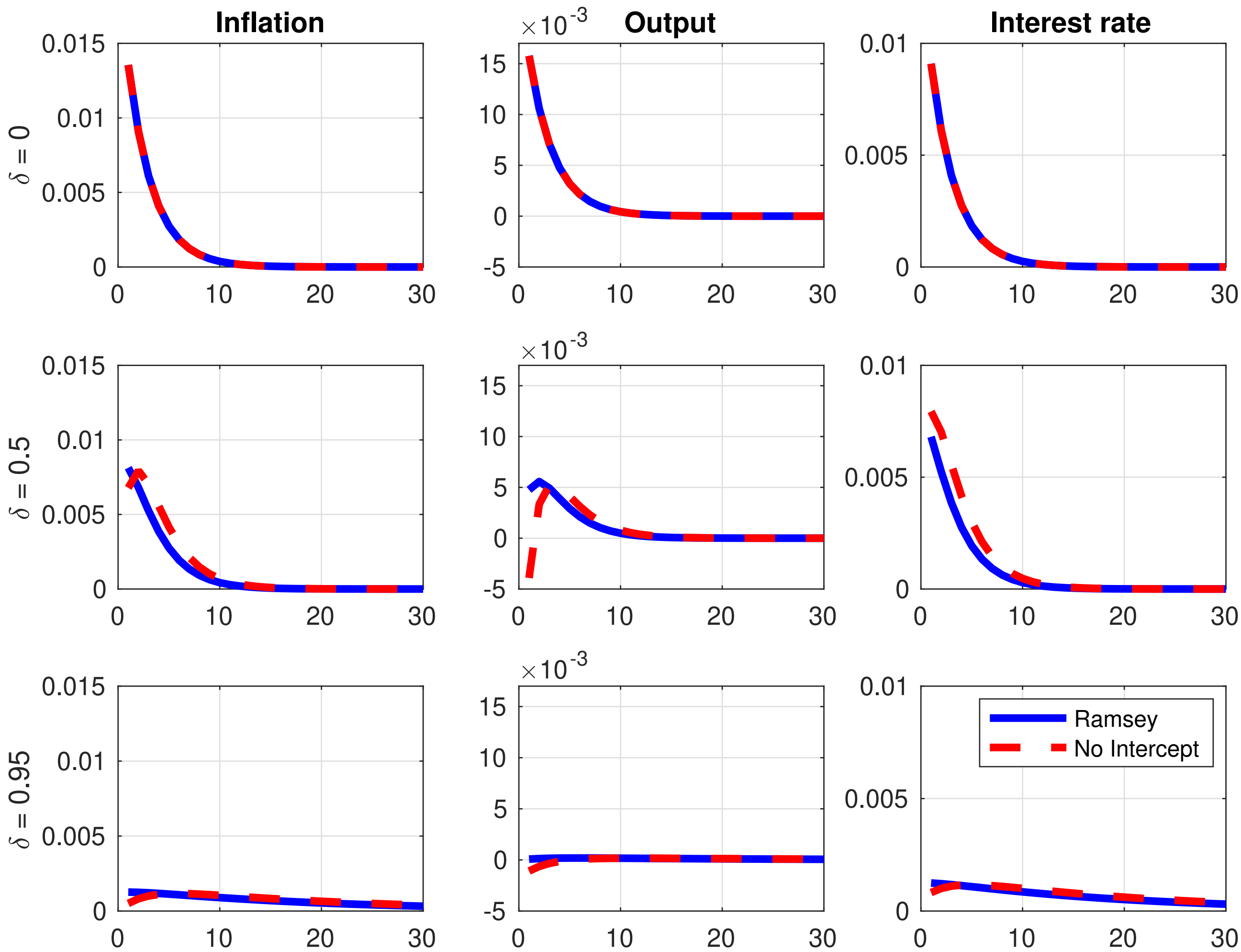
4.3.2 Simple inflation targeting rules / The (un)importance of the indirect channel.

Simple rules as in Propositions 3 and 4 without the stochastic intercepts can approximate well the optimal policy outcome under the plausible calibration $\delta = 0.95$. Building on this finding, we now investigate whether even simpler interest rate rules, that focus on the direct impact of inflation, can sustain this outcome.

In Figure 5 we show the responses of macroeconomic variables to spending shocks under the dual mandate objective function. The bottom panel considers the case of the microfounded loss

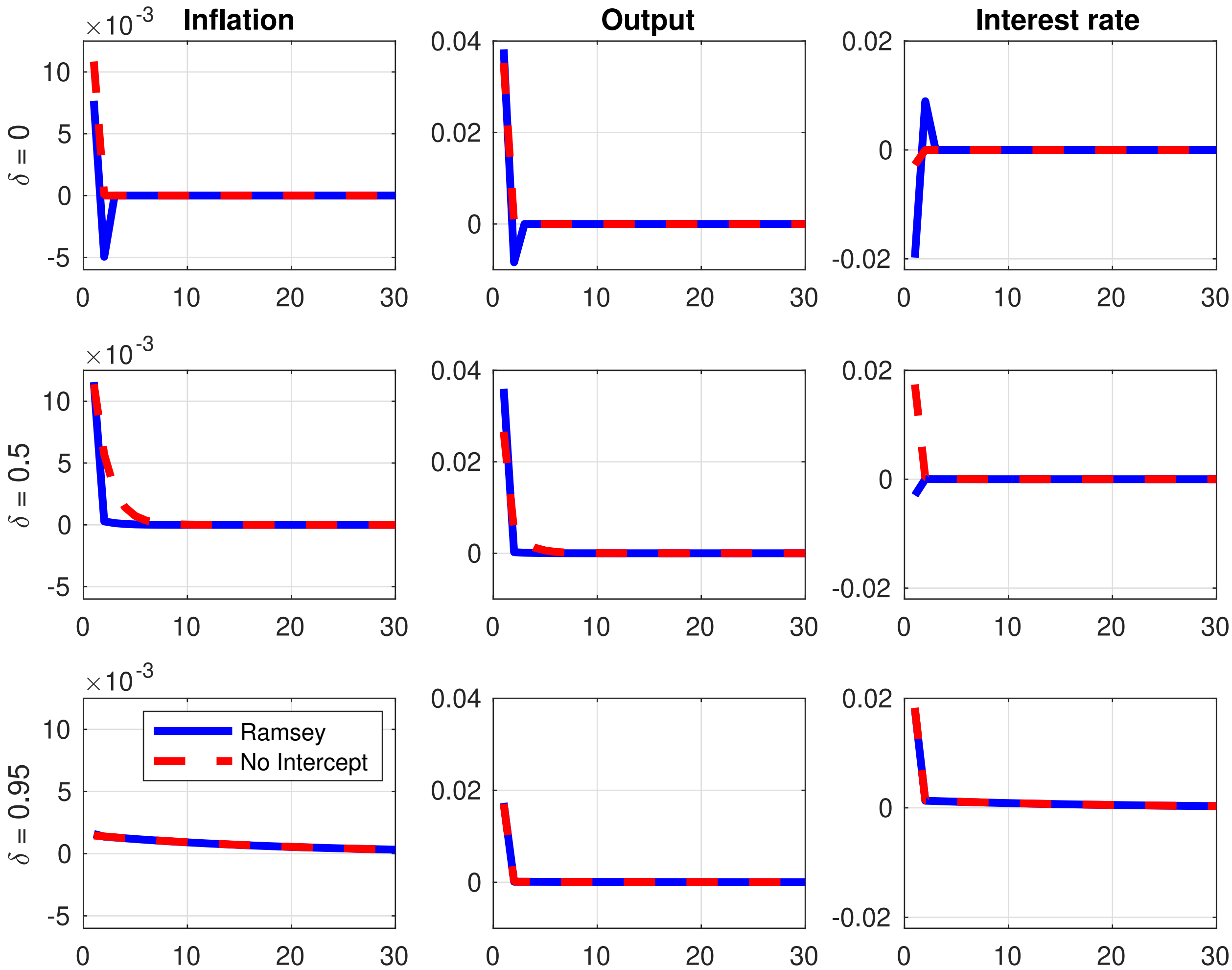
[34] Evidently, when $\delta = 0$ there is no margin for the planner to smooth the response of output by trading off less inflation in t for more inflation in the future, since only date t inflation can satisfy the intertemporal budget constraint.

Figure 3: **The role of stochastic intercepts:** $\lambda_Y > 0, \sigma = 0$.



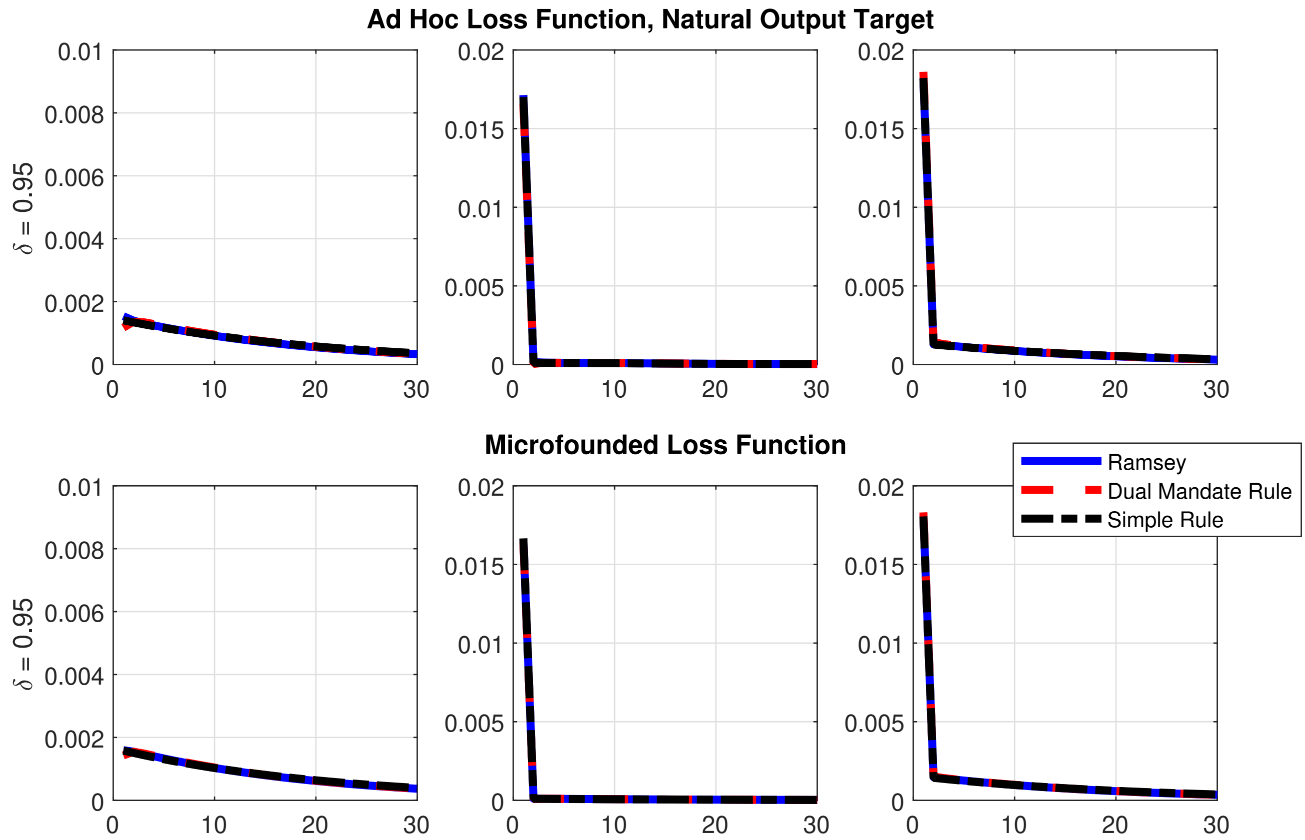
Notes: The figure plots the impulse responses of inflation, output, and the nominal interest rate following a government spending shock, in the full Ramsey solution (solid lines) and in a model without stochastic intercepts (dashed lines). We assume $\lambda_Y = 0.5$ and $\sigma = 0$. The top panel shows the case of short term debt, the middle and bottom panels set $\delta = 0.5, 0.95$ respectively.

Figure 4: **The role of stochastic intercepts:** $\lambda_Y = 0, \sigma > 0$.



Notes: The figure plots the impulse responses of inflation, output, and the nominal interest rate following a government spending shock, in the full Ramsey solution (solid lines) and in a model without stochastic intercepts (dashed lines). We assume $\lambda_Y = 0$ and $\sigma = 1$. The top panel shows the case of short term debt, the middle and bottom panels set $\delta = 0.5, 0.95$ respectively.

Figure 5: **Rules vs Ramsey**



Notes: The figure compares the optimal policy impulse responses of inflation, output, and the nominal interest rate (blue solid lines) with analogous objects when policy is set according to the rule in Proposition 5 (Dual Mandate Policy, red dashed line) and the simple inflation targeting rule $\hat{i}_t = \hat{r}_t^n + \delta\hat{\pi}_t$ (black dashed line). We assume $\delta = 0.95$ in all graphs. The top panel focuses on the case of an ad hoc objective function and monetary policy targets natural output. The bottom panel assumes the microfounded loss function. In both cases $\lambda_Y = \frac{\sigma\frac{\bar{Y}}{\bar{C}}+\gamma_h}{\theta} \approx 0.12$. We assumed $\sigma = \gamma_h = 1$

function (approximation around the efficient steady state) whereas in the top panel we consider an ad hoc loss function with natural output targeting.[35] The Ramsey optimal policy is depicted using the blue solid lines. The dashed red lines utilize the optimal rule in Proposition 5 without the stochastic intercept, and the black lines consider an even simpler policy function $\hat{i}_t = \widetilde{r}_t + \delta\hat{\pi}_t$. As can be seen from the Figure, the impulse responses produced by the three models almost completely overlap.

The finding that the 'Dual Mandate' rule of Proposition 5 continues to provide a close approximation of the Ramsey policy outcome, when we have omitted the stochastic intercept term, should be unsurprising in light of the results of the previous paragraph. Stochastic intercepts are not a significant feature of optimal policy when debt is long term, or in the presence of an output smoothing objective and therefore, what we found previously, generalizes to the canonical model with the dual mandate loss function.

The finding that an even simpler rule $\hat{i}_t = \widetilde{r}_t + \delta\hat{\pi}_t$, also provides a good fit of Ramsey

[35]The appendix extends this graph to the case of the constant output target.

policy merits more emphasis. We have seen that when debt is long term, distorting output intertemporally to ensure the solvency of the intertemporal budget, is not optimal. In other words, the indirect output channel becomes a less significant margin of policy, the main driving force is the direct impact of inflation on the real value of debt. Moreover, a strong incentive to smooth output fluctuations, also weakens the indirect effect. Our finding is explained by the combination of these forces.

In short, our substantive result in this section is that for a plausible calibration of the maturity structure of debt, a simple inflation targeting rule focusing on the direct channel of inflation is enough to approximate closely the Ramsey outcome. Though we will subsequently present several robustness checks to further test the validity of this result in alternative modelling environments, before concluding this paragraph it is worthwhile to make a couple comments concerning our key finding. First, our result illustrates that the average maturity of debt is the only relevant moment for the optimal inflation coefficient. We have however constrained our focus on a particular debt structure, assuming geometrically decaying payments. It turns out, however, that this assumption fits the US data well (see for example de Lannoy et al. (2022) for data on the payment profiles of US debt). Thus, our assumption appears not to be restrictive for the US. Second, with the parameter values we assumed in the calibration of this section, we get $\delta + \frac{\overline{Y}}{\overline{C}}\frac{\sigma}{\kappa_1}(1-\delta)(\beta\delta - 1) = 0.9484$ (the inflation coefficient in Proposition 4) which is indeed very close to $\delta = 0.95$. However, our finding generalizes to a large range of parameter values. We can (for example) set $\sigma = 5$ (a commonly assumed upper bound in macro models) or $\theta = 100$ (targeting a flatter Phillips curve) and we will continue finding that the simple inflation targeting rule works well. Finally, assuming lower values for the coefficient δ (0.9 or even 0.8, see next paragraph) or repeating the analysis of this section to consider to preference shocks instead or spending shocks also leaves our main finding unchanged.

4.4 Macroeconomic volatility under Optimized rules and 'Ramsey rules'

How important is it for monetary policy to follow 'Ramsey' optimal rules when fiscal policy is active? In the appendix we compare the macroeconomic volatility implied by our rules with that deriving from ad hoc interest rate rules with optimized coefficients. More specifically, we posit the following specification of monetary policy, commonly assumed in DSGE models,

$$\hat{i}_t = \rho_i \hat{i}_{t-1} + (1-\rho_i)(\phi_Y \hat{Y}_t + \phi_\pi \hat{\pi}_t)$$

and find the coefficients ϕ_Y, ϕ_π, ρ_i that minimize the loss function.[36]

Our results are as follows: First, 'Ramsey rules' (including stochastic intercepts) significantly outperform rules with optimized coefficients. At the minimum distance (when $\delta = 0.7$), the loss function under the ad hoc rule with optimized coefficients is 13 percent higher than under the

[36] The calibration of the model is discussed in detail in the appendix, but basically we borrowed the stochastic processes for spending and preference shocks from the estimated DSGE models literature (Smets and Wouters, 2007). The results that we drew from this exercise were however robust to numerous calibrations of the model.

Ramsey rule. The percentage differences are even more pronounced when debt is short term (> 100 percent) and when it is long term (for example when $\delta = 0.95$ approximately 70 percent). Second, removing stochastic intercepts does worsen the performance of the policy rules, but only in the case where debt is short term. For $\delta > 0.8$ we find that the rules without intercepts produce effectively the same outcome as the full Ramsey solution. Note that this means that the result of the previous paragraph extends to an average maturity greater than one year and one quarter (which is already a very low number compared with the empirical moments in OECD economies).

Third, when debt is short term (a case studied extensively in previous literature) the dual mandate rule without a stochastic intercept performs considerably better than the optimized coefficients rule.

5 Extensions

Before concluding the paper in the next section we briefly consider a few extensions of our baseline model, discussing the robustness of our findings to alternative model structures. We first extend our derivations to persistent shocks and to supply side factors (markup shocks). We then extend our findings by replacing the assumption that taxes are lump sum with distortionary taxation. Finally, we briefly state what optimal policy would look like in the case where the government can issue state contingent bonds (complete markets). For brevity all the derivations that pertain to these models are relegated to the online appendix.

5.1 Alternative Shocks and Shock Structures

Persistent shocks We derived, for tractability, our analytical results assuming that shocks are i.i.d. Our derivations can be generalized to the case where spending and preference shocks follow first order autoregressive processes. It is then possible to show that the interest rate rules in Propositions 3 to 5 will continue being optimal, the coefficients attached to inflation will not change, the only part of these rules that needs to be adjusted is the term $\widetilde{r}_t$ ($\widetilde{r}^n_t$).

For example, consider the version of the model studied in subsection 3.2.2. The optimal interest rate rule with general first order autoregressive shocks is :

$$\hat{i}_t = \underbrace{\hat{\xi}_t(1-\rho_\xi) + \sigma(\frac{\overline{G}}{\overline{C}} - \frac{\sigma\kappa_2}{\kappa_1}\frac{\overline{Y}}{\overline{C}})\hat{G}_t(1-\rho_G)}_{\widetilde{r}_t = \widetilde{r}^n_t} + \left(\delta + \frac{\overline{Y}}{\overline{C}}\frac{\sigma}{\kappa_1}(1-\delta)(\delta\beta - 1)\right)\hat{\pi}_t - f\Delta\psi_{gov,t} \tag{34}$$

where ρ_ξ and ρ_G are the persistence coefficients for spending and preference factors. The inflation coefficient is unaffected by shock persistence and parameter f is exactly the same as in Proposition 4. Thus, the only change that shock persistence brings to the optimal rule is through the real rate target. We can easily show that this applies to all of the policy functions we derived in this paper.

The principle behind this change is the following: The leading term $\widetilde{r}_t$ needs to account for shock persistence since monetary policy uses this term to eliminate the shocks from the Euler equation. The optimal inflation coefficients and the stochastic intercepts are however determined by the debt constraint and the persistence of the shocks will not matter for how inflation will optimally adjust to satisfy this constraint. Shock persistence will only affect the size of the response of inflation and output, not the shape of the path of these variables.

Finally, in the online appendix, we run several experiments to test the robustness of our findings in Section 4 to non i.i.d. shocks. We found that assuming persistent shock processes does not change any of our results.

Supply side shocks In the appendix, we extend our analysis to include supply-side shocks, which we model as markup shocks that shift the Phillips curve. The central features of the optimal Ramsey rules remain unchanged, which is unsurprising given that the usual New Keynesian inflation-output trade-off was already accounted for in our formulas. Consequently, the optimal interest rate rules we derive retain the same inflation coefficients. However, supply shocks must be incorporated into the expression for the target rate $\widetilde{r}_t$ when $\lambda_Y > 0$. We provide an analytical derivation of the relevant formula.

Moreover, when evaluating the performance of the Ramsey rules in the presence of supply shocks across different maturity structures, we discovered a striking result. For the dual mandate objective, the maturity structure of debt has only a moderate impact on optimal policy. Our explanation for this outcome is the following: supply-side shocks create a steep trade-off between output and inflation, even within the standard New Keynesian model. They do not directly affect the debt constraint, thus prompting the planner to address them in the usual New Keynesian manner. Any imbalances of the intertemporal debt constraint resulting from these shocks can be managed with slight adjustments in inflation. The bulk of the volatility in inflation and output, however, does not derive from the debt constraint.

5.2 Real rate tracking

A key property of the optimal interest rate policies that we derived in this paper is that the nominal rate needs to track the appropriate (consistent with the output target) measure of the real interest rate $\widetilde{r}_t$. Real rate tracking is a common feature of optimal policy in the canonical New Keynesian model (see for example Woodford (2003a); Holden (2024)), however implementing the target $\widetilde{r}_t$ pragmatically requires to possess accurate estimates of the underlying shock processes and shock realizations.

Holden (2024) proposes (as an alternative target) that monetary policy track the real interest rate that can be inferred from the Fisher equation ($\widetilde{r}_t := \hat{\imath}_t - E_t\hat{\pi}_{t+1}$). He describes an approach to back this real rate from the yields of inflation indexed government bonds.

In the appendix we investigate whether with this alternative target, our optimal interest rate rules can continue providing a good approximation of the Ramsey outcome. We find that indeed they do, and more specifically our key result, that with an inflation coefficient equal to δ a simple rule is nearly optimal under the dual mandate loss function, continues to hold.

5.3 Distortionary Taxation

We briefly discuss the implications of replacing the assumption that taxes are lump sum with distortionary taxation. Distortionary taxes do not change any of the conclusions we drew from our analysis; the optimal rules are very similar to the rules we derived previously.

With distortionary taxes, the threshold for passive fiscal policy is given by:

$$\widetilde{\phi}_\tau \equiv \frac{\overline{b}(1-\beta)}{\overline{R}(1-\beta\delta)\left(\frac{1}{1-\overline{\tau}^d} - \frac{\overline{\tau}^d}{1-\overline{\tau}^d}(1+\frac{1}{\gamma_h+\sigma\frac{\overline{Y}}{\overline{C}}})\right)}$$

where $\overline{R}$ denotes the steady state revenue of the government and $\overline{\tau}^d$ is the distortionary tax rate. Parameters γ_h, σ become important for the threshold $\widetilde{\phi}_\tau$ since they determine the response of output to shocks and consequently that of fiscal revenue.

To illustrate how the presence of distortionary taxes may affect the optimal policy rules we derived, let us focus on the simplest possible scenario $\sigma = \lambda_Y = 0$. In the active fiscal case the optimal interest rate rule is:

$$\hat{i}_t = \xi_t + \delta\hat{\pi}_t - \delta\frac{\overline{R}(1+\gamma_h)}{\kappa_1}\Delta\psi_{gov,t} \tag{35}$$

(see appendix).

Relative to Proposition 2, assuming distortionary taxation introduces an additional element to policy, the final term on the RHS of (35). This term relates to the effect of output on fiscal revenue. An increase in inflation will increase current output and thus increase the revenue and the surplus. The optimal policy internalizes the new revenue effect, by keeping the nominal rate slightly lower in t after a positive spending shock.[37]

It turns out that this effect is not substantial. One can therefore drop the stochastic intercept term from the optimal policy rule and end up with very similar dynamics for macroeconomic variables. For the sake of brevity, we leave it to the appendix to plot the impulse response functions for this model. We also derive optimal policy rules for each of the other versions of the model we considered. We prove that our previous results continue to hold.

5.4 Complete Markets

Our results in this paper were derived under the assumption that debt is not state contingent. Methodologically, our paper is thus very close to numerous papers studying optimal policy under incomplete markets (e.g. Faraglia et al. (2013, 2019); Schmitt-Grohé and Uribe (2004); Lustig et al. (2008); Leeper and Zhou (2021) among others). In the appendix we extend our analysis to the case of complete markets, assuming that government debt is state contingent. We ask: What will optimal interest rate rules look like in this model when fiscal policy is active? Our

[37] Notice that this effect is valid only when debt is not short term. With short debt, only period t inflation can finance debt and the magnitude of the response of inflation is pinned down by the intertemporal budget.

finding is that the optimal policy continues to follow a passive money rule, which tracks the real rate. However, any response to inflation $\phi_\pi < 1$ can implement the optimal policy outcome. With complete markets, debt acts as a shock absorber. Since shocks effectively do not impact the debt constraint we get $\Delta\psi_{gov,t} = 0$ in equilibrium. Then, inflation does not need to adjust to ensure the solvency of debt and any passive rule can implement the outcome.

This is a limiting result that concerns only the case of complete markets. Whenever markets cannot be completed and $\Delta\psi_{gov,t} \neq 0$ the rules that we derived in this paper are applicable.

6 Conclusion

We presented a framework of optimal monetary policy when the central bank may need to take into account the government budget constraint and is thus concerned with the solvency of debt. Our model is tractable and enables us to derive optimal interest rate rules analytically. One of our substantive finding is that simple inflation targeting rules that track the real interest rate are sufficient to implement the Ramsey policy outcomes. A second key result is that for a debt maturity structure that approximates the US data, a simple (and practically relevant) inflation targeting rule setting the inflation coefficient equal to $1 - \frac{1}{\text{Maturity}}$ is nearly optimal.

A few of extensions of this work seem to us fruitful for future research. First, using the framework we proposed to extend the analysis to the case of regime fluctuations, seems a meaningful next step. More specifically, it would be interesting to study optimal policy in the context of a model in which the work of Davig and Leeper (2007) showed that the generalized Taylor principle is required for the determinacy of the equilibrium. In this context, optimal monetary policy will probably not be purely passive or purely active, pressumably a combination of the policy rules we derived here will be the optimum. Second, our analytical results were derived in the baseline New Keynesian framework (augmented with a fiscal block) and we have not explored optimal policy in environments featuring wage rigidities, inflation inertia or the zero lower bound constraint. We believe that the methodological approach that we followed in this paper to derive analytically optimal rules is applicable to models including these features. Lastly, our approach should also be applicable to models in which the Ramsey planner sets jointly monetary and fiscal variables. Thus, our findings should be useful also in the context of optimal coordinated policies.

References

Aiyagari, S. R., A. Marcet, T. J. Sargent, and J. Seppälä (2002): "Optimal taxation without state-contingent debt," *Journal of Political Economy*, 110, 1220–1254.

Angeletos, G.-M. (2002): "Fiscal policy with noncontingent debt and the optimal maturity structure," *The Quarterly Journal of Economics*, 117, 1105–1131.

Bassetto, M. (2002): "A game–theoretic view of the fiscal theory of the price level," *Econometrica*, 70, 2167–2195.

BENIGNO, P. AND M. WOODFORD (2007): "Optimal inflation targeting under alternative fiscal regimes," *Series on Central Banking, Analysis, and Economic Policies, no. 11.*

BHANDARI, A., D. EVANS, M. GOLOSOV, AND T. J. SARGENT (2017): "Fiscal policy and debt management with incomplete markets," *The Quarterly Journal of Economics*, 132, 617–663.

BI, H. AND M. KUMHOF (2011): "Jointly optimal monetary and fiscal policy rules under liquidity constraints," *Journal of Macroeconomics*, 33, 373–389.

BIANCHI, F. AND C. ILUT (2017): "Monetary/fiscal policy mix and agents' beliefs," *Review of economic Dynamics*, 26, 113–139.

BIANCHI, F. AND L. MELOSI (2017): "Escaping the great recession," *American Economic Review*, 107, 1030–58.

——— (2019): "The dire effects of the lack of monetary and fiscal coordination," *Journal of Monetary Economics*, 104, 1–22.

BOUAKEZ, H., R. OIKONOMOU, AND R. PRIFTIS (2018): "Optimal debt management in a liquidity trap," *Journal of Economic Dynamics and Control*, 93, 5–21.

BUERA, F. AND J. P. NICOLINI (2004): "Optimal maturity of government debt without state contingent bonds," *Journal of Monetary Economics*, 51, 531–554.

CANZONERI, M., R. CUMBY, AND B. DIBA (2010): "The interaction between monetary and fiscal policy," *Handbook of monetary economics*, 3, 935–999.

CHAFWEHÉ, B., C. DE BEAUFFORT, AND R. OIKONOMOU (2022): "Optimal Monetary Policy Rules in the Fiscal Theory of the Price Level," Tech. rep., LIDAM Discussion Paper IRES 2022-07 , Université catholique de Louvain.

CHARI, V. V. AND P. J. KEHOE (1999): "Optimal fiscal and monetary policy," *Handbook of macroeconomics*, 1, 1671–1745.

COCHRANE, J. H. (1998): "A frictionless view of US inflation," *NBER macroeconomics annual*, 13, 323–384.

——— (2001): "Long-term debt and optimal policy in the fiscal theory of the price level," *Econometrica*, 69, 69–116.

——— (2018): "Stepping on a rake: The fiscal theory of monetary policy," *European Economic Review*, 101, 354–375.

DAVIG, T. AND E. M. LEEPER (2007): "Generalizing the Taylor principle," *American Economic Review*, 97, 607–635.

de Lannoy, L. R. A., A. Bhandari, D. Evans, M. Golosov, and T. J. Sargent (2022): "Managing Public Portfolios," Tech. rep., National Bureau of Economic Research.

Del Negro, M. and C. A. Sims (2015): "When does a central bank's balance sheet require fiscal support?" *Journal of Monetary Economics*, 73, 1–19.

Eggertsson, G. B. (2008): "Great Expectations and the End of the Depression," *American Economic Review*, 98, 1476–1516.

Faraglia, E., A. Marcet, R. Oikonomou, and A. Scott (2013): "The impact of debt levels and debt maturity on inflation," *The Economic Journal*, 123, F164–F192.

——— (2016): "Long Term Government Bonds," Mimeo.

——— (2019): "Government debt management: The long and the short of it," *The Review of Economic Studies*, 86, 2554–2604.

Galí, J. (2015): *Monetary policy, inflation, and the business cycle: an introduction to the new Keynesian framework and its applications*, Princeton University Press.

Giannoni, M. P. (2014): "Optimal interest-rate rules and inflation stabilization versus price-level stabilization," *Journal of Economic Dynamics and Control*, 41, 110–129.

Giannoni, M. P. and M. Woodford (2003): "Optimal interest-rate rules: I. General theory," Tech. rep., National Bureau of Economic Research.

Holden, T. (2024): "Robust real rate rules," *Econometrica*, (forthcoming).

Jarociński, M. and B. Maćkowiak (2018): "Monetary-fiscal interactions and the euro area's malaise," *Journal of International Economics*, 112, 251–266.

Kirsanova, T. and S. Wren-Lewis (2012): "Optimal fiscal feedback on debt in an economy with nominal rigidities," *The Economic Journal*, 122, 238–264.

Kumhof, M., R. Nunes, and I. Yakadina (2010): "Simple monetary rules under fiscal dominance," *Journal of Money, Credit and Banking*, 42, 63–92.

Leeper, E. M. (1991): "Equilibria under 'active'and 'passive'monetary and fiscal policies," *Journal of monetary Economics*, 27, 129–147.

Leeper, E. M. and C. Leith (2016): "Understanding inflation as a joint monetary–fiscal phenomenon," in *Handbook of Macroeconomics*, Elsevier, vol. 2, 2305–2415.

Leeper, E. M., N. Traum, and T. B. Walker (2017): "Clearing up the fiscal multiplier morass," *American Economic Review*, 107, 2409–54.

Leeper, E. M. and X. Zhou (2021): "Inflation's role in optimal monetary-fiscal policy," *Journal of Monetary Economics*, 124, 1–18.

Lucas, R. E. and N. L. Stokey (1983): "Optimal fiscal and monetary policy in an economy without capital," *Journal of monetary Economics*, 12, 55–93.

Lustig, H., C. Sleet, and Ş. Yeltekin (2008): "Fiscal hedging with nominal assets," *Journal of Monetary Economics*, 55, 710–727.

Marcet, A. and A. Scott (2009): "Debt and deficit fluctuations and the structure of bond markets," *Journal of Economic Theory*, 144, 473–501.

McKay, A. and C. Wolf (2022): "Optimal policy rules in hank," *mimeo*.

Orphanides, A. and J. C. Williams (2007): "Robust monetary policy with imperfect knowledge," *Journal of monetary Economics*, 54, 1406–1435.

Reis, R. (2016): "Funding quantitative easing to target inflation," .

Rotemberg, J. J. (1982): "Sticky prices in the United States," *Journal of Political Economy*, 90, 1187–1211.

Sargent, T. J., N. Wallace, et al. (1981): "Some unpleasant monetarist arithmetic," *Federal reserve bank of minneapolis quarterly review*, 5, 1–17.

Schmitt-Grohé, S. and M. Uribe (2000): "Price level determinacy and monetary policy under a balanced-budget requirement," *Journal of Monetary Economics*, 45, 211–246.

——— (2004): "Optimal fiscal and monetary policy under sticky prices," *Journal of economic Theory*, 114, 198–230.

——— (2007): "Optimal simple and implementable monetary and fiscal rules," *Journal of monetary Economics*, 54, 1702–1725.

Sims, C. A. (1994): "A simple model for study of the determination of the price level and the interaction of monetary and fiscal policy," *Economic theory*, 4, 381–399.

——— (2013): "Paper money," *American Economic Review*, 103, 563–84.

Siu, H. E. (2004): "Optimal fiscal and monetary policy with sticky prices," *Journal of Monetary Economics*, 51, 575–607.

Smets, F. and R. Wouters (2007): "Shocks and frictions in US business cycles: A Bayesian DSGE approach," *American economic review*, 97, 586–606.

Svensson, L. (1999): "Price level targeting versus inflation targeting: a free lunch?" *Journal of Money, Credit and Banking*, 31, 277–295.

Svensson, L. E. (2003): "What is wrong with Taylor rules? Using judgment in monetary policy through targeting rules," *Journal of Economic Literature*, 41, 426–477.

WOODFORD, M. (1994): "Monetary policy and price level determinacy in a cash-in-advance economy," *Economic theory*, 4, 345–380.

——— (1995): "Price-level determinacy without control of a monetary aggregate," in *Carnegie-Rochester conference series on public policy*, Elsevier, vol. 43, 1–46.

——— (2001a): "Fiscal requirements for price stability," .

——— (2001b): "The Taylor rule and optimal monetary policy," *American Economic Review*, 91, 232–237.

——— (2003a): "Interest and prices," .

——— (2003b): "Optimal interest-rate smoothing," *The Review of Economic Studies*, 70, 861–886.

Online Appendix

A Proofs of Propositions and Derivations in Sections 3 and 4

We provide proofs for the Propositions 3 and 4. We also derive the analytical formulae shown in Section 4 of the paper (the impulse responses of inflation).

A.1 Proof of Proposition 3 and derivations for Case 1: $\lambda_Y > 0$, $\sigma = 0$.

The FONC for inflation and output combined give us the following condition:

$$-\hat{\pi}_t - \frac{\lambda_Y}{\kappa_1}\Delta\hat{Y}_t + \frac{\bar{b}}{1-\beta\delta}\sum_{k=0}^{\infty}\delta^k\Delta\psi_{gov,t-l} = 0$$

Using the Phillips curve, we can write:

$$-\hat{\pi}_t - \frac{\lambda_Y}{\kappa_1^2}(\hat{\pi}_t - \beta E_t\hat{\pi}_{t+1}) + \frac{\lambda_Y}{\kappa_1^2}\left(\hat{\pi}_{t-1} - \beta E_{t-1}\hat{\pi}_t\right) + \frac{\bar{b}}{1-\beta\delta}\sum_{k=0}^{\infty}\delta^k\Delta\psi_{gov,t-l} = 0$$

Define:

$$\zeta_t \equiv \left(\hat{\pi}_t - E_{t-1}\hat{\pi}_t\right) + \frac{\kappa_1^2}{\beta\lambda_Y}\frac{\bar{b}}{1-\beta\delta}\sum_{k=0}^{\infty}\delta^k\Delta\psi_{gov,t-l} = 0$$

Then, inflation evolves according to:

$$E_t\hat{\pi}_{t+1} - \left(1 + \frac{1}{\beta} + \frac{\kappa_1^2}{\lambda_Y\beta}\right)\hat{\pi}_t + \frac{1}{\beta}\hat{\pi}_{t-1} = -\zeta_t \tag{36}$$

We will now resolve the above difference equation. Letting $\widetilde{\kappa} = \frac{\kappa_1^2}{\lambda_Y\beta}$, the characteristic polynomial is $\lambda^2 - (1 + \frac{1}{\beta} + \widetilde{\kappa})\lambda + \frac{1}{\beta}$. The two roots are:

$$\widetilde{\lambda}_{1,2} = \frac{1}{2}\left((1 + \frac{1}{\beta} + \widetilde{\kappa}) \pm \sqrt{(1 + \frac{1}{\beta} + \widetilde{\kappa})^2 - \frac{\widetilde{4}}{\beta}}\right)$$

It is simple to show that one root is stable and one unstable. Let $\widetilde{\lambda}_1$ denote the stable root. (43) can be written as:

$$\hat{\pi}_t = \frac{1}{\widetilde{\lambda}_2}E_t\hat{\pi}_{t+1} + \frac{1}{\widetilde{\lambda}_2}\frac{1}{1-\widetilde{\lambda}_1 L}\zeta_t = \frac{1}{\widetilde{\lambda}_2}\frac{1}{1-\widetilde{\lambda}_1 L}\sum_{j\geq 0}\frac{1}{\widetilde{\lambda}_2^j}E_t\zeta_{t+j} \tag{37}$$

(for the usual boundary condition that inflation does not explode).

Let us compute the term

$$\sum_{j\geq 0}\frac{1}{\widetilde{\lambda}_2^j}E_t\zeta_{t+j} = \sum_{j\geq 0}\frac{1}{\widetilde{\lambda}_2^j}E_t\left[\left(\hat{\pi}_{t+j} - E_{t+j-1}\hat{\pi}_{t+j}\right) + \widetilde{\kappa}\frac{\bar{b}}{1-\beta\delta}\sum_{k=0}^{\infty}\delta^k\Delta\psi_{gov,t+j-l}\right]$$

When $\Delta\psi_{gov,t} \neq 0$ (in an equilibrium with active fiscal policy), the final term on the RHS is

$$\widetilde{\kappa}\frac{\bar{b}}{1-\beta\delta}\sum_{j\geq 0}\frac{1}{\widetilde{\lambda}_2^j}E_t\left[\sum_{k=0}^{\infty}\delta^k\Delta\psi_{gov,t+j-l}\right] = \widetilde{\kappa}\frac{\bar{b}}{1-\beta\delta}\frac{1}{1-\frac{\delta}{\widetilde{\lambda}_2}}\frac{1}{1-\delta L}\Delta\psi_{gov,t}$$

(this follows from the random walk property of the multiplier). Moreover, it clearly holds that:

$$\sum_{j\geq 0}\frac{1}{\widetilde{\lambda}_2^j}E_t\Big(\hat{\pi}_{t+j} - E_{t+j-1}\hat{\pi}_{t+j}\Big) = \hat{\pi}_t - E_{t-1}\hat{\pi}_t$$

Putting everything together and using (44) we have:

$$\hat{\pi}_t = \widetilde{\lambda}_1\hat{\pi}_{t-1} + \frac{1}{\widetilde{\lambda}_2}(\hat{\pi}_t - E_{t-1}\hat{\pi}_t) + \frac{\widetilde{\kappa}}{\widetilde{\lambda}_2}\frac{\bar{b}}{1-\beta\delta}\frac{1}{1-\frac{\delta}{\widetilde{\lambda}_2}}\frac{1}{1-\delta L}\Delta\psi_{gov,t} \tag{38}$$

A.1.1 Proof of Proposition 3.

To derive the interest rate rules use (38). Consider first the the case where $\Delta\psi_{gov,t} = 0$ (passive fiscal policy).

Then,

$$E_t\hat{\pi}_{t+1} = \widetilde{\lambda}_1\hat{\pi}_t + \frac{1}{\widetilde{\lambda}_2}E_t\Big(\hat{\pi}_{t+1} - E\pi_{t+1}\Big) = \widetilde{\lambda}_1\hat{\pi}_t$$

and clearly $E_{t-1}\hat{\pi}_t = \widetilde{\lambda}_1\hat{\pi}_{t-1}$. Then since $\zeta_t = 0$ from (36) optimal inflation solves

$$\hat{\pi}_{t+1} - \Big(\widetilde{\lambda}_1 + \widetilde{\lambda}_2\Big)\hat{\pi}_t + \widetilde{\lambda}_1\widetilde{\lambda}_2\hat{\pi}_{t-1} = 0 \tag{39}$$

(expectations can be dropped because inflation is clearly not random). The unique solution is $\hat{\pi}_t = 0$ for all t. Standard arguments imply uniqueness of the equilibrium when:

$$\hat{i}_t = \hat{\xi}_t + \Big(\widetilde{\lambda}_1 + \widetilde{\lambda}_2\Big)\hat{\pi}_t - \widetilde{\lambda}_1\widetilde{\lambda}_2\hat{\pi}_{t-1}$$

Now consider the case where $\Delta\psi_{gov,t} \neq 0$. From (38) we have

$$E_t\hat{\pi}_{t+1} = \widetilde{\lambda}_1\hat{\pi}_t + \frac{\widetilde{\kappa}}{\widetilde{\lambda}_2}\frac{\bar{b}}{1-\beta\delta}\frac{1}{1-\frac{\delta}{\widetilde{\lambda}_2}}\underbrace{E_t\frac{1}{1-\delta L}\Delta\psi_{gov,t+1}}_{=\frac{\delta}{1-\delta L}\Delta\psi_{gov,t}}$$

and also

$$E_{t-1}\hat{\pi}_t = \widetilde{\lambda}_1\hat{\pi}_{t-1} + \frac{\widetilde{\kappa}}{\widetilde{\lambda}_2}\frac{\bar{b}}{1-\beta\delta}\frac{1}{1-\frac{\delta}{\widetilde{\lambda}_2}}\frac{\delta}{1-\delta L}\Delta\psi_{gov,t-1}$$

We thus get

$$\hat{\pi}_t - E_{t-1}\hat{\pi}_t = \frac{1}{\widetilde{\lambda}_2}\left(\hat{\pi}_t - E_{t-1}\hat{\pi}_t\right) + \frac{\widetilde{\kappa}}{\widetilde{\lambda}_2}\frac{\overline{b}}{1-\beta\delta}\frac{1}{1-\frac{\delta}{\widetilde{\lambda}_2}}\Delta\psi_{gov,t}$$

$$\rightarrow \hat{\pi}_t - E_{t-1}\hat{\pi}_t = \frac{\widetilde{\lambda}_2}{\widetilde{\lambda}_2 - 1}\frac{\widetilde{\kappa}}{\lambda_2}\frac{\overline{b}}{1-\beta\delta}\frac{1}{1-\frac{\delta}{\widetilde{\lambda}_2}}\Delta\psi_{gov,t}$$

It is now easy to derive the optimal interest rate rule. Let $\widetilde{o} = \frac{\widetilde{\kappa}}{\lambda_2}\frac{\overline{b}}{1-\beta\delta}\frac{1}{1-\frac{\delta}{\widetilde{\lambda}_2}}$

$$\hat{i}_t = \hat{\xi}_t + E_t\hat{\pi}_{t+1} = \hat{\xi}_t + \widetilde{\lambda}_1\hat{\pi}_t + \widetilde{o}\frac{\delta}{1-\delta L}\Delta\psi_{gov,t} = \hat{\xi}_t + \widetilde{\lambda}_1\hat{\pi}_t + \delta\left(\hat{\pi}_t - \widetilde{\lambda}_1\hat{\pi}_{t-1} - \frac{\widetilde{o}}{\widetilde{\lambda}_2 - 1}\Delta\psi_{gov,t}\right)$$

■

A.1.2 Additional derivations: The path of optimal inflation.

We now provide additional derivations for the impulse responses shown in Section 4.

The case where fiscal policy is passive is trivial since inflation and output are at steady state in all periods. Consider the case where fiscal policy is active. Focus on the case a shock in either spending or preferences can hit the economy in t and all shocks before or after t are 0. Thus, $\Delta\psi_{gov,t} \neq 0$ but $\Delta\psi_{gov,t+j} = 0$ for $j \neq 0$. Moreover, let $\hat{b}_{t-1} = 0$. The intertemporal consolidated budget in this model can be written as:

$$-\overline{G}\hat{G}_t + (\overline{s} - \overline{b})\hat{\xi}_t = -\overline{b}\sum_{j\geq 0}(\beta\delta)^j\sum_{l=0}^{j}\hat{\pi}_{t+l}$$

From the above derivations we can show that:

$$\hat{\pi}_t = \widetilde{\lambda}_1\hat{\pi}_{t-1} + \frac{1}{\widetilde{\lambda}_2 - 1}\widetilde{o}\Delta\psi_{gov,t} + \widetilde{o}\Delta\psi_{gov,t}$$

(given $\Delta\psi_{gov,t-1} = \Delta\psi_{gov,t-2} = ...0$.) Also,

$$\hat{\pi}_{t+1} = \widetilde{\lambda}_1\hat{\pi}_t + \widetilde{o}\delta\Delta\psi_{gov,t} = \widetilde{\lambda}_1\hat{\pi}_{t-1} + \widetilde{o}(\delta + \widetilde{\lambda}_1)\Delta\psi_{gov,t} + \widetilde{o}\frac{\widetilde{\lambda}_1}{\widetilde{\lambda}_2 - 1}\Delta\psi_{gov,t}$$

$$...$$

$$\hat{\pi}_{t+l} = \widetilde{\lambda}_1^{l+1}\hat{\pi}_{t-1} + \widetilde{o}\left(\delta^l + \delta^{l-1}\widetilde{\lambda}_1 + \delta^{l-2}\widetilde{\lambda}_1^2 + ... + \delta\widetilde{\lambda}_1^{l-1} + \widetilde{\lambda}^l\right)\Delta\psi_{gov,t} + \widetilde{o}\frac{\widetilde{\lambda}_1^l}{\widetilde{\lambda}_2 - 1}\Delta\psi_{gov,t}$$

Noting that $\left(\delta^l + \delta^{l-1}\widetilde{\lambda}_1 + \delta^{l-2}\widetilde{\lambda}_1^2 + ... + \delta\widetilde{\lambda}_1^{l-1} + \widetilde{\lambda}_1^l\right) = \frac{\delta^{l+1} - \widetilde{\lambda}_1^{l+1}}{\delta - \widetilde{\lambda}_1}$ (in the more plausible case

$\widetilde{\lambda}_1 \neq \delta$) and also by assumption $\hat{\pi}_{t-1} = 0$, we can write the consolidated budget constraint as:

$$-\overline{G}\hat{G}_t + (\overline{s} - \overline{b})\hat{\xi}_t = -\overline{b}\widetilde{o}\Delta\psi_{gov,t} \sum_{j\geq 0} (\beta\delta)^j \sum_{l=0}^{j} \left(\frac{\widetilde{\lambda}_1^l}{\widetilde{\lambda}_2 - 1} + \frac{\delta^{l+1} - \widetilde{\lambda}_1^{l+1}}{\delta - \widetilde{\lambda}_1} \right)$$

The final result is:

$$\Delta\psi_{gov,t} = (1 - \beta\delta)(1 - \beta\delta\widetilde{\lambda}_1) \frac{\overline{G}\hat{G}_t + (\overline{b} - \overline{s})\hat{\xi}_t}{\overline{b}\widetilde{o}(\frac{1}{1-\beta\delta^2} + \frac{1}{\widetilde{\lambda}_2 - 1})}$$

Now use the above derivations to derive the impulse responses. We have:

$$\hat{\pi}_t = \frac{\widetilde{\lambda}_2}{\widetilde{\lambda}_2 - 1} \widetilde{o}\Delta\psi_{gov,t}$$

$$\hat{\pi}_{t+j} = \widetilde{o}\frac{\delta^{j+1} - \widetilde{\lambda}_1^{j+1}}{\delta - \widetilde{\lambda}_1} \Delta\psi_{gov,t} + \widetilde{o}\frac{\widetilde{\lambda}_1^j}{\widetilde{\lambda}_2 - 1}\Delta\psi_{gov,t}, \quad j \geq 1$$

The case $\lambda_Y = 0$. Let us calculate the limit when $\lambda_Y = 0$. In this case we have $\widetilde{\kappa} \to \infty$ and it is easy to show that $\widetilde{\lambda}_1 \to 0$ and $\widetilde{\lambda}_2 \to \infty$. Also: $\lim_{\lambda_Y \to 0} \widetilde{o} = \lim_{\lambda_Y \to 0} \frac{\widetilde{\kappa}}{\widetilde{\lambda}_2} \frac{\overline{b}}{1-\beta\delta} \frac{1}{1-\frac{\delta}{\widetilde{\lambda}_2}} = \frac{\overline{b}}{1-\beta\delta}$. We thus have:

$$\hat{\pi}_{t+j} = \frac{\overline{b}}{1 - \beta\delta}\delta^j\Delta\psi_{gov,t}, \quad j \geq 0$$

and

$$\Delta\psi_{gov,t} = (1 - \beta\delta)^2(1 - \beta\delta^2)\frac{\overline{G}\hat{G}_t + (\overline{b} - \overline{s})\hat{\xi}_t}{\overline{b}^2}$$

A.2 Proof of Proposition 4 and derivations for Case 2: $\sigma > 0$ $\lambda_Y = 0$.

We first derive the optimal rules in the case where $\overline{G} = \kappa_2 = 0$ The FONC for inflation are:

$$\hat{\pi}_t = \frac{\overline{b}}{1 - \beta\delta} \sum_{l=0}^{\infty} \delta^l \Delta\psi_{gov,t-l} + \sigma\frac{\overline{Y}}{\overline{C}\kappa_1}\overline{b}\sum_{l=0}^{\infty}\delta^l \left(\Delta\psi_{gov,t-l} - \Delta\psi_{gov,t-l-1} \right) + \frac{\omega_Y}{\kappa_1}\Delta\psi_{gov,t} \tag{40}$$

To find the optimal interest rate rule in the case where fiscal policy is active we combined the Euler equation and the Phillips curve:

$$\hat{i}_t = \sigma\left(E_t\hat{Y}_{t+1} - \hat{Y}_t\right) + E_t\hat{\pi}_{t+1} + \hat{\xi}_t = \frac{\sigma}{\kappa_1}E_t\left(\hat{\pi}_{t+1} - \beta\hat{\pi}_{t+2} - \hat{\pi}_t + \beta\hat{\pi}_{t+1}\right) + E_t\hat{\pi}_{t+1} + \hat{\xi}_t$$

We can now derive $E_t\hat{\pi}_{t+1}$ as follows:

$$E_t\hat{\pi}_{t+1} = \frac{\bar{b}\delta}{1-\beta\delta}\sum_{l=0}^{\infty}\delta^l\Delta\psi_{gov,t-l} + \delta\frac{\sigma}{\kappa_1}\bar{b}\sum_{l=0}^{\infty}\delta^l\Big(\Delta\psi_{gov,t-l} - \Delta\psi_{gov,t-l-1}\Big) - \frac{\sigma}{\kappa_1}\bar{b}\Delta\psi_{gov,t} =$$
$$\delta\hat{\pi}_t - \delta\frac{\omega_Y}{\kappa_1}\Delta\psi_{gov,t} - \frac{\sigma}{\kappa_1}\bar{b}\Delta\psi_{gov,t}$$

Moreover, it is simple to show that

$$E_t\hat{\pi}_{t+2} = \delta^2\hat{\pi}_t - \delta^2\frac{\omega_Y}{\kappa_1}\Delta\psi_{gov,t} - \delta\frac{\sigma}{\kappa_1}\bar{b}\Delta\psi_{gov,t}$$

Making use of this we get:

$$\hat{i}_t = \hat{\xi}_t + \Big(\delta + \frac{\sigma}{\kappa_1}(1-\delta)(\delta\beta-1)\Big)\hat{\pi}_t - \underbrace{(\delta\frac{\omega_Y}{\kappa_1} + \frac{\sigma}{\kappa_1}\bar{b})(1+\frac{\sigma}{\kappa_1}+\beta\frac{\sigma}{\kappa_1}(1-\delta))}_{:=\widetilde{\omega}}\Delta\psi_{gov,t}$$

Now let us turn to the passive fiscal policy case. Since the shock is a demand shock, we have that $\hat{\pi}_t = \hat{Y}_t = 0$ (divine coincidence). The interest rate rule that can implement this outcome is $\hat{i}_t = \hat{\xi}_t + \phi_\pi\hat{\pi}_t$ where $\phi_\pi > 1$.

A.2.1 Proof of Proposition 4: Optimal interest rate rules when $\overline{G} > 0$.

We now derive the optimal rule when $\overline{G} > 0$.

$$\hat{i}_t = \sigma\frac{\overline{Y}}{\overline{C}}\Big(E_t\hat{Y}_{t+1} - \hat{Y}_t\Big) + E_t\hat{\pi}_{t+1} + \hat{\xi}_t + \sigma\frac{\overline{G}}{\overline{C}}\hat{G}_t$$
$$= \frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}E_t\Big(\hat{\pi}_{t+1} - \beta\hat{\pi}_{t+2} - \hat{\pi}_t + \beta\hat{\pi}_{t+1}\Big) + E_t\hat{\pi}_{t+1} + \hat{\xi}_t + \sigma\frac{\overline{G}}{\overline{C}}\hat{G}_t - \frac{\sigma\kappa_2}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\hat{G}_t$$

Moreover,

$$E_t\hat{\pi}_{t+1} = \frac{\bar{b}\delta}{1-\beta\delta}\sum_{l=0}^{\infty}\delta^l\Delta\psi_{gov,t-l} + \delta\frac{\overline{Y}}{\overline{C}}\frac{\sigma}{\kappa_1}\bar{b}\sum_{l=0}^{\infty}\delta^l\Big(\Delta\psi_{gov,t-l} - \Delta\psi_{gov,t-l-1}\Big) - \frac{\overline{Y}}{\overline{C}}\frac{\sigma}{\kappa_1}\bar{b}\Delta\psi_{gov,t} =$$
$$\delta\hat{\pi}_t - \delta\frac{\omega_Y}{\kappa_1}\Delta\psi_{gov,t} - \frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\bar{b}\Delta\psi_{gov,t}$$

and

$$E_t\hat{\pi}_{t+2} = \delta^2\hat{\pi}_t - \delta^2\frac{\omega_Y}{\kappa_1}\Delta\psi_{gov,t} - \delta\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\bar{b}\Delta\psi_{gov,t}$$

We therefore get:

$$\hat{i}_t = \hat{\xi}_t + \sigma\frac{\overline{G}}{\overline{C}}\hat{G}_t - \frac{\sigma\kappa_2}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\hat{G}_t + \Big(\delta + \frac{\overline{Y}}{\overline{C}}\frac{\sigma}{\kappa_1}(1-\delta)(\delta\beta-1)\Big)\hat{\pi}_t - f\Delta\psi_{gov,t} \tag{41}$$

where $f := \Big(1 + \frac{\overline{Y}}{\overline{C}}\frac{\sigma}{\kappa_1}(1+\beta-\beta\delta)\Big)\Big(\delta\frac{\omega_Y}{\kappa_1} + \frac{\overline{Y}}{\overline{C}}\frac{\sigma}{\kappa_1}\bar{b}\Big)$ ■

A.2.2 The optimal path of inflation

We now derive the optimal inflation path for this model shown in Section 4. The LHS of the consolidated budget constraint under the parameter values assumed here, is:

$$\sum_{j\geq 0}\beta^j\overline{s}\hat{S}_{t+j} = \sum_{j\geq 0}\beta^j\left[-\overline{G}\hat{G}_{t+j} + (\overline{\tau}-\overline{G})\sigma\frac{\overline{G}}{\overline{C}}\hat{G}_{t+j} - (\overline{\tau}-\overline{G})\sigma\frac{\overline{Y}}{\overline{C}}\hat{Y}_{t+j} + (\overline{\tau}-\overline{G})\hat{\xi}_{t+j}\right]$$

$$= -\overline{G}\hat{G}_t + (\overline{\tau}-\overline{G})\sigma\frac{\overline{G}}{\overline{C}}\hat{G}_t + (\overline{\tau}-\overline{G})\hat{\xi}_t - \sum_{j\geq 0}\beta^j\left[(\overline{\tau}-\overline{G})\sigma\frac{\overline{Y}}{\overline{C}}\hat{Y}_{t+j}\right]$$

Consider the last term. We have:

$$-(\overline{\tau}-\overline{G})\sigma\frac{\overline{Y}}{\overline{C}}\sum_{j\geq 0}\beta^j\hat{Y}_{t+j} = -(\overline{\tau}-\overline{G})\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\sum_{j\geq 0}\beta^j\left[\hat{\pi}_{t+j} + \kappa_2\hat{G}_{t+j} - \beta\hat{\pi}_{t+j+1}\right]$$

$$= -(\overline{\tau}-\overline{G})\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\left(\hat{\pi}_t + \kappa_2\hat{G}_t\right)$$

We therefore get:

$$\sum_{j\geq 0}\beta^j\overline{s}\hat{S}_{t+j} = -\overline{G}\hat{G}_t + (\overline{\tau}-\overline{G})\sigma\frac{\overline{G}}{\overline{C}}\hat{G}_t + (\overline{\tau}-\overline{G})\hat{\xi}_t - (\overline{\tau}-\overline{G})\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\left(\hat{\pi}_t + \kappa_2\hat{G}_t\right)$$

The RHS of the intertemporal constraint is:

$$\bar{b}\sum_{j\geq 0}(\beta\delta)^j\left(-\sigma(\frac{\overline{Y}}{\overline{C}}\hat{Y}_{t+j} - \frac{\overline{G}}{\overline{C}}\hat{G}_{t+j}) - \sum_{l=0}^{j}\hat{\pi}_{t+l}\right) + \bar{b}\hat{\xi}_t$$

Let us separately derive each of the components. The first is:

$$\bar{b}\sum_{j\geq 0}(\beta\delta)^j\left(-\sigma(\frac{\overline{Y}}{\overline{C}}\hat{Y}_{t+j} - \frac{\overline{G}}{\overline{C}}\hat{G}_{t+j})\right)$$

$$= -\sigma\bar{b}\sum_{j\geq 0}(\beta\delta)^j\left(\frac{\overline{Y}}{\overline{C}}\hat{Y}_{t+j}\right) + \sigma\bar{b}\frac{\overline{G}}{\overline{C}}\hat{G}_t - \frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\bar{b}\sum_{j\geq 0}(\beta\delta)^j\left(\hat{\pi}_{t+j} + \kappa_2\hat{G}_{t+j} - \beta\hat{\pi}_{t+j+1}\right) + \sigma\bar{b}\frac{\overline{G}}{\overline{C}}\hat{G}_t$$

$$= -\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\bar{b}\sum_{j\geq 0}(\beta\delta)^j\left(\hat{\pi}_{t+j} - \beta\hat{\pi}_{t+j+1}\right) + \sigma\bar{b}\hat{G}_t\left(\frac{\overline{G}}{\overline{C}} - \frac{\kappa_2}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\right)$$

We can now substitute out the term $\hat{\pi}_{t+j} - \beta\hat{\pi}_{t+j+1}$ using formula (29) in text. Let $\nu = \frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}(\bar{b} - (\overline{\tau}-\overline{G}))$ we can write (29) as:

$$\hat{\pi}_{t+j} - \beta\hat{\pi}_{t+j+1} = \left\{\begin{array}{ll} \left[\frac{\bar{b}}{1-\beta\delta}(1-\beta\delta) + \nu - \beta\frac{\sigma}{\kappa_1}(1-\delta)\frac{\overline{Y}}{\overline{C}}\bar{b}\right]\Delta\psi_{gov,t} & j=0 \\ \left[\frac{\bar{b}}{1-\beta\delta}\delta^j - \frac{\sigma}{\kappa_1}(1-\delta)\delta^{j-1}\frac{\overline{Y}}{\overline{C}}\bar{b}\right](1-\beta\delta)\Delta\psi_{gov,t} & j\geq 1 \end{array}\right\}$$

Making use of this result we can write:

$$-\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\bar{b}\sum_{j\geq 0}(\beta\delta)^j\left(\hat{\pi}_{t+j}-\beta\hat{\pi}_{t+j+1}\right) = -\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\bar{b}\left[\frac{\bar{b}}{1-\beta\delta}(1-\beta\delta)+\nu-\beta\frac{\sigma}{\kappa_1}(1-\delta)\frac{\overline{Y}}{\overline{C}}\bar{b}\right]\Delta\psi_{gov,t}$$

$$-\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\bar{b}(1-\beta\delta)\Delta\psi_{gov,t}\sum_{j\geq 1}(\beta\delta)^j\left[\frac{\bar{b}}{1-\beta\delta}\delta^j-\frac{\sigma}{\kappa_1}(1-\delta)\delta^{j-1}\frac{\overline{Y}}{\overline{C}}\bar{b}\right]=$$

$$-\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\bar{b}\Delta\psi_{gov,t}\Bigg\{(1-\beta\delta)\left[\frac{\bar{b}}{1-\beta\delta}\frac{\beta\delta^2}{1-\beta\delta^2}-\frac{\sigma}{\kappa_1}(1-\delta)\frac{\overline{Y}}{\overline{C}}\bar{b}\frac{\beta\delta}{1-\beta\delta^2}\right]$$

$$+\left[\frac{\bar{b}}{1-\beta\delta}(1-\beta\delta)+\nu-\beta\frac{\sigma}{\kappa_1}(1-\delta)\frac{\overline{Y}}{\overline{C}}\bar{b}\right]\Bigg\}$$

$$=-\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\bar{b}\Delta\psi_{gov,t}\left\{\left[\frac{\bar{b}}{1-\beta\delta^2}+\nu-\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\bar{b}(1-\delta)\left((1-\beta\delta)\frac{\beta\delta}{1-\beta\delta^2}+\beta\right)\right]\right\}$$

Consider now the term:

$$-\bar{b}\sum_{j\geq 0}(\beta\delta)^j\left(\sum_{l=0}^{j}\hat{\pi}_{t+l}\right) = \left[-\frac{\bar{b}}{1-\beta\delta}\nu-\frac{\bar{b}^2}{1-\beta\delta}\sum_{j\geq 0}(\beta\delta)^j\frac{1-\delta^{j+1}}{1-\delta}+\frac{\sigma}{\kappa_1}(1-\delta)\frac{\overline{Y}}{\overline{C}}\bar{b}^2\sum_{j\geq 1}(\beta\delta)^j\frac{1-\delta^j}{1-\delta}\right]\Delta\psi_{gov,t}$$

$$=-\left[\frac{\bar{b}}{1-\beta\delta}\nu+\frac{\bar{b}^2}{(1-\beta\delta)^2(1-\beta\delta^2)}-\frac{\sigma}{\kappa_1}(1-\delta)\frac{\overline{Y}}{\overline{C}}\frac{\bar{b}^2(\beta\delta)^2}{(1-\beta\delta)(1-\beta\delta^2)}\right]\Delta\psi_{gov,t}$$

We can thus write the intertemporal constraint as:

$$-\left[\mu_1+\mu_2-\bar{s}\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\left(\frac{\bar{b}}{1-\beta\delta}+\nu\right)\right]\Delta\psi_{gov,t} = -\overline{G}\hat{G}_t+\left(\bar{s}-\bar{b}\right)\sigma\left(\frac{\overline{G}}{\overline{C}}-\frac{\kappa_2}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\right)\hat{G}_t+\left(\bar{s}-\bar{b}\right)\hat{\xi}_t \tag{42}$$

where $$\mu_1=\left[\frac{\bar{b}}{1-\beta\delta}\nu+\frac{\bar{b}^2}{(1-\beta\delta)^2(1-\beta\delta^2)}-\frac{\sigma}{\kappa_1}\left(1-\delta\right)\frac{\overline{Y}}{\overline{C}}\frac{\bar{b}^2(\beta\delta)^2}{(1-\beta\delta)(1-\beta\delta^2)}\right]$$

$$\mu_2=\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\bar{b}\left[\frac{\bar{b}}{1-\beta\delta^2}+\nu-\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\bar{b}\left(1-\delta\right)\left((1-\beta\delta)\frac{\beta\delta}{1-\beta\delta^2}+\beta\right)\right]$$

and $\bar{s}=\bar{\tau}-\overline{G}$.

Finally, consider the $\hat{G}_t$ terms in (42). We have $\frac{\kappa_2}{\kappa_1}=\frac{\sigma\frac{\overline{G}}{\overline{C}}}{(\gamma_h+\sigma\frac{\overline{Y}}{\overline{C}})}$ and so:

$$-\overline{G}\hat{G}_t+\left(\bar{s}-\bar{b}\right)\sigma\left(\frac{\overline{G}}{\overline{C}}-\frac{\kappa_2}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\right)\hat{G}_t=-\overline{G}\hat{G}_t+\left(\bar{s}-\bar{b}\right)\sigma\frac{\overline{G}}{\overline{C}}\left(1-\frac{1}{(\gamma_h+\sigma\frac{\overline{Y}}{\overline{C}})}\frac{\overline{Y}}{\overline{C}}\right)\hat{G}_t$$

B The nonlinear model

We present here the nonlinear equations of our baseline New Keynesian model.

Households Households maximize

$$E_0\sum_{t=0}^{\infty}\beta^t\xi_t\left(\frac{C_t^{1-\sigma}}{1-\sigma}-\chi\frac{h_t^{1+\gamma_h}}{1+\gamma_h}\right)$$

subject to

$$P_tC_t + P_{t,s}B_{t,S} + P_{t,L}B_{t,\delta} \leqslant (1-\tau_t)W_th_t - P_tT_t + P_tD_t + B_{t-1,S} + (1+\delta P_{t,L})B_{t-1,\delta}$$

where C_t denotes consumption and h_t denotes hours worked. D_t represents firms' profits redistributed to households, and P_t denotes the aggregate price level. $B_{t,\delta}$ is a long-term government bond, a perpetuity with coupon payments decaying at the rate $0 \leqslant \delta < 1$ and price $P_{t,L}$. $B_{t,s}$ denotes short-term bonds with price is $P_{t,S}$. We assume that short debt is in zero net supply. ξ_t is the preference shock. $0 \leq \tau_t \leq 1$ denotes the (distortionary) tax rate on labour, and T_t the level of lump-sum taxes.

The first order conditions of the household's problem are:

$$\begin{aligned} P_{t,s}\xi_tC_t^{-\sigma} &= \beta E_t\xi_{t+1}\frac{C_{t+1}^{-\sigma}}{\pi_{t+1}} \\ P_{t,L}\xi_tC_t^{-\sigma} &= \beta E_t\xi_{t+1}\frac{C_{t+1}^{-\sigma}}{\pi_{t+1}}(1+\delta P_{t+1,L}) \\ h_t^{\gamma_h}C_t^{\sigma} &= (1-\tau_t)\frac{W_t}{P_t} \end{aligned}$$

where $\pi_t \equiv \frac{P_t}{P_{t-1}}$ is the gross inflation rate.

Firms Production takes place in monopolistically competitive firms which operate technologies with labour as the sole input. The final good is a CES aggregate of the intermediate goods $Y_t(j)$:

$$Y_t = \Big(\int_0^1 Y_t(j)^{\frac{1+\eta}{\eta}}dj\Big)^{\frac{\eta}{1+\eta}}$$

where η governs the elasticity of substitution between differentiated goods. Firms set prices to maximize profits subject to the demand curve

$$Y_t(j) = \Big(\frac{P_t(j)}{P_t}\Big)^{\eta}Y_t$$

and given price adjustment costs, modelled as in Rotemberg (1982). The dynamic profit maximization program is:

$$\begin{aligned} \max_{P_t(j)} \quad & E_t\sum_{s=0}^{\infty}Q_{t,t+s}\Big(\frac{P_{t+s}(j)}{P_{t+s}}Y_{t+s}(j) - \frac{W_{t+s}(j)}{P_{t+s}}Y_{t+s}(j) - AC_{t+s}(j)\Big) \\ \text{s.t.} \quad & Y_{t+s}(j) = \Big(\frac{P_{t+s}(j)}{P_{t+s}}\Big)^{\eta}Y_{t+s} \\ & AC_{t+s}(j) = \frac{\theta}{2}\Big(\frac{P_{t+s}(j)}{P_{t+s-1}(j)} - \pi\Big)^2Y_{t+s} \end{aligned}$$

where $Q_{t,t+s} \equiv \beta^s$ is the discount factor of households and W_{t+s} is the wage rate, that is equal to the marginal cost of production. (43) is the quadratic adjustment costs incurred by firms.

Focusing on a symmetric equilibrium, the first order condition from the firm's dynamic program, gives us the following non-linear Phillips Curve:

$$\theta(\pi_t-\pi)\pi_t = 1 + \eta(1-\frac{W_t}{P_t}) + \beta\theta E_t\frac{C_t^{\sigma}}{C_{t+1}^{\sigma}}\frac{Y_{t+1}}{Y_t}(\pi_{t+1}-\pi)\pi_{t+1}$$

Fiscal policy Government spending G_t evolves exogenously according to an AR(1) process in logs:

$$\hat{G}_t = \rho_g \hat{G}_{t-1} + \epsilon_{g,t}$$

where $\hat{G}_t \equiv \log G_t - \log \overline{G}$.

The labor tax is set by the fiscal authority according to the simple rule described in text (in log-linear form). The flow government budget constraint can be written as:

$$P_{t,L} b_{t,\delta} = (1 + \delta P_{t,L}) \frac{b_{t-1,\delta}}{\pi_t} + G_t - T_t$$

where $b_{t,\delta} \equiv \frac{B_{t,\delta}}{P_t}$ denotes real long-term government debt.

Log-linearization Making use of the labor supply condition $h_t^{\gamma_h} C_t^{\sigma} = \frac{W_t}{P_t}$, as well as the resource constraint $h_t = Y_t = C_t + G_t + \int AC(j)dj$ to dispense with W_t, C_t and h_t, we get the following linear New Keynesian Phillips Curve:

$$\hat{\pi}_t = \kappa_1 \hat{Y}_t - \kappa_2 \hat{G}_t + \beta E_t \hat{\pi}_{t+1}$$

where κ_1, κ_2 were defined in text.

Defining $i_t \equiv -\log P_{t,S}$, log-linearizing the Euler equation for short bonds (and again making use of the resource constraint) we get:

$$\hat{i}_t = E_t\left(\hat{\pi}_{t+1} - \hat{\xi}_{t+1} + \hat{\xi}_t - \sigma \left[\frac{\overline{Y}}{\overline{C}} (\hat{Y}_t - \hat{Y}_{t+1}) - \frac{\overline{G}}{\overline{C}} (\hat{G}_t - \hat{G}_{t+1}) \right] \right)$$

Finally, making use of the Euler equation for long bonds and iterating forward and making use of the resource constraint we get $P_{t,L} = \sum_{j \geq 1} E_t \beta^j \delta^{j-1} \frac{\xi_{t+j}}{\xi_t} \frac{(Y_{t+j} - G^{-\sigma}_{t+j})}{(Y_t - G^{-\sigma}_{t)} \prod_{l=1}^{j} \pi_{t+l}}$. Using this to substitute out $P_{L,t}$ from the government budget constraint we obtain:

$$\sum_{j \geq 1} E_t \beta^j \delta^{j-1} \frac{\xi_{t+j}}{\xi_t} \frac{(Y_{t+j} - G_{t+j})^{-\sigma}}{(Y_t - G_t)^{-\sigma} \prod_{l=1}^{j} \pi_{t+l}} b_{t,\delta} =$$
$$\left(1 + \delta \sum_{j \geq 1} E_t \beta^j \delta^{j-1} \frac{\xi_{t+j}}{\xi_t} \frac{(Y_{t+j} - G_{t+j})^{-\sigma}}{(Y_t - G_t)^{-\sigma} \prod_{l=1}^{j} \pi_{t+l}} \right) \frac{b_{t-1,\delta}}{\pi_t} - S_t$$

where $S_t = \tau_t h_t w_t - G_t$ or $S_t = T_t - G_t$ depending on whether the fiscal instrument is the labour tax or the lump-sum tax. Log-linearizing this equation we get:

$$\frac{\beta \overline{b}}{1 - \beta\delta} \hat{b}_{t,\delta} + \overline{b} \sum_{j=1}^{\infty} \beta^j \delta^{j-1} \left[E_t \left(-\sigma \left(\frac{\overline{Y}}{\overline{C}} \hat{Y}_{t+j} - \frac{\overline{G}}{\overline{C}} \hat{G}_{t+j}\right) - \sum_{l=1}^{j} \hat{\pi}_{t+l} + \hat{\xi}_{t+j} \right) \right]$$
$$= -\overline{S} \hat{S}_t - \overline{b}\sigma \left(\frac{\overline{Y}}{\overline{C}} \hat{Y}_t - \frac{\overline{G}}{\overline{C}} \hat{G}_t \right) + \overline{b} \hat{\xi}_t$$
$$+ \frac{\overline{b}}{1 - \beta\delta} (\hat{b}_{t-1,\delta} - \hat{\pi}_t) + \delta \overline{b} \sum_{j=1}^{\infty} \beta^j \delta^{j-1} E_t \left(-\sigma \left(\frac{\overline{Y}}{\overline{C}} \hat{Y}_{t+j} - \frac{\overline{G}}{\overline{C}} \hat{G}_{t+j}\right) - \sum_{l=1}^{j} \hat{\pi}_{t+l} + \hat{\xi}_{t+j} \right)$$

which is the equation stated in the main text.

C Optimal policy with dual mandates/ microfounded loss function

In this subsection we present the following results: First, we derive the interest rate rule for a dual mandate objective function in the canonical New Keynesian model. This is Proposition 5 in the main text. Second, we derive a quadratic loss function based on a second order approximation of household utility. Third, we repeat our derivations for the dual mandate under the welfare based objective. Fourth, we explore the optimal policies in the case where supply side shocks can hit the economy. Finally, we show the IRFS for this model under Ramsey policy and under simpler inflation targeting rules.

C.1 Optimal interest rates in the canonical model with a dual mandate objective function/ Proof of Proposition 5

We first consider the case of targeting steady state output. Moreover, we assume that the NK Phillips curve is given by

$$\hat{\pi}_t = \kappa_1 \hat{Y}_t + \hat{\mu}_t + \beta E_t \hat{\pi}_{t+1}$$

The term $\hat{\mu}_t$ picks up all the shock terms that may appear in the Phillips curve. It can be interpreted as a cost push shock, or (following the specification of the baseline model in text) it may represent the term $-\kappa_2 \hat{G}_t$ (the income effect on labour supply induced by spending). We will derive our formula for the optimal interest rate using the notation $\hat{\mu}_t$. Then, it will be straightforward to add spending shocks explicitly.

The Ramsey optimality condition when $\sigma, \lambda_Y > 0$ is given by:

$$0 = \hat{\pi}_t + \frac{\lambda_Y \Delta \hat{Y}_t}{\kappa_1} - \frac{\sigma}{\kappa_1} \frac{\overline{Y}}{\overline{C}} \bar{b} \sum_{l=0}^{\infty} \delta^l \Big(\Delta\psi_{gov,t-l} - \Delta\psi_{gov,t-l-1} \Big) - \frac{\omega_Y}{\kappa_1} \Delta\psi_{gov,t} - \frac{\bar{b}}{(1-\beta\delta)} \sum_{l=0}^{\infty} \delta^l \Delta\psi_{gov,t-l}$$

Using the Phillips curve, we can write:

$$\begin{aligned} -\hat{\pi}_t - \frac{\lambda_Y}{\kappa_1^2} \Big(\hat{\pi}_t - \hat{\mu}_t - \beta E_t \hat{\pi}_{t+1} \Big) + \frac{\lambda_Y}{\kappa_1^2} \Big(\hat{\pi}_{t-1} - \hat{\mu}_{t-1} - \beta E_{t-1} \hat{\pi}_t \Big) \\ + \frac{\bar{b}}{1-\beta\delta} \sum_{k=0}^{\infty} \delta^k \Delta\psi_{gov,t-l} + \frac{\sigma}{\kappa_1} \frac{\overline{Y}}{\overline{C}} \bar{b} \sum_{l=0}^{\infty} \delta^l \Big(\Delta\psi_{gov,t-l} - \Delta\psi_{gov,t-l-1} \Big) + \frac{\omega_Y}{\kappa_1} \Delta\psi_{gov,t} = 0 \end{aligned}$$

Define:

$$\begin{aligned} \zeta_t \equiv \Big(\hat{\pi}_t - E_{t-1} \hat{\pi}_t \Big) + \frac{\kappa_1^2}{\beta \lambda_Y} \Bigg[\frac{\bar{b}}{1-\beta\delta} \sum_{k=0}^{\infty} \delta^k \Delta\psi_{gov,t-l} \\ + \frac{\sigma}{\kappa_1} \frac{\overline{Y}}{\overline{C}} \bar{b} \sum_{l=0}^{\infty} \delta^l \Big(\Delta\psi_{gov,t-l} - \Delta\psi_{gov,t-l-1} \Big) + \frac{\omega_Y}{\kappa_1} \Delta\psi_{gov,t} \Bigg] + \frac{1}{\beta} \Delta\hat{\mu}_t \end{aligned}$$

Then, inflation evolves according to:

$$E_t \hat{\pi}_{t+1} - \left(1 + \frac{1}{\beta} + \frac{\kappa_1^2}{\lambda_Y \beta} \right) \hat{\pi}_t + \frac{1}{\beta} \hat{\pi}_{t-1} = -\zeta_t \tag{43}$$

We will now resolve the above difference equation. Letting $\widetilde{\kappa} = \frac{\kappa_1^2}{\lambda_Y \beta}$, the characteristic polynomial is $\lambda^2 - (1 + \frac{1}{\beta} + \widetilde{\kappa})\lambda + \frac{1}{\beta}$. The two roots are:

$$\widetilde{\lambda}_{1,2} = \frac{1}{2}\left((1 + \frac{1}{\beta} + \widetilde{\kappa}) \pm \sqrt{(1 + \frac{1}{\beta} + \widetilde{\kappa})^2 - \widetilde{\frac{4}{\beta}}}\right)$$

It is simple to show that one root is stable and one unstable. Let $\widetilde{\lambda}_1$ denote the stable root. (43) can be written as:

$$\hat{\pi}_t = \frac{1}{\widetilde{\lambda}_2} E_t \hat{\pi}_{t+1} + \frac{1}{\widetilde{\lambda}_2} \frac{1}{1 - \widetilde{\lambda}_1 L} \zeta_t = \frac{1}{\widetilde{\lambda}_2} \frac{1}{1 - \widetilde{\lambda}_1 L} \sum_{j \geq 0} \frac{1}{\widetilde{\lambda}_2^j} E_t \zeta_{t+j} \tag{44}$$

(for the usual boundary condition that inflation does not explode).

Let us compute the term

$$\sum_{j \geq 0} \frac{1}{\widetilde{\lambda}_2^j} E_t \zeta_{t+j} = \sum_{j \geq 0} \frac{1}{\widetilde{\lambda}_2^j} E_t \Bigg[\Big(\hat{\pi}_{t+j} - E_{t+j-1} \hat{\pi}_{t+j} \Big) + \widetilde{\kappa} \frac{\bar{b}}{1 - \beta\delta} \sum_{k=0}^{\infty} \delta^k \Delta\psi_{gov,t+j-l}$$
$$+ \widetilde{\kappa} \frac{\sigma}{\kappa_1} \frac{\overline{Y}}{\overline{C}} \bar{b} \sum_{l=0}^{\infty} \delta^l \Big(\Delta\psi_{gov,t+j-l} - \Delta\psi_{gov,t+j-l-1} \Big) + \widetilde{\kappa} \frac{\omega_Y}{\kappa_1} \Delta\psi_{gov,t+j} \Bigg]$$

When $\Delta\psi_{gov,t} \neq 0$ (in an equilibrium with active fiscal policy), the second term on the RHS is

$$\widetilde{\kappa} \frac{\bar{b}}{1 - \beta\delta} \sum_{j \geq 0} \frac{1}{\widetilde{\lambda}_2^j} E_t \left[\sum_{k=0}^{\infty} \delta^k \Delta\psi_{gov,t+j-l} \right] = \widetilde{\kappa} \frac{\bar{b}}{1 - \beta\delta} \frac{1}{1 - \frac{\delta}{\widetilde{\lambda}_2}} \frac{1}{1 - \delta L} \Delta\psi_{gov,t}$$

(this follows from the random walk property of the multiplier). The final term is

$$\sum_{j \geq 0} \frac{1}{\widetilde{\lambda}_2^j} E_t \left[\widetilde{\kappa} \frac{\omega_Y}{\kappa_1} \Delta\psi_{gov,t+j} \right] = \widetilde{\kappa} \frac{\omega_Y}{\kappa_1} \Delta\psi_{gov,t}$$

and the third term is

$$\sum_{j \geq 0} \frac{1}{\widetilde{\lambda}_2^j} E_t \left[\widetilde{\kappa} \frac{\sigma}{\kappa_1} \frac{\overline{Y}}{\overline{C}} \bar{b} \sum_{l=0}^{\infty} \delta^l (\Delta\psi_{gov,t+j-l} - \Delta\psi_{gov,t+j-l-1}) \right] =$$
$$\widetilde{\kappa} \frac{\sigma}{\kappa_1} \frac{\overline{Y}}{\overline{C}} \bar{b} \left(\frac{1}{1 - \frac{\delta}{\widetilde{\lambda}_2}} \frac{1}{1 - \delta L} (\Delta\psi_{gov,t} - \Delta\psi_{gov,t-1}) + \frac{1}{\widetilde{\lambda}_2} \frac{1}{1 - \frac{\delta}{\widetilde{\lambda}_2}} \Delta\psi_{gov,t} \right)$$

Moreover, it clearly holds that:

$$\sum_{j \geq 0} \frac{1}{\widetilde{\lambda}_2^j} E_t \Big(\hat{\pi}_{t+j} - E_{t+j-1} \hat{\pi}_{t+j} \Big) = \hat{\pi}_t - E_{t-1} \hat{\pi}_t$$

Finally,

$$\sum_{j \geq 0} \frac{1}{\widetilde{\lambda}_2^j} E_t \frac{1}{\beta} \Delta\hat{\mu}_{t+j} = \frac{1}{\beta} \frac{1}{1 - \frac{\rho_\mu}{\widetilde{\lambda}_2}} (1 - \frac{1}{\widetilde{\lambda}_2}) \hat{\mu}_t - \frac{1}{\beta} \hat{\mu}_{t-1}$$

We can now use the above results to derive te optimal path for inflation in this model. Substituting into (44) we get:

$$\hat{\pi}_t = \widetilde{\lambda}_1 \hat{\pi}_{t-1} + \frac{1}{\widetilde{\lambda}_2}\Big(\hat{\pi}_t - E_{t-1}\hat{\pi}_t\Big) + \frac{\widetilde{\kappa}}{\widetilde{\lambda}_2}\frac{\bar{b}}{1-\beta\delta}\frac{1}{1-\frac{\delta}{\widetilde{\lambda}_2}}\frac{1}{1-\delta L}\Delta\psi_{gov,t} + \frac{\widetilde{\kappa}}{\widetilde{\lambda}_2}\frac{\omega_Y}{\kappa_1}\Delta\psi_{gov,t}$$

$$+\frac{\widetilde{\kappa}}{\widetilde{\lambda}_2}\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\bar{b}\Big(\frac{1}{1-\frac{\delta}{\widetilde{\lambda}_2}}\frac{1}{1-\delta L}(\Delta\psi_{gov,t} - \Delta\psi_{gov,t-1}) + \frac{1}{\widetilde{\lambda}_2}\frac{1}{1-\frac{\delta}{\widetilde{\lambda}_2}}\Delta\psi_{gov,t}\Big) + \frac{1}{\beta\widetilde{\lambda}_2}\frac{1}{1-\frac{\rho_\mu}{\widetilde{\lambda}_2}}(1-\frac{1}{\widetilde{\lambda}_2})\hat{\mu}_t - \frac{1}{\beta}\hat{\mu}_{t-1}$$

and obviously

$$(1-\frac{1}{\widetilde{\lambda}_2})(\hat{\pi}_t - E_{t-1}\hat{\pi}_t) = \frac{\widetilde{\kappa}}{\widetilde{\lambda}_2}\frac{\bar{b}}{1-\beta\delta}\frac{1}{1-\frac{\delta}{\widetilde{\lambda}_2}}\Delta\psi_{gov,t}+$$

$$\frac{\widetilde{\kappa}}{\widetilde{\lambda}_2}\frac{\omega_Y}{\kappa_1}\Delta\psi_{gov,t} + \frac{\widetilde{\kappa}}{\widetilde{\lambda}_2}\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\bar{b}\Big(\frac{1}{1-\frac{\delta}{\widetilde{\lambda}_2}}\Delta\psi_{gov,t} + \frac{1}{\widetilde{\lambda}_2}\frac{1}{1-\frac{\delta}{\widetilde{\lambda}_2}}\Delta\psi_{gov,t}\Big) + \frac{1}{\beta\widetilde{\lambda}_2}\frac{1}{1-\frac{\rho_\mu}{\widetilde{\lambda}_2}}(1-\frac{1}{\widetilde{\lambda}_2})(\hat{\mu}_t - \rho_\mu\hat{\mu}_{t-1}) \rightarrow$$

$$(\hat{\pi}_t - E_{t-1}\hat{\pi}_t) = \psi\Delta\psi_{gov,t} + \frac{1}{\beta\widetilde{\lambda}_2}\frac{1}{1-\frac{\rho_\mu}{\widetilde{\lambda}_2}}(\hat{\mu}_t - \rho_\mu\hat{\mu}_{t-1})$$

Inflation solution. Optimal inflation is given by:

$$\hat{\pi}_t = \widetilde{\lambda}_1 \hat{\pi}_{t-1} + \frac{1}{\widetilde{\lambda}_2}\psi\Delta\psi_{gov,t} + \frac{\widetilde{\kappa}}{\widetilde{\lambda}_2}\frac{\bar{b}}{1-\beta\delta}\frac{1}{1-\frac{\delta}{\widetilde{\lambda}_2}}\frac{1}{1-\delta L}\Delta\psi_{gov,t} + \frac{\widetilde{\kappa}}{\widetilde{\lambda}_2}\frac{\omega_Y}{\kappa_1}\Delta\psi_{gov,t}$$

$$+\frac{\widetilde{\kappa}}{\widetilde{\lambda}_2}\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\bar{b}\Big(\frac{1}{1-\frac{\delta}{\widetilde{\lambda}_2}}\frac{1}{1-\delta L}(\Delta\psi_{gov,t} - \Delta\psi_{gov,t-1}) + \frac{1}{\widetilde{\lambda}_2}\frac{1}{1-\frac{\delta}{\widetilde{\lambda}_2}}\Delta\psi_{gov,t}\Big) + \frac{1}{\beta\widetilde{\lambda}_2}\frac{1}{1-\frac{\rho_\mu}{\widetilde{\lambda}_2}}(\hat{\mu}_t - \hat{\mu}_{t-1}) \tag{45}$$

The solution reveals that inflation displays persistence $\widetilde{\lambda}_1$ reacts directly to shocks to the Phillips curve (the term $\frac{1}{\beta\widetilde{\lambda}_2}\frac{1}{1-\frac{\rho_\mu}{\widetilde{\lambda}_2}}(\hat{\mu}_t - \hat{\mu}_{t-1})$) and also to the multipliers attached to the consolidated budget constraint. The first two elements are standard features of the NK model. Shocks to the Phillips curve induce a trade-off between output and inflation and under the optimal policy this is resolved by making inflation partially absorb the shock, depending on the weight attached to output stabilization in the policy objective function. If λ_Y is a very small number, then $\widetilde{\lambda}_2$ approaches infinity and cost-push shocks do not exert any influence on inflation. Conversely, if λ_Y is infinite (equivalent to the planner only seeking to stabilize the output gap) then $\widetilde{\lambda}_2 \rightarrow \frac{1}{\beta}$ and all of the effect of the cost-push shock is absorbed by inflation.

Moreover, attaching a larger weight to output stabilization yields a more persistent inflation process. The intuition behind this property is simple: Since from the Phillips curve output variability is proportional to the variability of the changes in inflation, the planner makes inflation react persistently to shocks in order to smooth the output target. In the limit, when policy only cares about smoothing output fluctuations, then $\widetilde{\lambda}_1 \rightarrow 1$ and inflation displays a unit root.

Finally, the terms $\Delta\psi_{gov}$ capture the effect of shocks being filtered through the consolidated budget on inflation. These terms also induce an inflation-output trade-off (hence the dependence on parameters $\widetilde{\lambda}_1, \widetilde{\lambda}_2$) but, as expected, this trade-off now depends also on the debt maturity structure. Innovations to $\Delta\psi_{gov}$ can result from both demand and supply side shocks.

Expected inflation. Given this solution it is straightforward to show that the expected inflation rates in $t+1$ and $t+2$ are given by.

$$E_t\hat{\pi}_{t+1} = \widetilde{\lambda}_1\hat{\pi}_t + \underbrace{\left[\frac{\widetilde{\kappa}}{\widetilde{\lambda}_2}\frac{\bar{b}}{1-\beta\delta}\frac{1}{1-\frac{\delta}{\widetilde{\lambda}_2}}\delta + \frac{\widetilde{\kappa}}{\widetilde{\lambda}_2}\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\bar{b}\left(\frac{1}{1-\frac{\delta}{\widetilde{\lambda}_2}}(\delta-1)\right)\right]}_{\equiv\widetilde{\zeta}}\frac{\Delta\psi_{gov,t}}{1-\delta L} + \underbrace{\frac{1}{\beta\widetilde{\lambda}_2}\frac{(\rho_\mu-1)}{1-\frac{\rho_\mu}{\widetilde{\lambda}_2}}}_{\widetilde{\nu}_1}\hat{\mu}_t$$

and

$$E_t\hat{\pi}_{t+2} = \widetilde{\lambda}_1 E_t\hat{\pi}_{t+1} + \delta\widetilde{\zeta}\frac{\Delta\psi_{gov,t}}{1-\delta L} + \rho_\mu\widetilde{\nu}_1\hat{\mu}_t = \widetilde{\lambda}_1^2\hat{\pi}_t + (\delta+\widetilde{\lambda}_1)\widetilde{\zeta}\frac{\Delta\psi_{gov,t}}{1-\delta L} + (\rho_\mu+\widetilde{\lambda}_1)\widetilde{\nu}_1\hat{\mu}_t$$

where we used for convenience the notation $\widetilde{\nu}_1$ and $\widetilde{\zeta}$ to summarize the more complex algebraic expressions.

The Ramsey rule. We make use of these results inside the Euler equation to derive the optimal interest rate rule. Assuming for now that $\hat{\mu}_t$ is a pure cost push shock (hence does not affect directly the Euler equation) we have

$$\begin{aligned}\hat{i}_t &= \sigma\frac{\overline{Y}}{\overline{C}}\left(E_t\hat{Y}_{t+1} - \hat{Y}_t\right) + E_t\hat{\pi}_{t+1}\\ &= \frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}E_t\left(\hat{\pi}_{t+1} - \beta\hat{\pi}_{t+2} - \hat{\pi}_t + \beta\hat{\pi}_{t+1}\right) + E_t\hat{\pi}_{t+1} + \frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}(1-\rho_\mu)\hat{\mu}_t\end{aligned}$$

Gathering terms and using the previous formulae to substitute out expected inflation we get:

$$\begin{aligned}\hat{i}_t = &\left((1+\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}(1+\beta))\widetilde{\lambda}_1 - 1 - \beta\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\widetilde{\lambda}_1^2\right)\hat{\pi}_t + \frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}(1-\rho_\mu)\hat{\mu}_t +\\ &[(1+\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}(1+\beta)) - \beta\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}(\rho_\mu+\widetilde{\lambda}_1)]\widetilde{\nu}_1\hat{\mu}_t + \widetilde{\zeta}[(1+\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}(1+\beta)) - \beta\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}(\delta+\widetilde{\lambda}_1)]\frac{\Delta\psi_{gov,t}}{1-\delta L}\end{aligned}$$

Moreover, from (45) it is simple to show that

$$\frac{\widetilde{\zeta}}{\delta}\frac{\Delta\psi_{gov,t}}{1-\delta L} = \hat{\pi}_t - \widetilde{\lambda}_1\hat{\pi}_{t-1} - \widetilde{f}\Delta\psi_{gov,t} - \frac{1}{\beta\widetilde{\lambda}_2}\frac{1}{1-\frac{\rho_\mu}{\widetilde{\lambda}_2}}(\hat{\mu}_t - \hat{\mu}_{t-1})$$

where $\widetilde{f}$ sums the coefficients of all terms $\Delta\psi_{gov,t}$.

The optimal interest rate rule is:

$$\begin{aligned}\hat{i}_t = &\left((1+\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}(1+\beta))\widetilde{\lambda}_1 - 1 - \beta\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\widetilde{\lambda}_1^2\right)\hat{\pi}_t + \frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}(1-\rho_\mu)\hat{\mu}_t +\\ &[(1+\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}(1+\beta)) - \beta\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}(\rho_\mu+\widetilde{\lambda}_1)]\widetilde{\nu}_1\hat{\mu}_t\\ &+[(1+\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}(1+\beta)) - \beta\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}(\delta+\widetilde{\lambda}_1)]\delta[\hat{\pi}_t - \widetilde{\lambda}_1\hat{\pi}_{t-1} - \widetilde{f}\Delta\psi_{gov,t} - \frac{1}{\beta\widetilde{\lambda}_2}\frac{1}{1-\frac{\rho_\mu}{\widetilde{\lambda}_2}}(\hat{\mu}_t - \hat{\mu}_{t-1})]\end{aligned}$$

Spending and preference shocks. It is simple to modify the above formula to introduce spending and preference shocks. Let $\hat{\mu}_t = -\kappa_2\hat{G}_t$ (from the Phillips curve assumed in text).

Then the optimal rule is:

$$\hat{i}_t = \left((1+\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}(1+\beta))\widetilde{\lambda}_1 - 1 - \beta\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\widetilde{\lambda}_1^2\right)\hat{\pi}_t + \hat{\xi}_t(1-\rho_\xi) + \sigma(\frac{\overline{G}}{\overline{C}} - \frac{\kappa_2}{\kappa_1}\frac{\overline{Y}}{\overline{C}})(1-\rho_G)\hat{G}_t +$$

$$[(1+\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}(1+\beta)) - \beta\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}(\rho_\mu + \widetilde{\lambda}_1)]\widetilde{\nu}_1\hat{G}_t$$

$$+[(1+\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}(1+\beta)) - \beta\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}(\delta + \widetilde{\lambda}_1)]\delta[\hat{\pi}_t - \widetilde{\lambda}_1\hat{\pi}_{t-1} - \widetilde{f}\Delta\psi_{gov,t} - \frac{1}{\beta\widetilde{\lambda}_2}\frac{1}{1-\frac{\rho_\mu}{\widetilde{\lambda}_2}}(\hat{G}_t - \hat{G}_{t-1})]$$

which can be rearranged to obtain the formula in Proposition 5.

■

C.2 The micro-founded loss function

We now derive the microfounded objective function using a second-order approximation of household utility function. To do so, we use a model version in which the first best allocation is reached at the steady-state. We thus subsidize output at rate $\varkappa = \frac{-1}{\eta+1} > 0$.

The competitive equilibrium in the model with the output subsidy is summarized by:

$$C_t^{-\sigma} = \beta R_t E_t \frac{C_{t+1}^{-\sigma}}{\pi_{t+1}}$$

$$P_{t,L}C_t^{-\sigma} = \beta E_t(1+\delta P_{t+1,L})\frac{C_{t+1}^{-\sigma}}{\pi_{t+1}}$$

$$\chi h_t^{\phi} C_t^{\sigma} = w_t$$

$$\theta(\pi_t - \pi)\pi_t = \eta(1-w_t) + \beta\theta E_t \frac{u_{c,t+1}}{u_{c,t}}\frac{Y_{t+1}}{Y_t}(\pi_{t+1}-\pi)\pi_{t+1}$$

$$C_t + G_t + \frac{\theta}{2}(\pi_t - \pi)^2 Y_t = Y_t = h_t$$

$$P_{t,L}b_{t,\delta} = (1+\delta P_{t,L})\frac{b_{t-1,\delta}}{\pi_t} + G_t + \varkappa Y_t - T_t$$

$$\chi^{\frac{-1}{\sigma}}(Y_t^n)^{\frac{-\phi}{\sigma}} + G_t = Y_t^n$$

To save notation we abstract from preference shocks. Y_t^n denotes the natural output level which, in log-linear form, is given by

$$\hat{Y}_t^n = \frac{\sigma\overline{G}}{\sigma\overline{Y} + \phi\overline{C}}\hat{G}_t$$

Taking a second-order approximation of the utility function $U_t = u\Big(Y_t(1-\frac{\theta}{2}(\pi_t-\pi)^2) - G_t\Big) - v(Y_t)$ (we use the resource constraint to substitute C_t and the equilibrium $Y_t = h_t$ to substitute hours) around the steady-state we get:

$$U_t \approx u_c\overline{Y}\hat{Y}_t + \frac{1}{2}(u_{cc}\overline{Y}^2 + u_c\overline{Y})\hat{Y}_t^2 - \frac{1}{2}\theta u_c \pi^2\overline{Y}\hat{\pi}_t^2 - u_{cc}\overline{YG}\hat{Y}_t\hat{g}_t - v_h\overline{Y}\hat{Y}_t - \frac{1}{2}(v_{hh}\overline{Y}^2 + v_h\overline{Y})\hat{Y}_t + \text{t.i.p}$$

where "t.i.p" groups terms that are independent of policy. In an efficient steady-state, $u_c = v_n$.

This gives:

$$U_t \approx \frac{1}{2}(u_{cc}\overline{Y} - v_{hh}\overline{Y})\overline{Y}\hat{Y}_t^2 - \frac{1}{2}\theta u_c \pi^2 \overline{Y}\hat{\pi}_t^2 - u_{cc}\overline{YG}\hat{Y}_t\hat{G}_t + \text{t.i.p}$$

Using (46) we obtain:

$$U_t \approx \frac{1}{2}(u_{cc}\overline{Y} - v_{hh}\overline{Y})\overline{Y}\hat{Y}_t^2 - \frac{1}{2}\theta u_c \pi^2 \overline{Y}\hat{\pi}_t^2 - u_{cc}\overline{Y}\frac{\sigma\overline{Y} + \phi\overline{C}}{\sigma}\hat{Y}_t\hat{Y}_t^n + \text{t.i.p}$$

Using $u_{cc} = -\sigma\overline{C}^{-\sigma-1}$ and $v_{hh} = \phi\chi\overline{Y}^{\phi-1} = \phi\overline{C}^{-\sigma}/\overline{Y}$, we get:

$$U_t \approx -\frac{1}{2}\left[\overline{C}^{-\sigma-1}\overline{Y}(\sigma\overline{Y} + \phi\overline{C})\left(\hat{Y}_t - \hat{Y}_t^n\right)^2 + \theta\pi^2\overline{C}^{-\sigma}\overline{Y}\hat{\pi}_t^2\right] + \text{t.i.p}$$

Rescaling (and using $\pi = 1$), we get the loss function:

$$L = \frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t\left(\hat{\pi}_t^2 + \frac{\sigma\frac{\overline{Y}}{\overline{C}} + \phi}{\theta}(\hat{Y}_t - \hat{Y}_t^n)^2\right) \tag{46}$$

Setting $\gamma_h = \phi$ (our notation in text) the optimal output stabilization weight is equal to $\lambda_Y = \frac{\sigma\frac{\overline{Y}}{\overline{C}} + \gamma_h}{\theta}$.

C.3 Optimal interest rates with the microfounded loss function.

We now derive the optimal interest rate rule in the case where the planner's objective function is given by (46).

For simplicity, we will denote $\widetilde{Y}_t = \hat{Y}_t - \hat{Y}_t^n$ the output gap. The constraint set of the planner is

$$\hat{i}_t = \sigma\frac{Y}{C}(E_t\widetilde{Y}_{t+1} - \widetilde{Y}_t) + E_t\hat{\pi}_{t+1} + \hat{r}_t^n$$

$$\hat{\pi}_t = \kappa_1\widetilde{Y}_t + \beta E_t\hat{\pi}_{t+1} + \hat{\mu}_t$$

$$\hat{p}_{t,\delta} = -\hat{i}_t + \beta\delta E_t\hat{p}_{t+1,\delta}$$

$$\overline{b}\overline{p}_\delta(\hat{b}_{t,\delta} + \hat{p}_{t,\delta}) = -S\hat{S}_t + \overline{b}(1 + \delta\overline{p}_\delta)(\hat{b}_{t-1,\delta} - \hat{\pi}_t) + \overline{p}_\delta\delta\overline{b}\hat{p}_{t,\delta}$$

where now $\kappa_1 = -\frac{\eta}{\theta}(\gamma_h + \sigma\frac{\overline{Y}}{\overline{C}})$. $\hat{r}_t^n =\equiv -\frac{\gamma_h\overline{G}}{\overline{Y} + \frac{\gamma_h}{\sigma}\overline{C}}(E_t\hat{G}_{t+1} - \hat{G}_t) - E_t(\hat{\xi}_{t+1} - \xi_t)$ denotes the *natural rate of interest*. Notice that while government expenditures do not show up in the Phillips curve (they are included in the natural output definition) variable $\hat{\mu}_t$ has been added as a shifter and it represents a standard cost push shock.[38]

The government's surplus is now given by:

$$S\hat{S}_t = -\omega_1\overline{G}\hat{G}_t - \omega_2\widetilde{Y}_t$$

[38]It is simple to show that adding this shock does not change our derivations for the welfare function, the natural output or the natural interest rate.

We thus focus on the active fiscal scenario in which debt is not back by surpluses. Moreover, parameter ω_2 is not zero since we assume that the government sets a subsidy to eliminate distortions from monopolistic competition as it is standard in the literature. Specifically, $\omega_2 = \varkappa Y\left(1+\gamma_h+\frac{\sigma\overline{Y}}{\overline{C}}\right) > 0$ and also $\omega_1 = 1+\frac{\varkappa\overline{Y}}{\overline{Y}+\frac{\gamma_h}{\sigma}\overline{C}} > 0$

As discussed in text, we can simplify the Ramsey program noting that $\hat{i}_t$ can be set to satisfy the Euler equation. Moreover, subsituting out the bond price we can write the consolidated budget constraint as:

$$\bar{b}\frac{\beta}{1-\beta\delta}\left(\hat{b}_{t,\delta}-\sigma\sum_{j\geq 1}(\beta\delta)^{j-1}\frac{\overline{Y}}{\overline{C}}(E_t\widetilde{Y}_{t+j}-\widetilde{Y}_{t+j-1})-\sum_{j\geq 1}(\beta\delta)^{j-1}E_t\hat{\pi}_{t+j}-\sum_{j\geq 1}(\beta\delta)^{j-1}\hat{r}^n_{t+j-1}\right)=$$
$$(1+\omega_1)G\hat{G}_t+\omega_2\widetilde{Y}_t+\bar{b}\frac{1}{1-\beta\delta}\left(\hat{b}_{t-1,\delta}-\hat{\pi}_t\right)$$
$$-\frac{\delta\beta}{1-\beta\delta}\bar{b}\left(\sigma\sum_{j\geq 1}(\beta\delta)^{j-1}\frac{\overline{Y}}{\overline{C}}(E_t\widetilde{Y}_{t+j}-\widetilde{Y}_{t+j-1})+\sum_{j\geq 1}(\beta\delta)^{j-1}E_t\hat{\pi}_{t+j}+\sum_{j\geq 1}(\beta\delta)^{j-1}\hat{r}^n_{t+j-1}\right)$$

Given multipliers $\psi_{\pi,t}$ for the Phillips curve constraint and $\psi_{gov,t}$ for the budget constraint we can state the optimality condition for inflation as:

$$-\hat{\pi}_t+\Delta\psi_{\pi,t}+\frac{\bar{b}}{1-\beta\delta}\sum_{j\geq 0}\delta^j\Delta\psi_{gov,t-j}=0$$

The first order condition for output is given by:

$$-\lambda_Y\widetilde{Y}_t-\psi_{\pi,t}\kappa_1-\omega_2\psi_{gov,t}-\sigma\frac{\bar{b}}{1-\beta\delta}(1-\delta)\frac{\overline{Y}}{\overline{C}}\sum_{j\geq 1}\delta^{j-1}\psi_{gov,t-j}+\sigma\frac{\overline{Y}}{\overline{C}}\frac{\bar{b}}{1-\beta\delta}\beta(1-\delta)\sum_{j\geq 0}\delta^j\psi_{gov,t-j}=0$$

However,

$$-\sigma\frac{\bar{b}}{1-\beta\delta}(1-\delta)\frac{\overline{Y}}{\overline{C}}\sum_{j\geq 1}\delta^{j-1}\psi_{gov,t-j}+\sigma\frac{\overline{Y}}{\overline{C}}\frac{\bar{b}}{1-\beta\delta}\beta(1-\delta)\sum_{j\geq 0}\delta^j\psi_{gov,t-j}=$$
$$\sigma\bar{b}\frac{\overline{Y}}{\overline{C}}\sum_{j\geq 0}\delta^j\Delta\psi_{gov,t-j}+\sigma\frac{\overline{Y}}{\overline{C}}\frac{\bar{b}}{1-\beta\delta}(\beta-1)\psi_{gov,t}=\sigma\bar{b}\frac{\overline{Y}}{\overline{C}}\sum_{j\geq 0}\delta^j\Delta\psi_{gov,t-j}-\overline{S}\sigma\frac{\overline{Y}}{\overline{C}}\psi_{gov,t}$$

where the last equality follows from the steady state intertemporal budget $\frac{\bar{b}}{1-\beta\delta}=\frac{\overline{S}}{1-\beta}$. Thus, we can write the output FONC as

$$-\lambda_Y\widetilde{Y}_t-\psi_{\pi,t}\kappa_1-\omega_2\psi_{gov,t}+\sigma\bar{b}\frac{\overline{Y}}{\overline{C}}\sum_{j\geq 0}\delta^j\Delta\psi_{gov,t-j}-\overline{S}\sigma\frac{\overline{Y}}{\overline{C}}\psi_{gov,t}=0$$

Combining the two first order conditions we thus get:

$$-\hat{\pi}_t-\frac{\lambda_Y}{\kappa_1}\Delta\widetilde{Y}_t-\frac{\omega_2+\overline{S}\sigma\frac{\overline{Y}}{\overline{C}}}{\kappa_1}\Delta\psi_{gov,t}+\frac{\sigma}{\kappa_1}\bar{b}\frac{\overline{Y}}{\overline{C}}\sum_{j\geq 0}\delta^j\left(\Delta\psi_{gov,t-j}-\Delta\psi_{gov,t-j-1}\right)+\frac{\bar{b}}{1-\beta\delta}\sum_{j\geq 0}\delta^j\Delta\psi_{gov,t-j}=0 \tag{47}$$

Equation (47) is basically the same as the optimal trade-off equation we derived in text for the baseline model. The difference is that here the planner targets the natural output (hence the term

$\Delta\widetilde{Y}_t$ has replaced ΔY_t and now $\omega_Y = \frac{\omega_2 + \overline{S}\sigma\frac{\overline{Y}}{\overline{C}}}{\kappa_1}$ instead of $\frac{\overline{S}\sigma\frac{\overline{Y}}{\overline{C}}}{\kappa_1}$). However, this difference does not matter for the optimal inflation coefficients of the interest rate rules, it only matters for the real interest rate target and the constant multiplying the stochastic intercept. It is thus easy to follow the steps of the previous subsection and establish that the optimal interest rate rule is

$$\hat{i}_t = \hat{r}_t^n + \left((1 + \frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}(1+\beta))\widetilde{\lambda}_1 - 1 - \beta\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\widetilde{\lambda}_1^2\right)\hat{\pi}_t$$

$$+\delta\left[(1 + \frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}(1+\beta)) - \beta\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}(\delta + \widetilde{\lambda}_1)\right]\left[\hat{\pi}_t - \widetilde{\lambda}_1\hat{\pi}_{t-1} - \widetilde{f}\Delta\psi_{gov,t}\right] + \frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}(1-\rho_\mu)\hat{\mu}_t$$

$$+\left[(1 + \frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}(1+\beta)) - \beta\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}(\rho_\mu + \widetilde{\lambda}_1)\right]\widetilde{\nu}_1\hat{\mu}_t$$

$$-\delta\left[(1 + \frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}(1+\beta)) - \beta\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}(\delta + \widetilde{\lambda}_1)\right]\frac{1}{\beta\widetilde{\lambda}_2}\frac{1}{1 - \frac{\rho_\mu}{\widetilde{\lambda}_2}}(\hat{\mu}_t - \hat{\mu}_{t-1})$$

where the expression for $\widetilde{f}$ is the same as in subsection C.1 but now $\frac{\omega_Y}{\kappa_1} = \frac{\omega_2 + \overline{S}\sigma\frac{\overline{Y}}{\overline{C}}}{\kappa_1}$.

C.4 Impulse responses

We complement the results we showed in the main text with additional graphs plotting the responses of macroeconomic variables to shocks.

Persistent Shocks to Spending Figure 6 plots the IRFS under Ramsey and the optimal rules when the output target is the steady state output. The top 2 panels of the Figure assume $\lambda_Y = 0$ and the bottom panels set $\lambda_Y = 0.12$ ($\approx \frac{1}{\theta}(\sigma\frac{\overline{Y}}{\overline{C}} + \gamma_h)$). The maturity of debt δ varies across the panels. To produce these graphs we assumed that spending follows a first order autoregressive process with a persistence coefficient equal to 0.9.

Each of the plots contains three IRFS. Ramsey corresponds to the optimal Ramsey policy (equivalently an optimal rule with stochastic intercepts). The 'Dual Mandate Rule' corresponds to the optimal policy derived in Proposition 5 without stochastic intercepts.[39] Finally, 'Simple Rule' corresponds to the case where monetary policy follows $\hat{i}_t = \widetilde{r}_t + \delta\hat{\pi}_t$.

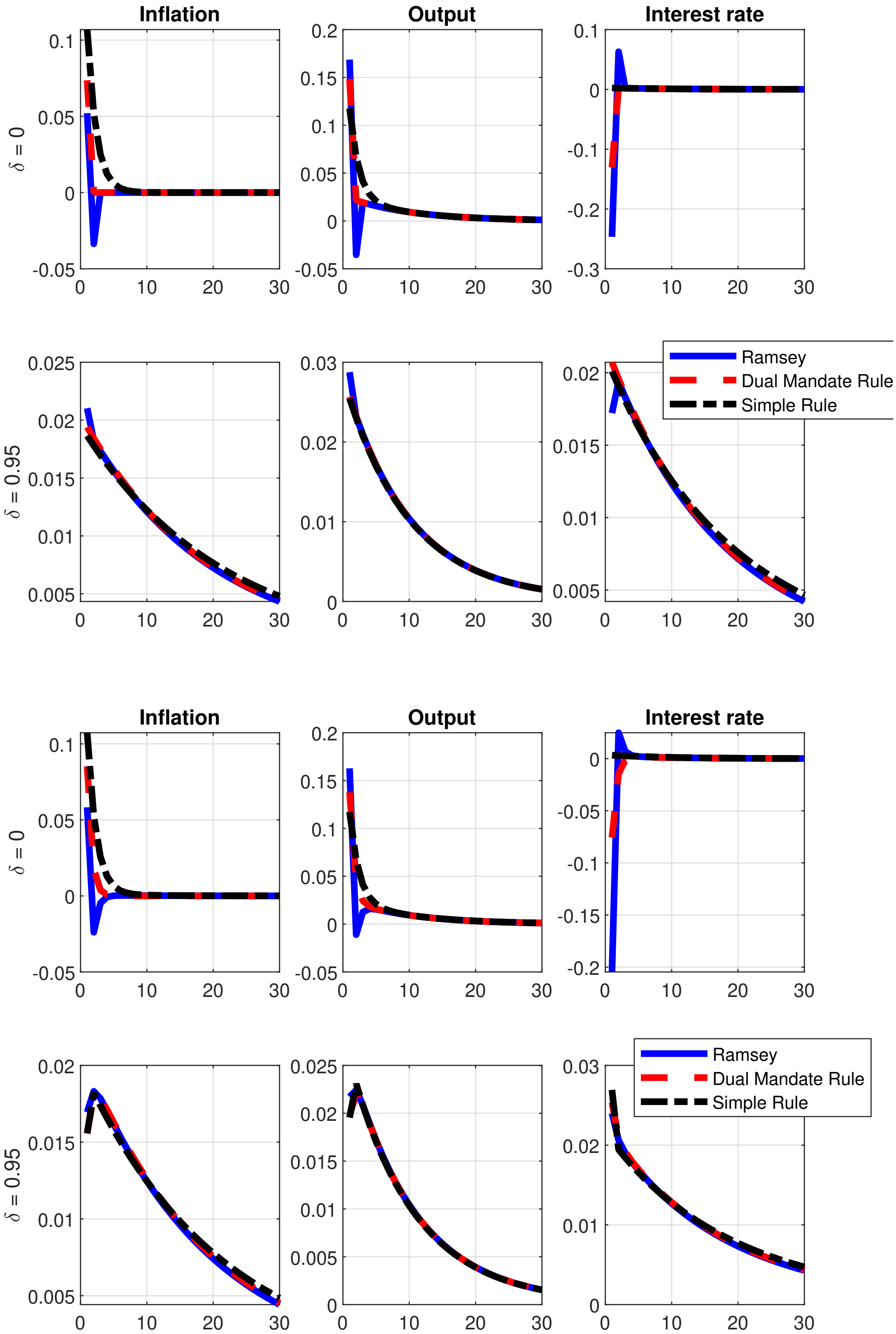
As can be seen from the graphs the dual mandate policy matches almost perfectly the impulse response functions when debt is long term. With short debt however the fit is not good. These patterns confirm our main findings for the case of persistent shocks.

In Figure 7 we conduct the same exercise, but now assume $\lambda_Y = 0.5$. The dual mandate rule matches almost perfectly the Ramsey IRFS with long debt. The 'Simple Rule' (setting the inflation coefficient equal to δ) also results in a good fit. With a constant output target government spending becomes a shifter in the Phillips curve, (when there is an income effect on labour supply). Therefore, a disturbance in government spending simultaneously shocks all three equations of the model (Euler, Phillips and consolidated budget). Even so, the optimal rules that we derived in this paper can approximate the Ramsey outcome closely when government debt is long term.

Shocks to preferences We now revisit the analysis from Section 4 to present the impulse responses of macroeconomic variables to preference shocks. As in Section 4, we consider i.i.d. shocks and separately examine the cases $\sigma = 0, \lambda_Y > 0$ and $\sigma > 0, \lambda_Y = 0$. We have analytically derived the response of inflation to spending and preference shocks in these models (equations

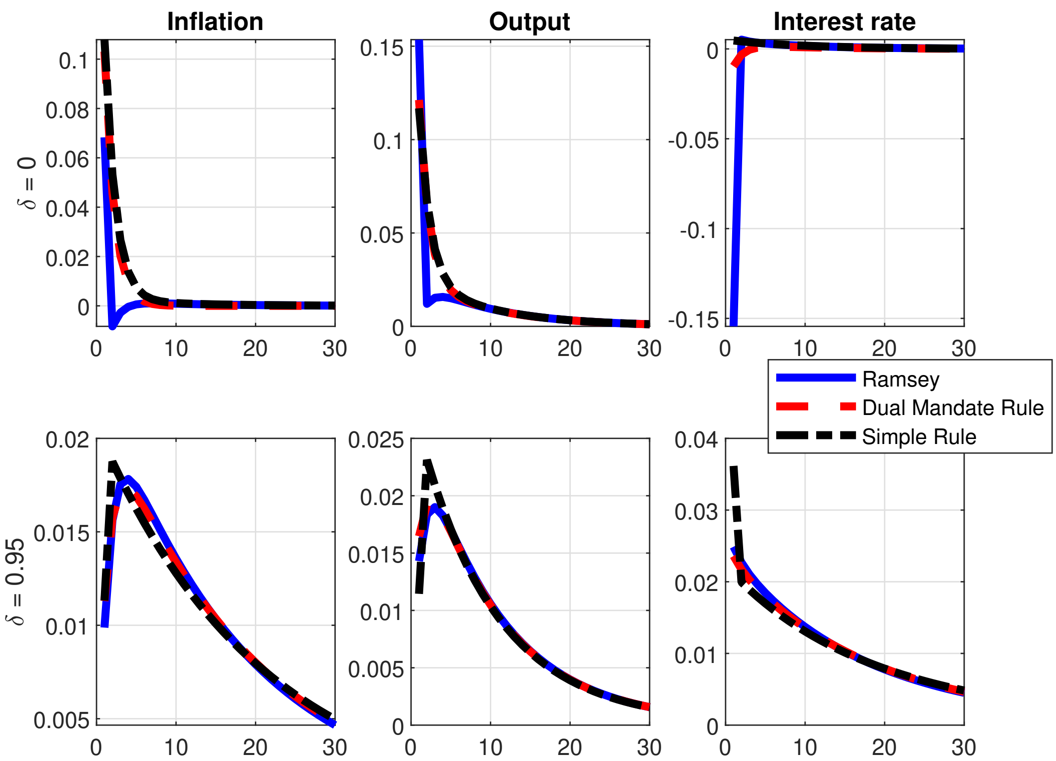
[39]For $\lambda_Y = 0$ Proposition 5 defines the same policy function as Proposition 4, when $\widetilde{\lambda_1} = 0$.

Figure 6: **Rules vs Ramsey: Steady State Output Target I**



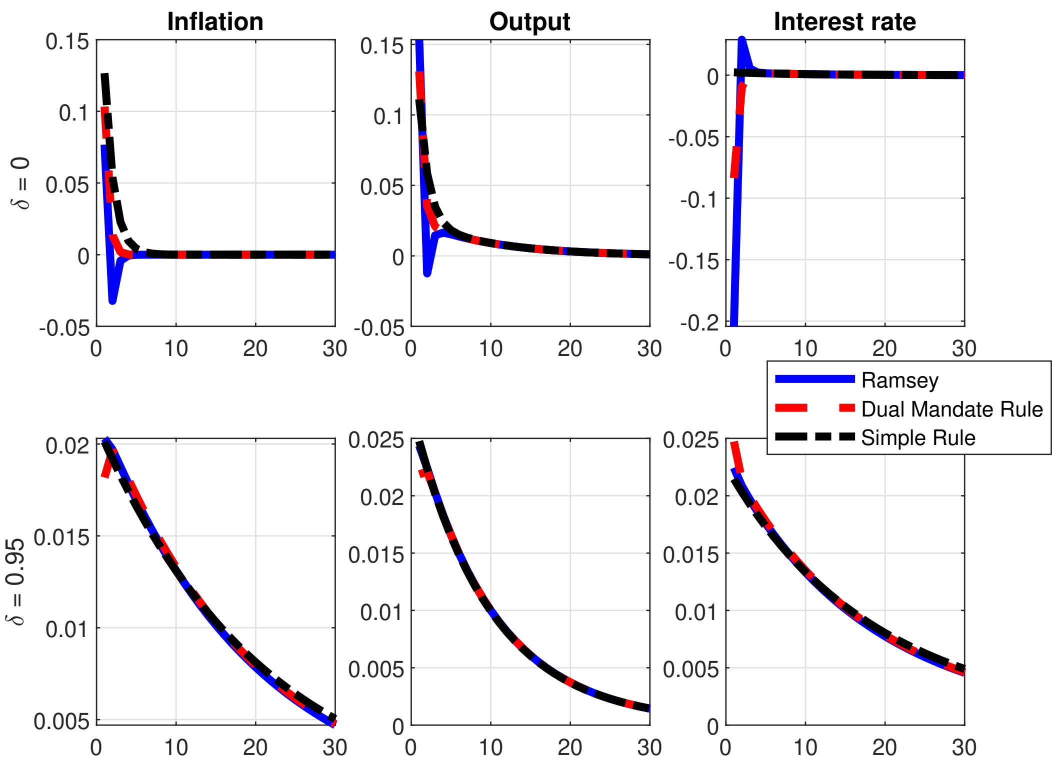
Notes: The figure compares the optimal policy impulse responses of inflation, output, and the nominal interest rate with the optimal interest rate rules when we omit stochastic intercepts. We set $\lambda_Y = 0$ in the top panel and $\lambda_Y = \frac{1}{\theta}(\sigma \frac{\overline{Y}}{\overline{C}} + \gamma_h) \approx 0.12$ in the bottom panel.

Figure 7: **Rules vs Ramsey: Steady State Output Target II**



Notes: The figure compares the optimal policy impulse responses of inflation, output, and the nominal interest rate with the optimal interest rate rules when we omit stochastic intercepts. We set $\lambda_Y = 0.5$.

Figure 8: **Rules vs Ramsey: Micro-founded loss function**



Notes: The figure compares the Ramsey impulse responses of inflation, output, and the nominal interest rate with the analogous objects under the optimal inflation targeting rules.

(28) and (30) respectively). For convenience, we provide these results again.

$$\Delta\psi_{gov,t} = \widetilde{\psi}(\overline{G}\hat{G}_t + (\bar{b} - \bar{s})\hat{\xi}_t)$$

in the Fisherian model with output smoothing and

$$\Delta\psi_{gov,t} = \widetilde{\epsilon}\left[(\overline{G} + (\bar{b} - \bar{s})\sigma\frac{\overline{G}}{\overline{C}})\hat{G}_t + (\bar{b} - \bar{s})\hat{\xi}_t\right]$$

in the canonical NK model without output smoothing. The expressions for $\widetilde{\psi} > 0$ and $\widetilde{\epsilon} > 0$ were provided in Appendix A.

Assume that $\hat{\xi}_t < 0$. Noting that $\bar{s} < \bar{b}$ when $\delta < 1$, the above expressions tell us that the shock will make $\Delta\psi_{gov,t}$ negative, therefore inflation will drop following the shock.

To understand why a negative $\hat{\xi}_t$ shock is deflationary, note that it has two opposing effects on the intertemporal consolidated budget. On the one hand, the shock will increase the market value of debt outstanding in t through increasing real long bond prices. On the other hand, it will also increase the present value of surpluses that compensate for debt. When $\delta < 1$ the second effect dominates and the shock needs to be 'financed' with deflation for the intertemporal budget to hold. When long bonds are consols, $\delta = 1$, the two effects will cancel out and inflation will be zero. [40]

The formulae in equations (27) and (29) in text showed that the dynamic path of inflation in response to a change in $\Delta\psi_{gov,t}$ induced by a preference shock is exactly that from a spending shock. Therefore, we do not need to discuss again these solutions. In Figures 9 and 10 we plot impulse responses. The patterns are indeed very similar to the analogous objects for the spending shock we showed in text, the signs have flipped since we now consider a negative shock.

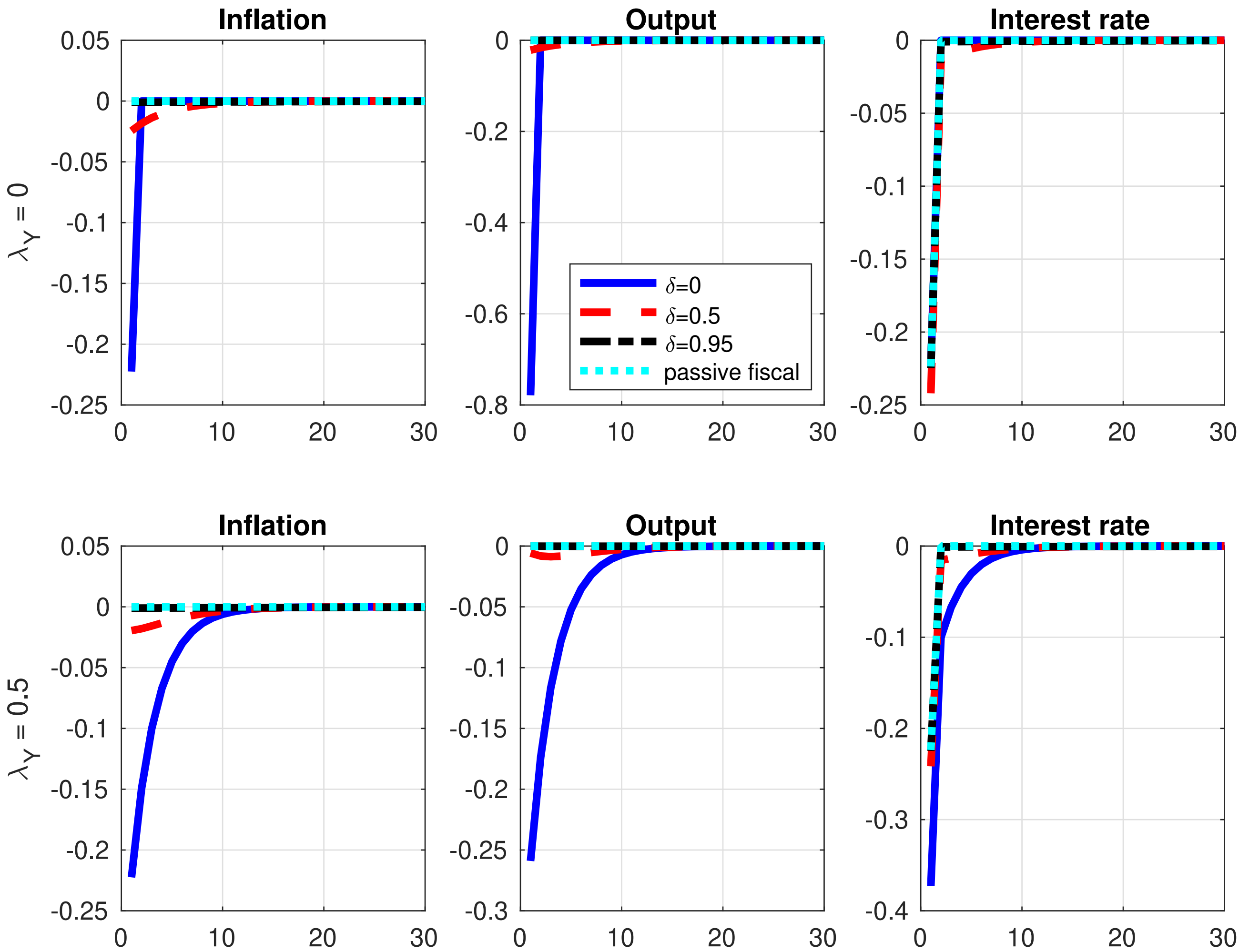
In Figure 11 we present the Impulse Response Functions (IRFs) for a persistent preference shock. The top two panels depict results for $\lambda_Y = 0$, while the bottom panels show results for $\lambda_Y = 0.5$. Under long-term debt, the Ramsey policy is closely approximated by both the 'Dual Mandate' and 'Simple' rules. The discrepancies observed with the 'Dual Mandate Rule' are minor and temporary, especially when considering the scale of the graphs. Monetary policy proves highly effective in stabilizing macroeconomic variables in response to the preference shock when debt is long-term. Conversely, with short-term debt, the interest rate rules perform poorly compared to the Ramsey outcome.

Finally, it is important to note that for preference shocks, the constant output target scenario considered in Figure 11 closely aligns with the outcome derived from a microfounded loss function. For the sake of brevity, we have not included this case in the graphs.

Cost Push Shocks. We now show the responses of macroeconomic variables to cost push shocks. The top two panels in Figure 12 consider $\lambda_Y = 0$. The bottom panels set λ_Y equal to the microfounded weight. As it is evident from the graphs, the optimal interest rate rules match closely the Ramsey responses, when debt is long term. Assuming short maturity debt however, compromises the fit. We thus conclude that the optimal interest rate rules can approximate the Ramsey outcome also in the case of supply side shocks.

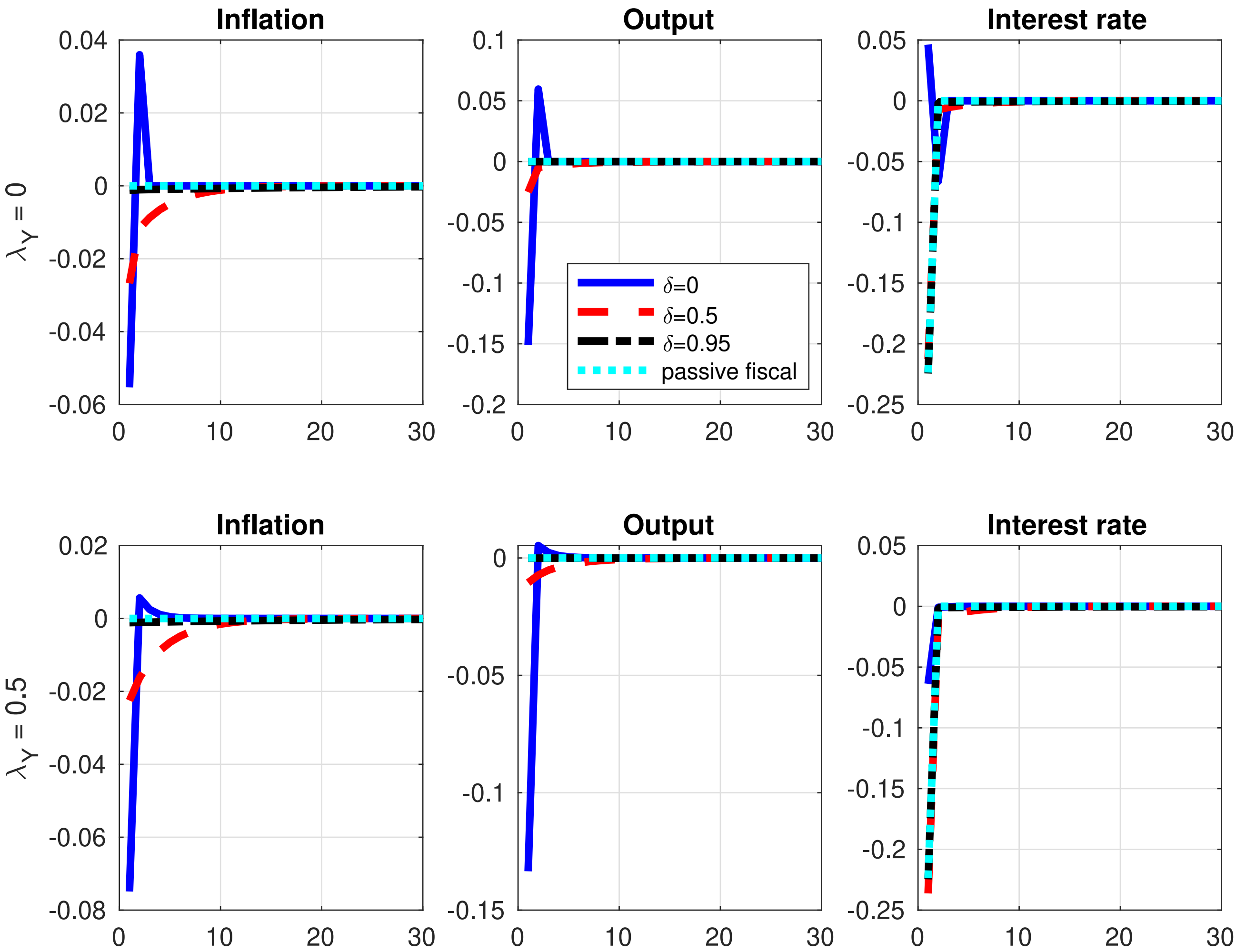
[40] We prove this assertion analytically in Appendix F.

Figure 9: **Impulse response functions, ξ shock**



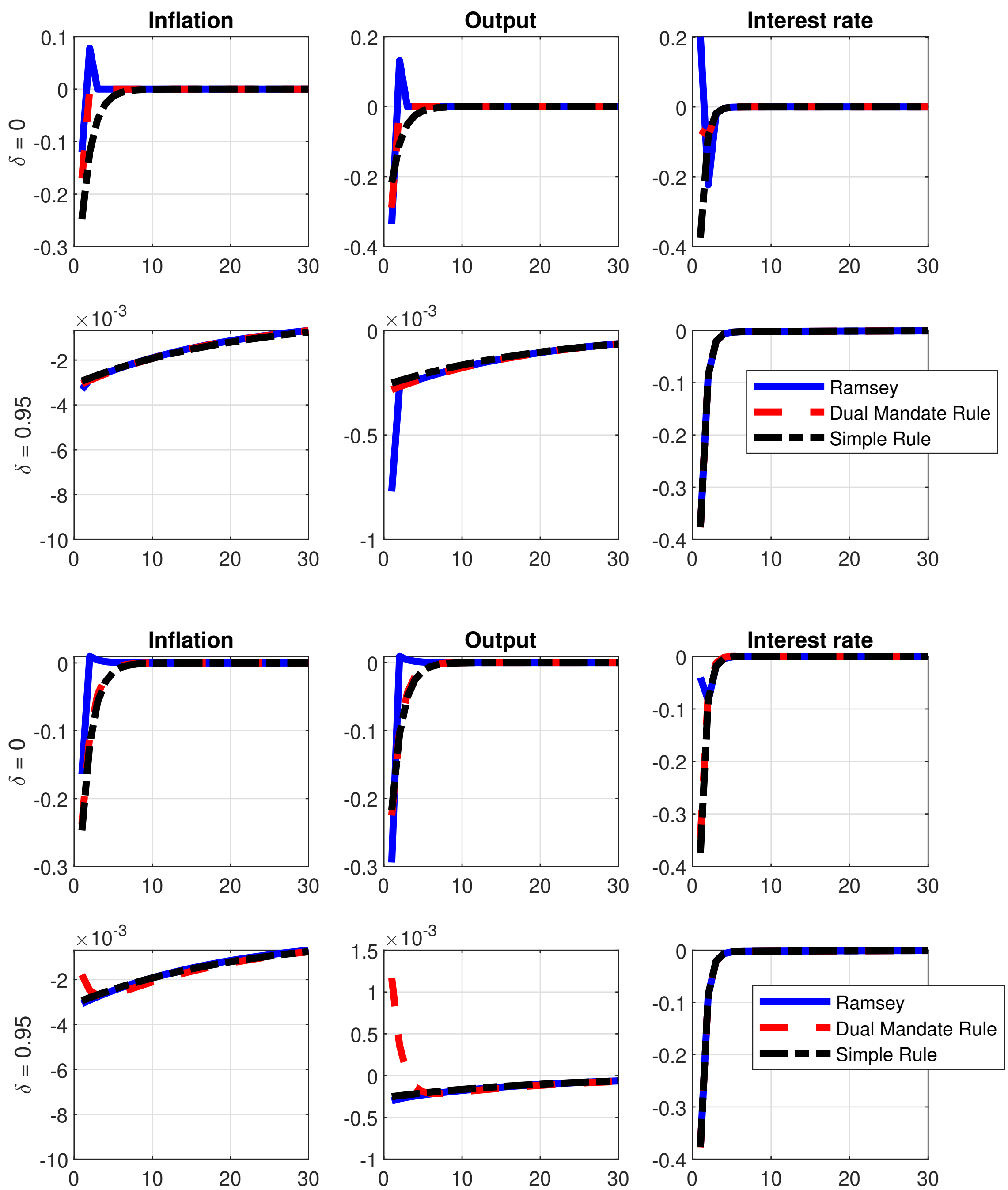
Notes: The figure displays the impulse responses of inflation, output, and the nominal interest rate following a negative preference shock (ξ), in the case where $\lambda_i = \sigma = 0$. Top panels assume $\lambda_Y = 0$, while in the bottom panels we set $\lambda_Y = 0.5$. In each plot, the solid blue line depicts impulse responses in the case where government debt is short term ($\delta = 0$); the dashed red lines and dash-dotted black lines plot the responses of variables when $\delta = 0.5$ and $\delta = 0.95$, respectively. The dotted cyan line considers the case where fiscal policy is passive ($\phi_{\tau,b} > \tilde{\phi}_\tau$).

Figure 10: **Impulse response functions with $\sigma > 0$ (ξ shock)**



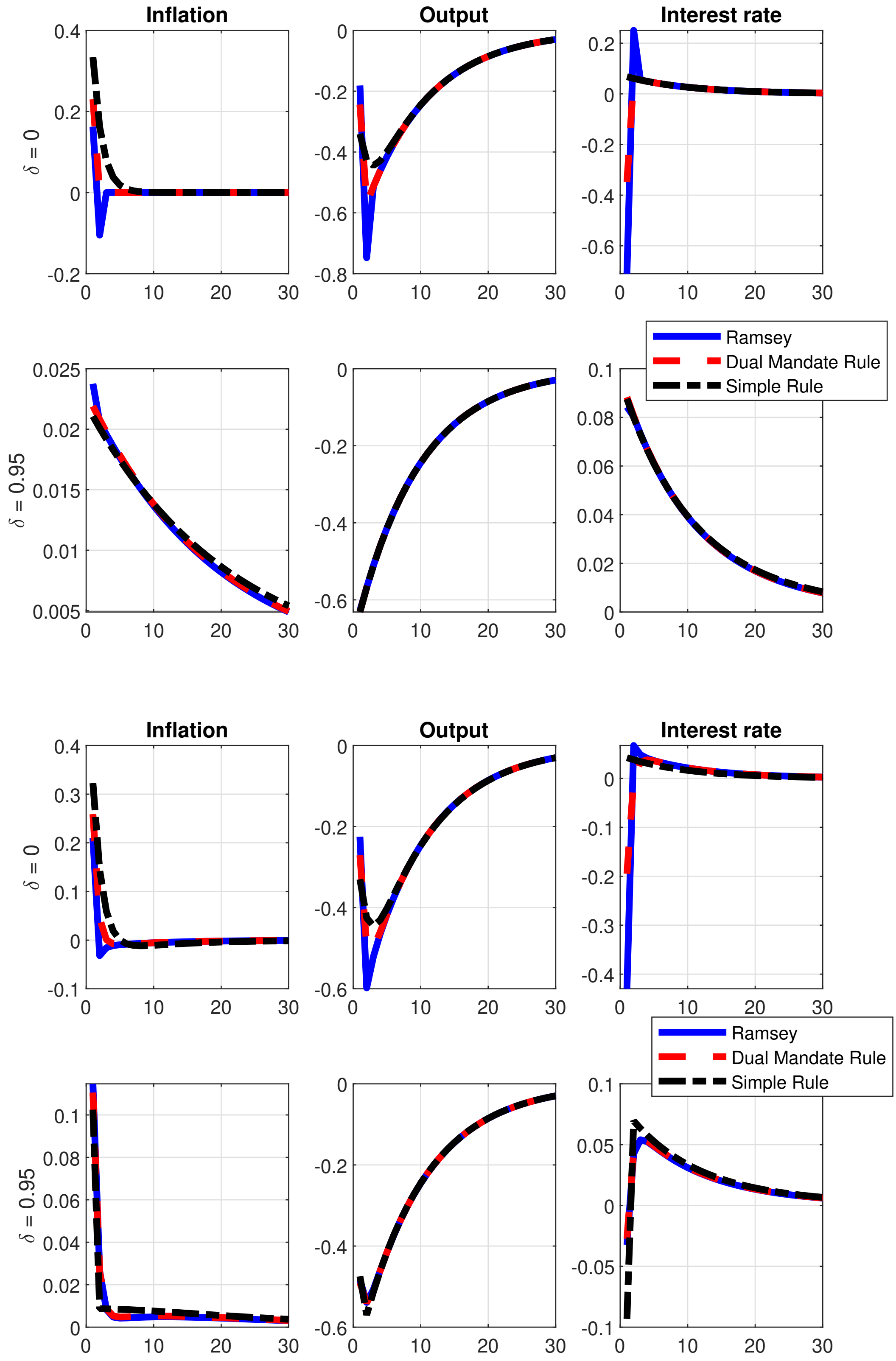
Notes: The figure displays the impulse responses of inflation, output, and the nominal interest rate following a negative demand shock (ξ), in the case where $\lambda_i = 0$ and $\sigma = 1$. Top panels assume $\lambda_Y = 0$, while in the bottom panels we set $\lambda_Y = 0.5$. In each plot, the solid blue line depicts impulse responses in the case where government debt is short term ($\delta = 0$); the dashed red lines and dash-dotted black lines plot the responses of variables when $\delta = 0.5$ and $\delta = 0.95$, respectively. The dotted cyan line considers the case where fiscal policy is passive ($\phi_{\tau,b} > \tilde{\phi}_\tau$).

Figure 11: **Rules vs Ramsey: Preference Shocks**



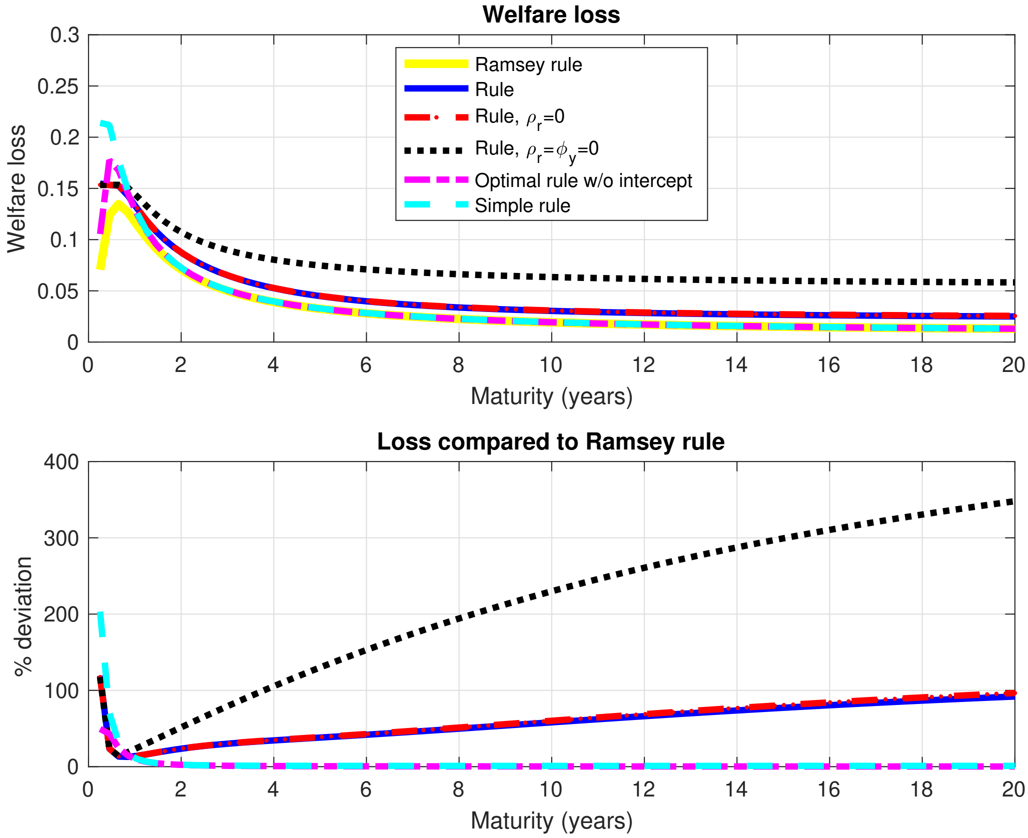
Notes: The figure compares the optimal policy impulse responses of inflation, output, and the nominal interest rate with the optimal interest rate rules when we omit stochastic intercepts. We set $\lambda_Y = 0$ in the top panel and $\lambda_Y = 0.5$ in the bottom panel.

Figure 12: **Rules vs Ramsey: Cost Push Shocks**



Notes: The figure compares the optimal policy impulse responses of inflation, output, and the nominal interest rate with the optimal interest rate rules when we omit stochastic intercepts. We set $\lambda_Y = 0$ in the top panel and $\lambda_Y = 0.12$ in the bottom panel.

Figure 13: **Welfare losses: Ramsey vs. interest rate rules with demand shocks**



Notes: The Figures compare the performance of the Ramsey rules, the Dual Mandate Rule without intercept (Optimal rule w/o intercept), the 'Simple rule' $\hat{i}_t = \widetilde{r}_t + \delta\hat{\pi}_t$ with the performance of ad hoc rules (48) with optimized coefficients, as a function of debt maturity in years ($4 \times \frac{1}{1-\delta}$). The graphs labeled 'Rule' optimizes s ρ_i, ϕ_π and ϕ_Y in (48). The 'Rule $\rho_i = 0$' constrains ρ_i to zero. 'Rule $\rho_i = \phi_Y = 0$' optimizes over ϕ_π.

The top graph shows the absolute welfare losses. The bottom graph expresses the losses relative to the Ramsey solution.

C.5 Ad hoc interest rate rules vs Ramsey rules.

In this subsection we compare the performance of ad hoc interest rate rules and Ramsey optimal rules in the active fiscal regime. We show that Ramsey optimal rules outperform (by a considerable margin) ad hoc rules in terms of reducing macroeconomic volatility.

More specifically, we assume that the planner minimizes the microfounded loss function derived in Section C.2. The optimal Ramsey rule for this model was derived in C.3. We compare the outcome under the optimal policy equilibrium with that of an equilibrium where interest rates are set according to:

$$\hat{i}_t = \rho_i \hat{i}_{t-1} + \phi_\pi \hat{\pi}_t + \phi_Y \widetilde{Y}_t \tag{48}$$

Parameters ρ_i, ϕ_π and ϕ_Y are set optimally so that the loss function is minimized.

Rules of the form (48) are commonly used in DSGE models.[41] The main differences with the optimal Ramsey rule is that in the latter, $\hat{i}_t$ tracks the natural rate of interest and a stochastic intercept term that is a function of the contemporaneous shocks.

[41]See Bianchi and Ilut (2017); Bianchi and Melosi (2019); Leeper and Leith (2016) among others. See also Schmitt-Grohé and Uribe (2007) for an exercise with optimized interest rate rules in the context of the fiscal theory.

We solve the models calibrating parameters to the values reported in text (Table 1). We further assume that spending and preference factors follow processes $\hat{x}_t = \rho_x \hat{x}_{t-1} + \hat{v}_{x,t}$ for $x = \xi, G$. We calibrate the first order autcorrelation coefficients and the variances of the shocks using the estimates of Smets and Wouters (2007). Thus $\rho_G = 0.97$, $\rho_\xi = 0.22$ $\sigma_G = 0.53$ and $\sigma_\xi = 0.23$.

Figure (13) plots the values of the loss function for different calibrations of the debt maturity. The solid blue line is the Ramsey rule outcome, the dashed red is the outcome under rule (48) when coefficients ρ_i, ϕ_π and ϕ_Y are optimized separately for each value of δ considered. Moreover, the black line constrains $\rho_i = 0$ whereas the green line assumes $\rho_i = \phi_Y = 0$, focusing on a simple inflation targeting rule when ϕ_π is optimal.

Ramsey rules lead to considerably smaller losses than ad hoc rules. When debt is short term (quarterly), the Ramsey losses are more than 2 times smaller than the losses implied by the rules with optimized coefficients. The losses are also smaller in the case of the dual mandate rule (Proposition 5) when we drop the stochastic intercepts. However, a simple rule with inflation coefficient equal to δ performs worse than any of the other models considered.

The reasons for these outcomes should be clear from our discussion in text. When debt is short term, stochastic intercepts are an important ingredient of Ramsey rules and therefore dropping them worsens the performance of the model. Moreover, in the presence of strong indirect output effects, setting the inflation coefficient equal to δ (in this case 0) is far from optimal. (In fact, the optimal inflation coefficients ought to be negative!). Note that this explains the relative success of the dual mandate policy. In this case inflation coefficients are not constrained (they are optimal) and also monetary policy tracks the real interest rate.

Next consider the performance of these models under plausible maturity structures of debt (with debt maturity exceeding 3-4 years, or say equal to 5 years to match the US data moment). It is evident that the outcomes under the 'dual mandate' and 'simple' rules essentially coincide with the Ramsey outcome. However, rules with optimized coefficients perform much worse and moreover, as we lengthen the maturity structure, the gap gets progressively bigger. Obviously, the key difference between the optimal rules and those with optimized coefficients in the case of long debt maturity, is the presence of explicit real rate tracking in the former, but not in the latter.

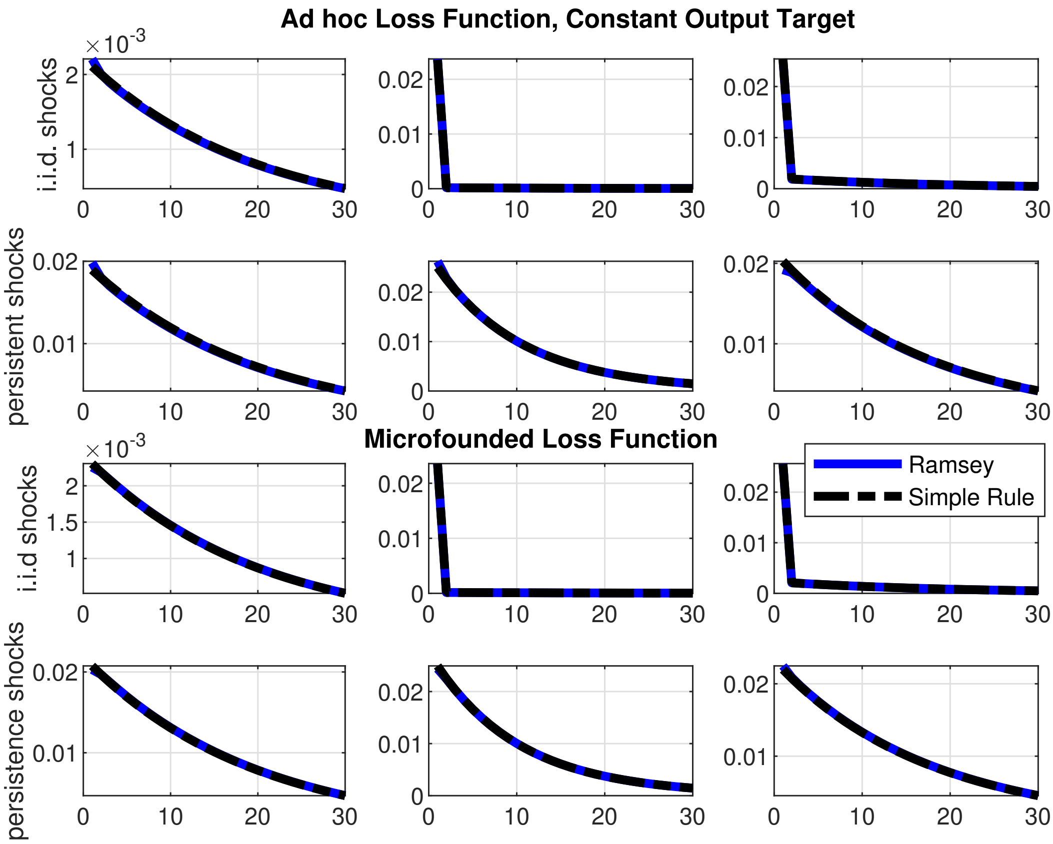
These findings generalize to alternative calibrations of the shock processes, which for the sake of brevity we do not report here. Based on the findings of this subsection we conclude that optimal rules dominate ad hoc DSGE rules, in terms of minimizing the losses of the central bank.

C.6 Real rate tracking.

We now show that our main result that optimal policy can be approximated by a simple rule setting the inflation coefficient equal to δ is robust towards assuming that the real interest rate that is tracked is the one that is backed out from the Fisher equation. Obviously, this property holds in the case of the Fisherian models we analyzed in text and so we consider the canonical New Keynesian model with a dual mandate objective function. In Figure 14 we plot the impulse response functions under steady state output targeting (top 2 rows) and the microfounded loss function (rows 3 and 4). Rows 1 and 3 assume i.i.d shocks and 2 and 4 assumed persistent shocks. As can be seen from these graphs setting $\widetilde{r}_t = \hat{i}_t - E_t\hat{\pi}_{t+1}$ does not compromise the fit of the simple rule to the Ramsey outcome.

This property should not be surprising. The reader that followed our methodology used to derive the optimal interest rate rules in the previous sections, will have noticed that the appropriate $\widetilde{r}_t$ has been determined through collecting the shocks from the the Euler equation, wherein the Phillips curve was used to determine the optimal inflation coefficients. Hence, given the optimal inflation path, the terms $\sigma\frac{\overline{Y}}{\overline{C}}(\hat{Y}_{t+1} - \hat{Y}_t)$ in the real interest rate, are simply going to be equal to the shocks to the Phillips curve (when they are relevant). Our optimal rules are thus robust towards

Figure 14: **Tracking the real interest rate.**



Notes: The Figure shows the impulse response functions under the Ramsey policy and the simple rule $\hat{i}_t = \widetilde{r}_t + \delta\hat{\pi}_t$, when we assume that $\widetilde{r}_t$ is the real interest rate that satisfies the Fisher equation ($\widetilde{r}_t := \hat{i}_t - E_t\hat{\pi}_{t+1}$). All the graphs correspond to an average maturity of 5 years. The first and the third rows assume that i.i.d shocks under the ad hoc loss function (row 1) and the microfounded loss function (row 3). The second and fourth rows repeat these objects in the case of persistent shocks.

tracking the Fisher equation implied real interest rate.

D Interest rate smoothing objective.

We now provide additional results characterizing optimal policy in the case where the planner's objective function features inflation, output and interest rate smoothing. More specifically, as in Giannoni and Woodford (2003) we assume that the central bank sets inflation, output and interest rate sequences to maximize the following function:

$$-\frac{1}{2}\sum_{t=0}^{\infty}\beta^t E_0\left\{\hat{\pi}_t^2+\lambda_Y\hat{Y}_t^2+\lambda_i\hat{i}_t^2\right\} \tag{49}$$

for $\lambda_Y, \lambda_i \geq 0$.

Maximization of (49) is subject to the dynamic equations (1), (2) and (5) and given the tax rule (6).[42] We again solve for optimal policies with a Lagrangian. Letting $\psi_{\pi,t}$ be the multiplier attached to the Phillips curve constraint, $\psi_{i,t}$, and $\psi_{gov,t}$ the analogous multipliers attached to the Euler equation and the consolidated budget respectively, the first order conditions for the optimum are given by:

$$-\hat{\pi}_t+\Delta\psi_{\pi,t}-\frac{\psi_{i,t-1}}{\beta}+\frac{\bar{b}}{1-\beta\delta}\sum_{l=0}^{\infty}\delta^l\Delta\psi_{gov,t-l}=0 \tag{50}$$

$$-\lambda_Y\hat{Y}_t-\psi_{\pi,t}\kappa_1+\sigma\frac{\overline{Y}}{\overline{C}}(\psi_{i,t}-\frac{\psi_{i,t-1}}{\beta})+\sigma\frac{\overline{Y}}{\overline{C}}\bar{b}\sum_{l=0}^{\infty}\delta^l\Delta\psi_{gov,t-l}+\sigma\frac{\overline{Y}}{\overline{C}}(\overline{G}-\overline{\tau})\psi_{gov,t}=0 \tag{51}$$

$$-\lambda_i\hat{i}_t+\psi_{i,t}=0 \tag{52}$$

$$\frac{\bar{b}}{1-\beta\delta}\left(\psi_{gov,t}-E_t\psi_{gov,t+1}\right)+\phi_{\tau,b}\overline{\tau}E_t\psi_{gov,t+1}=0 \tag{53}$$

In what follows we characterize analytically interest rate rules for two scenarios. First, in a Fisherian model we can derive the optimal rule for any maturity of debt. Second, we can derive the optimal policy rule when $\sigma, \lambda_i > 0$ and $\delta = \lambda_Y = 0$. For other cases, deriving analytical results is not easy. We thus complement our analysis with numerically solved optimized interest rate rules.

D.1 A Fisherian model with output and interest rate smoothing objectives.

We concentrate on the case of active fiscal policy, hence $\Delta\psi_{gov,t} \neq 0$. Setting $\sigma = 0$ the system of first order conditions simplifies to

$$-\hat{\pi}_t-\frac{\lambda_Y}{\kappa_1}\Delta\hat{Y}_t-\frac{\lambda_i\hat{i}_{t-1}}{\beta}+\frac{\bar{b}}{1-\beta\delta}\sum_{l=0}^{\infty}\delta^l\Delta\psi_{gov,t-l}=0 \tag{54}$$

[42] Given optimal policies we can use (4) to solve for $\hat{p}_{t,\delta}$. In other words, we do not have to keep track of the bond price in the optimal policy program.

and using the Phillips curve and the Euler equation we can express this as a function of inflation and the Lagrange multiplier:

$$-\hat{\pi}_t - \frac{\lambda_Y}{\kappa_1^2}(\hat{\pi}_t - \beta E_t \hat{\pi}_{t+1}) + \frac{\lambda_Y}{\kappa_1^2}(\hat{\pi}_{t-1} - \beta E_{t-1}\hat{\pi}_t) - \frac{\lambda_i \hat{\xi}_{t-1}}{\beta} - \frac{\lambda_i \hat{\pi}_t}{\beta} + \frac{\lambda_i(\hat{\pi}_t - E_{t-1}\pi_t)}{\beta} + \\ + \frac{\bar{b}}{1-\beta\delta}\sum_{k=0}^{\infty}\delta^k \Delta\psi_{gov,t-l} = 0$$

Inflation evolves according to:

$$E_t\hat{\pi}_{t+1} - (1 + \widetilde{\kappa}\frac{\lambda_i}{\beta} + \widetilde{\kappa} + \frac{1}{\beta})\hat{\pi}_t + \frac{1}{\beta}\hat{\pi}_{t-1} = -\zeta_t$$

where

$$\zeta_t \equiv (1 + \widetilde{\kappa}\frac{\lambda_i}{\beta})(\hat{\pi}_t - E_{t-1}\hat{\pi}_t) - \widetilde{\kappa}\frac{\lambda_i \hat{\xi}_{t-1}}{\beta} + + \widetilde{\kappa}\frac{\bar{b}}{1-\beta\delta}\sum_{k=0}^{\infty}\delta^k \Delta\psi_{gov,t-l} = 0$$

and $\widetilde{\kappa} = \frac{\kappa_1^2}{\lambda_Y \beta}$.

The two roots of the characteristic polynomial are :

$$\widetilde{\lambda}_{1,2} = \frac{1}{2}\left((1 + \frac{1}{\beta} + \widetilde{\kappa} + \widetilde{\kappa}\frac{\lambda_i}{\beta}) \pm \sqrt{(1 + \frac{1}{\beta} + \widetilde{\kappa} + \widetilde{\kappa}\frac{\lambda_i}{\beta})^2 - \widetilde{\frac{4}{\beta}}}\right)$$

Let $\widetilde{\lambda}_1$ denote the stable root. Inflation solves :

$$\hat{\pi}_t = \frac{1}{\widetilde{\lambda}_2}E_t\hat{\pi}_{t+1} + \frac{1}{\widetilde{\lambda}_2}\frac{1}{1-\widetilde{\lambda}_1 L}\zeta_t = \frac{1}{\widetilde{\lambda}_2}\frac{1}{1-\widetilde{\lambda}_1 L}\sum_{j\geq 0}\frac{1}{\widetilde{\lambda}_2^j}E_t\zeta_{t+j} \tag{55}$$

We can write

$$\sum_{j\geq 0}\frac{1}{\widetilde{\lambda}_2^j}E_t\zeta_{t+j} = \sum_{j\geq 0}\frac{1}{\widetilde{\lambda}_2^j}E_t\left[(1 + \widetilde{\kappa}\frac{\lambda_i}{\beta})(\hat{\pi}_{t+j} - E_{t+j-1}\hat{\pi}_{t+j}) + \widetilde{\kappa}\frac{\bar{b}}{1-\beta\delta}\sum_{k=0}^{\infty}\delta^k\Delta\psi_{gov,t+j-l} - \widetilde{\kappa}\frac{\lambda_i\hat{\xi}_{t+j-1}}{\beta}\right] \\ = -\widetilde{\kappa}\frac{\lambda_i\hat{\xi}_{t-1}}{\beta} - \frac{1}{\widetilde{\lambda}_2}\widetilde{\kappa}\frac{\lambda_i\hat{\xi}_t}{\beta} + (1 + \widetilde{\kappa}\frac{\lambda_i}{\beta})(\hat{\pi}_t - E_{t-1}\hat{\pi}_t) + \widetilde{\kappa}\frac{\bar{b}}{1-\beta\delta}\frac{1}{1-\frac{\delta}{\widetilde{\lambda}_2}}\frac{1}{1-\delta L}\Delta\psi_{gov,t}$$

Putting everything together and using (55) we get:

$$\hat{\pi}_t = \widetilde{\lambda}_1\hat{\pi}_{t-1} + \frac{1}{\widetilde{\lambda}_2}(1 + \widetilde{\kappa}\frac{\lambda_i}{\beta})(\hat{\pi}_t - E_{t-1}\hat{\pi}_t) + \frac{\widetilde{\kappa}}{\widetilde{\lambda}_2}\frac{\bar{b}}{1-\beta\delta}\frac{1}{1-\frac{\delta}{\widetilde{\lambda}_2}}\frac{1}{1-\delta L}\Delta\psi_{gov,t} - \frac{1}{\widetilde{\lambda}_2}\widetilde{\kappa}\frac{\lambda_i\hat{\xi}_{t-1}}{\beta} - \frac{1}{\widetilde{\lambda}_2^2}\widetilde{\kappa}\frac{\lambda_i\hat{\xi}_t}{\beta} \tag{56}$$

and

$$(1 - \frac{1}{\widetilde{\lambda}_2}(1 + \widetilde{\kappa}\frac{\lambda_i}{\beta}))(\hat{\pi}_t - E_{t-1}\hat{\pi}_t) = \frac{\widetilde{\kappa}}{\widetilde{\lambda}_2}\frac{\bar{b}}{1-\beta\delta}\frac{1}{1-\frac{\delta}{\widetilde{\lambda}_2}}\Delta\psi_{gov,t} - \frac{1}{\widetilde{\lambda}_2^2}\widetilde{\kappa}\frac{\lambda_i\hat{\xi}_t}{\beta}$$

The optimal interest rate rule is stated in the following Proposition.

Proposition 6. *Assume $\sigma = 0$ and $\lambda_i, \lambda_Y > 0$. Assume further that fiscal policy is active. The optimal interest rate rule is*

$$\hat{i}_t = \delta \hat{i}_{t-1} + \widetilde{\lambda}_1 \hat{\pi}_t - \widetilde{\lambda}_1 \delta \hat{\pi}_{t-1} + \frac{\widetilde{\kappa}}{\widetilde{\lambda}_2} \frac{\overline{b}}{1-\beta\delta} \frac{\delta}{1-\frac{\delta}{\widetilde{\lambda}_2}} \Delta\psi_{gov,t} + (1 - \frac{1}{\widetilde{\lambda}_2} \widetilde{\kappa} \frac{\lambda_i}{\beta})(\hat{\xi}_t - \delta \hat{\xi}_{t-1})) \tag{57}$$

Proof. Use (56) to find $E_t \hat{\pi}_{t+1}$ and $\hat{i}_t = E_t \hat{\pi}_{t+1} + \hat{\xi}_t$ to obtain (57).

The optimal interest rates follow an inertial rule; $\hat{i}_t$ reacts to current and lagged inflation and to the lagged value of the nominal rate. This is clearly in accordance with the planner's objective when $\lambda_i, \lambda_Y > 0$. Furthermore, note that adjusting the nominal interest rate one for one with the real rate $\hat{\xi}_t$ is not optimal precisely because of the desire to smooth the path of $\hat{i}_t$. Therefore, in (57) both the current and the lagged value of the shock to preferences affect optimal policy. Lastly, the optimal policy tracks the stochastic intercept $\frac{\widetilde{\kappa}}{\widetilde{\lambda}_2} \frac{\overline{b}}{1-\beta\delta} \frac{\delta}{1-\frac{\delta}{\widetilde{\lambda}_2}} \Delta\psi_{gov,t}$. As in the previous analytical results $\Delta\psi_{gov,t}$ can be expressed as a function of the date t shocks. For brevity we omit this derivation.

To close this paragraph we state the optimal interest rate rule in the case where $\lambda_Y = 0$ and $\lambda_i > 0$. This rule cannot be found easily from (57) setting $\lambda_Y = 0$ and so for completeness we show it as a separate result.

Proposition 6'. *Assume $\lambda_i > 0$ and $\lambda_Y = \sigma = 0$. The optimal interest rate rule is:*

$$\hat{i}_t = \frac{\hat{\xi}_t}{1 + \lambda_i/\beta} + \frac{\delta}{1 + \lambda_i/\beta} \hat{\pi}_t + \frac{\delta \lambda_i/\beta}{1 + \lambda_i/\beta} \hat{i}_{t-1}$$

when $\phi_{\tau,b} = 0$, $\psi_{gov,t} \neq 0$ (active fiscal policy).

Proof: The proof is provided in paragraph D.4.1.

D.2 Optimal interest rate policy in the canonical NK model with active fiscal policy.

We now consider the canonical NK model with $\sigma > 0$. As we will illustrate in this subsection deriving analytical results for the optimal interest rate sequence when $\sigma, \lambda_i > 0$ is not easy. In contrast to the case without interest rate smoothing studied in text, under the interest rate smoothing objective, the Euler equation is a constraint in the planner's program and the first order conditions (50) to (53) feature the current and lagged values of the nominal interest rate. We will show by means of an analytical example (corresponding to the simplest case possible, $\lambda_Y = \delta = 0$) that the optimal inflation coefficient solves a non-linear equation whose roots are not feasible to find analytically. We will thus resort to the numerical solution of the model to characterize optimal interest rate rules more generally.

D.3 A partially analytical example

Consider first $\lambda_i, \sigma > 0$ but $\lambda_Y = 0$. Assume further that debt is short term, $\delta = 0$. For simplicity (and wlog) we will consider only the case of spending shocks. We will show that a simple inflation targeting rule of the form: $\hat{i}_t = \phi_G \hat{G}_t + \phi_\pi \hat{\pi}_t$ can fit the Ramsey optimal policy. Moreover, we will characterize the coefficients using the impulse response function. In other words, the rule $\hat{i}_t = \phi_G \hat{G}_t + \phi_\pi \hat{\pi}_t$ will fit the Ramsey outcome following a shock to spending. This simplifies our derivations considerably, however, the reader should note that our approach in this subsection is more restrictive than in the main text (where we were able to accomplish a general characterization of optimal interest rate policies). The analytics of this section will force all lagged variables of the model to be equal to zero.

Combining the first order conditions under the assumed parameter values gives:

$$-\hat{\pi}_t + \frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}(\lambda_i \Delta\hat{i}_t - \frac{\lambda_i}{\beta}\Delta\hat{i}_{t-1}) + \frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\bar{b}(\Delta\psi_{gov,t} - \Delta\psi_{gov,t-1}) + \frac{\omega_Y}{\kappa_1}\Delta\psi_{gov,t} - \frac{\lambda_i \hat{i}_{t-1}}{\beta} + \bar{b}\Delta\psi_{gov,t} = 0$$

Using the interest rate rule along with the initial conditions $\hat{i}_{t-1} = \hat{i}_{t-2}, \Delta\psi_{gov,t-1} = 0$ and the equilibrium $\Delta\psi_{gov,t} = \epsilon\hat{G}_t$ we can write:

$$-\hat{\pi}_t + \frac{\sigma\lambda_i}{\kappa_1}\frac{\overline{Y}}{\overline{C}}(\phi_G\hat{G}_t + \phi_\pi\hat{\pi}_t) + (\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\bar{b} + \frac{\omega_Y}{\kappa_1} + \bar{b})\epsilon\hat{G}_t = 0$$

Next the first order condition in $t+1$ becomes:

$$-\hat{\pi}_{t+1} + \frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}(\lambda_i\Delta\hat{i}_{t+1} - \frac{\lambda_i}{\beta}\hat{i}_t) - \frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\bar{b}\epsilon\hat{G}_t - \frac{\lambda_i\hat{i}_t}{\beta} = 0$$

or

$$\hat{\pi}_{t+1}\underbrace{(1 - \frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\lambda_i\phi_\pi)}_{\zeta_1} = -\underbrace{(\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\bar{b}\epsilon + \lambda_i(\frac{1}{\beta} + \frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}(1+\frac{1}{\beta}))\phi_G)}_{\zeta_2}\hat{G}_t - \underbrace{\lambda_i(\frac{1}{\beta} + \frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}(1+\frac{1}{\beta}))\phi_\pi}_{\zeta_3}\hat{\pi}_t$$

Finally, inflation in $t+2$ solves:

$$-\hat{\pi}_{t+2} + \frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}(\lambda_i\Delta\hat{i}_{t+2} - \frac{\lambda_i}{\beta}\Delta\hat{i}_{t+1}) - \frac{\lambda_i\hat{i}_{t+1}}{\beta} = 0$$

or

$$\hat{\pi}_{t+2}\zeta_1 = -\zeta_3\hat{\pi}_{t+1} + \frac{\lambda_i}{\beta}\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}(\phi_\pi\hat{\pi}_t + \phi_G\hat{G}_t) = \zeta_3(\frac{\zeta_3}{\zeta_1}\hat{\pi}_t + \frac{\zeta_2}{\zeta_1}\hat{G}_t) + \frac{\lambda_i}{\beta}\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}(\phi_\pi\hat{\pi}_t + \phi_G\hat{G}_t)$$

Dropping the shocks for simplicity and using the policy rule the Euler equation in t can be written as:

$$\phi_\pi\hat{\pi}_t = \left(\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}(1+\beta) + 1\right)\hat{\pi}_{t+1} - \frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\hat{\pi}_t - \beta\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\hat{\pi}_{t+2}$$

We thus get:

$$\boxed{\phi_\pi = -\left(\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}(1+\beta) + 1\right)\frac{\zeta_3}{\zeta_1} - \frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}} - \beta\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}(\frac{\zeta_3^2}{\zeta_1^2} + \frac{\lambda_i}{\beta}\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\frac{\phi_\pi}{\zeta_1})} \tag{58}$$

(58) is a a nonlinear equation in coefficient ϕ_π. Solving it analytically to obtain the value of the

coefficient is obviously not trivial. We thus illustrated that deriving complete analytical results in the canonical model with interest rate smoothing is not easy. We next turn to the numerical solution of the model to investigate the features of optimal policy rules when $\delta > 0$ and $\lambda_Y > 0$.

D.4 Numerical examples

To fix ideas it is useful to rearrange the first order conditions and derive the following expression for the nominal interest rate in the model.

Proposition 7. *The interest rate policy can be expressed as:*

$$\hat{i}_t = \underbrace{\widetilde{\phi}_\pi \hat{\pi}_t + \widetilde{\phi}_Y \Delta \hat{Y}_t + \widetilde{\phi}_i \hat{i}_{t-1} + \frac{1}{\beta}\Delta \hat{i}_{t-1}}_{\text{Giannoni and Woodford (2003)}} + \mathcal{D}_t \tag{59}$$

with $\widetilde{\phi}_\pi = \frac{\kappa_1 \overline{C}}{\lambda_i \sigma \overline{Y}}$, $\widetilde{\phi}_i = (1 + \frac{\kappa_1}{\beta\sigma}\frac{\overline{C}}{\overline{Y}})$ *and* $\widetilde{\phi}_Y = \frac{\lambda_Y \overline{C}}{\sigma \lambda_i \overline{Y}}$. *Moreover,*

$$\mathcal{D}_t = -\frac{\overline{C}}{\overline{Y}}\frac{\kappa_1}{\lambda_i \sigma}\frac{\overline{b}}{1-\beta\delta}\sum_{l=0}^{\infty}\delta^l \Delta\psi_{gov,t-l} - \frac{\overline{b}}{\lambda_i}\sum_{l=0}^{\infty}\delta^l\Big(\Delta\psi_{gov,t-l} - \Delta\psi_{gov,t-l-1}\Big) - \frac{(\overline{G}-\overline{\tau})}{\lambda_i}\Delta\psi_{gov,t}$$

Proof: The proof requires to simply combine the FONC and for the sake of brevity we omit it. ■

According to (59) the solution for the nominal interest rate can be decomposed in two distinct components .

The first (labeled Giannoni and Woodford (2003) in underbrace) is the standard optimal superinertial rule derived in Giannoni and Woodford (2003). It links interest rates to inflation, output growth and lagged values of the interest rate. The impact of these variables on $\hat{i}_t$ depends on the weights λ_i, λ_Y that capture the output and interest rate stabilization objectives of the central bank and on the structural parameters σ, κ_1 and β.[43]

The second component, $\mathcal{D}_t$, is a weighted sum of the current and lagged values of the growth of the multiplier $\Delta\psi_{gov}$. As discussed in text, the lagged values of the multiplier are the promises made by the planner to alter inflation and output following shocks that have hit the consolidated budget in the past. These terms enter in $\mathcal{D}_t$ because changes in inflation and output will influence the path of the nominal interest rate.

What we are interested in is to investigate optimal rules in which object $\mathcal{D}_t$ is not included in the specification and therefore $\hat{i}_t$ is a function of macroeconomic variables only, inflation output and lagged interest rates. As discussed previously, establishing such policies analytically is not easy and we thus need to turn to numerical experiments.

We assume that monetary policy sets the nominal interest rate according to:

$$\hat{i}_t = \phi_\pi \hat{\pi}_t + \phi_Y \hat{Y}_t + \phi_i \hat{i}_{t-1} + \phi_{\Delta i}\Delta \hat{i}_{t-1} \tag{60}$$

We will estimate the values of coefficients $\phi_\pi, \phi_Y, \phi_i, \phi_{\Delta i}$ so that the model when interest rates follow (60) approximates the optimal policy model.

[43]Thus, under passive fiscal policy the optimal rule is the one of Giannoni and Woodford (2003). This of course is not surprising since when the consolidated budget is irrelevant for optimal policy our model is essentially the three equation NK model with the same objective function as Giannoni and Woodford (2003).

Table 2 shows the estimates of the coefficients.[44] The table is split in 3 sub-tables. The first two correspond to the active fiscal policy model. We consider several calibrations: We set $\delta = 0.5$ in the left, and $\delta = 0.95$ in the middle. Moreover, in the top panel of each subtable we let $\lambda_Y = 0$, in which case we also constrain ϕ_Y to be equal to 0, whereas the bottom panel assumes $\lambda_Y = 0.5$, thus setting the weight to output stabilization in the policy objective to be half of the weight attached to inflation stabilization. Each of the columns of the subtables corresponds to a different calibration of the pair λ_i, σ.

For comparison, the right part of the Table shows the coefficients in the passive fiscal model corresponding to each calibration considered. Finally, the assumed values for the remaining model parameters are reported in Table 1 (see the notes of that table for a brief discussion of the calibration).

Several results stand out. First, note that (not surprisingly) under the estimated values of ϕ_i and $\phi_{\Delta i}$ monetary policy follows passive money rules.[45] In contrast to the *super-inertial* policy of Giannoni and Woodford, 2003 (where coefficients are such that the rule contains an explosive root), under active fiscal policy, simple inertial rules featuring only up to one lag of the nominal rate are approximately optimal.

Second, the estimated coefficients vary as we vary the values of parameters $\sigma, \lambda_i, \lambda_Y,$. A key determinant of the estimates is the debt maturity. This is easily noticeable in the top panels of the left and middle sub-tables. Assuming $\sigma = 1$ and $\lambda_i = 0.5$ yields an estimate ϕ_π of 0.21 when $\delta = 0.5$ and 0.43 when $\delta = 0.95$. Moreover, when we assume $\lambda_i = 1$ we have -0.10 and 0.52 for $\delta = 0.5$ and $\delta = 0.95$ respectively. Thus, higher debt maturity yields a stronger reaction of monetary policy to inflation, implying a more persistent response of inflation to the shock. This feature can be easily understood based on our results we derived in the paper.

To better illustrate the properties of optimal policy under the interest rate smoothing objective in Figure 15 we plot the IRFS, assuming $\lambda_Y = 0$ (top panel) and $\lambda_Y = 0.5$ (bottom). The blue lines are the baseline with no interest rate smoothing, the dashed and dotted lines correspond to $\lambda_i = 0.5$ and $\lambda_i = 1$ respectively. We further assume $\sigma = 1$ (our baseline value for this parameter). Concentrate on the second and fourth rows of the graph, assuming $\delta = .95$. Relative to the baseline, a positive coefficient λ_i implies that the interest rate response is hump shaped. Thus, interest rates increase less when the shock hits and continue increasing until roughly period 5. As can be seen from the left plots, this reaction of monetary policy leads inflation and output to react more strongly to the shock on impact. After 5 quarters, all macroeconomic variables begin to converge to the target values at rates at rates equal or close to δ.

It is quite evident that the response of the interest rate when $\delta = .95$ can be matched by an inertial rule with a positive inflation coefficient. This confirms the top panel of Table 2. Simple

[44]We produced these estimates through matching the impulse responses of the optimal policy model. We focused on matching the responses to the spending shock only. Note that (60) could produce a perfect fit in some of the cases reported in Table 2. In other cases this was not so and to improve the fit we added the spending shock as an additional argument to the rule. We therefore estimated

$$\hat{i}_t = \phi_G \hat{G}_t + \phi_\pi \hat{\pi}_t + \phi_Y \hat{Y}_t + \phi_i \hat{i}_{t-1} + \phi_{\Delta i} \Delta \hat{i}_{t-1}$$

This turned out being useful when $\delta = 0.95$. Presumably, we would need to add further lags of inflation, output and interest rates in (60) to avoid introducing $\hat{G}_t$. We did not want to extend to further lags, as we wanted to maintain as closely as possible the structure of Proposition 1. Coefficient ϕ_G was found to be small in magnitude and we do not report it in the Table.

[45]The rules obviously have to satisfy the passive policy requirement, otherwise the Blanchard-Kahn condition would not be satisfied and the model solution could not be found.

In the numerical examples shown in Table the estimated coefficients ϕ_π, ϕ_Y are smaller in magnitude than $\widetilde{\phi}_\pi, \widetilde{\phi}_Y$ in the Giannoni and Woodford, 2003 policy. This is not a necessary property of the passive money policy. Under the super-inertial rule, monetary policy is active for any positive values of coefficients ϕ_π, ϕ_Y. Thus ϕ_π, ϕ_Y could even be smaller than in the active fiscal scenario.

inertial rules would also likely produce a very god fit when we assume $\lambda_Y = 0.5$ (both output and interest rate smoothing) , but in Table 2 we instead explored rules targeting both inflation and output, to contrast with the results of Giannoni and Woodford, 2003.

We conclude that simple rules are approximately optimal in the case of the interest rate smoothing objective when $\delta = .95$. Establishing an explicit formula however is much more tedious than the dual mandate case considered in text.

D.4.1 Proof of Proposition 6'

Consider $\lambda_i > 0$ and $\lambda_Y = \sigma = 0$ and the case of active monetary policy. Conjecture that optimal policy is a rule of the form

$$\hat{i}_t = \theta\hat{\xi}_t + \widetilde{\lambda}_1\hat{\pi}_t + \lambda_2\hat{i}_{t-1} \tag{61}$$

From the FONC of the Ramsey program, inflation satisfies:

$$\hat{\pi}_t = -\frac{\lambda_i}{\beta}\hat{i}_{t-1} \tag{62}$$

and therefore $\hat{i}_t = \frac{1}{1+\frac{\lambda_i}{\beta}}\hat{\xi}_t$.

Now use (61) and the Euler equation to get:

$$\hat{i}_t = \frac{1}{1-\widetilde{\lambda}_2 L}\left(\theta\hat{\xi}_t + \widetilde{\lambda}_1\hat{\pi}_t\right) = \hat{\xi}_t + E_t\hat{\pi}_{t+1}$$

and when $\widetilde{\lambda}_1 + \widetilde{\lambda}_2 > 1$ we have

$$\hat{\pi}_t = E_t\sum_{j\geq 0}\frac{1}{(\widetilde{\lambda}_1+\widetilde{\lambda}_2)^j}\left(\frac{1-\theta}{(\widetilde{\lambda}_1+\widetilde{\lambda}_2)}\hat{\xi}_{t+j} - \frac{\widetilde{\lambda}_2}{(\widetilde{\lambda}_1+\widetilde{\lambda}_2)}\hat{\xi}_{t+j-1}\right) = \frac{1-\theta}{(\widetilde{\lambda}_1+\widetilde{\lambda}_2)}\hat{\xi}_t - \frac{\widetilde{\lambda}_2}{(\widetilde{\lambda}_1+\widetilde{\lambda}_2)}\hat{\xi}_{t-1} - \frac{\widetilde{\lambda}_2}{(\widetilde{\lambda}_1+\widetilde{\lambda}_2)^2}\hat{\xi}_t$$

where the final equality uses the assumption that shocks are i.i.d. From this equation, we can easily derive the conditions on parameters $\theta, \widetilde{\lambda}_1, \widetilde{\lambda}_2$ stated in the Proposition.

Now consider the case of passive monetary policy. Optimal inflation satisfies:

$$\hat{\pi}_t = -\frac{\lambda_i}{\beta}\hat{i}_{t-1} + \frac{\bar{b}}{1-\beta\delta}\sum_{l\geq 0}\delta^l\Delta\psi_{gov,t-l}$$

From the Euler equation we have:

$$\begin{aligned}\hat{i}_t = \hat{\xi}_t + E_t\left(-\frac{\lambda_i}{\beta}\hat{i}_t + \frac{\bar{b}}{1-\beta\delta}\sum_{l\geq 0}\delta^l\Delta\psi_{gov,t-l+1}\right) &= \hat{\xi}_t - \frac{\lambda_i}{\beta}\hat{i}_t + \frac{\delta\bar{b}}{1-\beta\delta}\sum_{l\geq 0}\delta^l\Delta\psi_{gov,t-l}\\ &= \hat{\xi}_t - \frac{\lambda_i}{\beta}\hat{i}_t + \delta\left(\hat{\pi}_t + \frac{\lambda_i}{\beta}\hat{i}_{t-1}\right)\end{aligned}$$

Thus :

$$\hat{i}_t = \frac{1}{1+\frac{\lambda_i}{\beta}}\hat{\xi}_t + \frac{\delta}{1+\frac{\lambda_i}{\beta}}\hat{\pi}_t + \frac{\frac{\lambda_i}{\beta}}{1+\frac{\lambda_i}{\beta}}\hat{i}_{t-1}$$

Table 2: **Model implied Taylor rules**

Active FP $\delta = 0.5$

	$\sigma = 1$		$\sigma = 2$	
	$\lambda_i = 0.5$	$\lambda_i = 1$	$\lambda_i = 0.5$	$\lambda_i = 1$
ϕ_π	0.214	-0.102	-0.131	-0.262
ϕ_Y	0	0	0	0
ϕ_i	0.378	0.273	0.254	0.052
ϕ_{Δ_i}	0.006	0.067	0.003	0.001
ϕ_π	0.533	0.516	0.449	0.352
ϕ_Y	0.367	0.233	0.234	0.072
ϕ_i	0.165	0.21	0.222	0.208
ϕ_{Δ_i}	0.025	0.032	-0.008	-0.034

Active FP $\delta = 0.95$

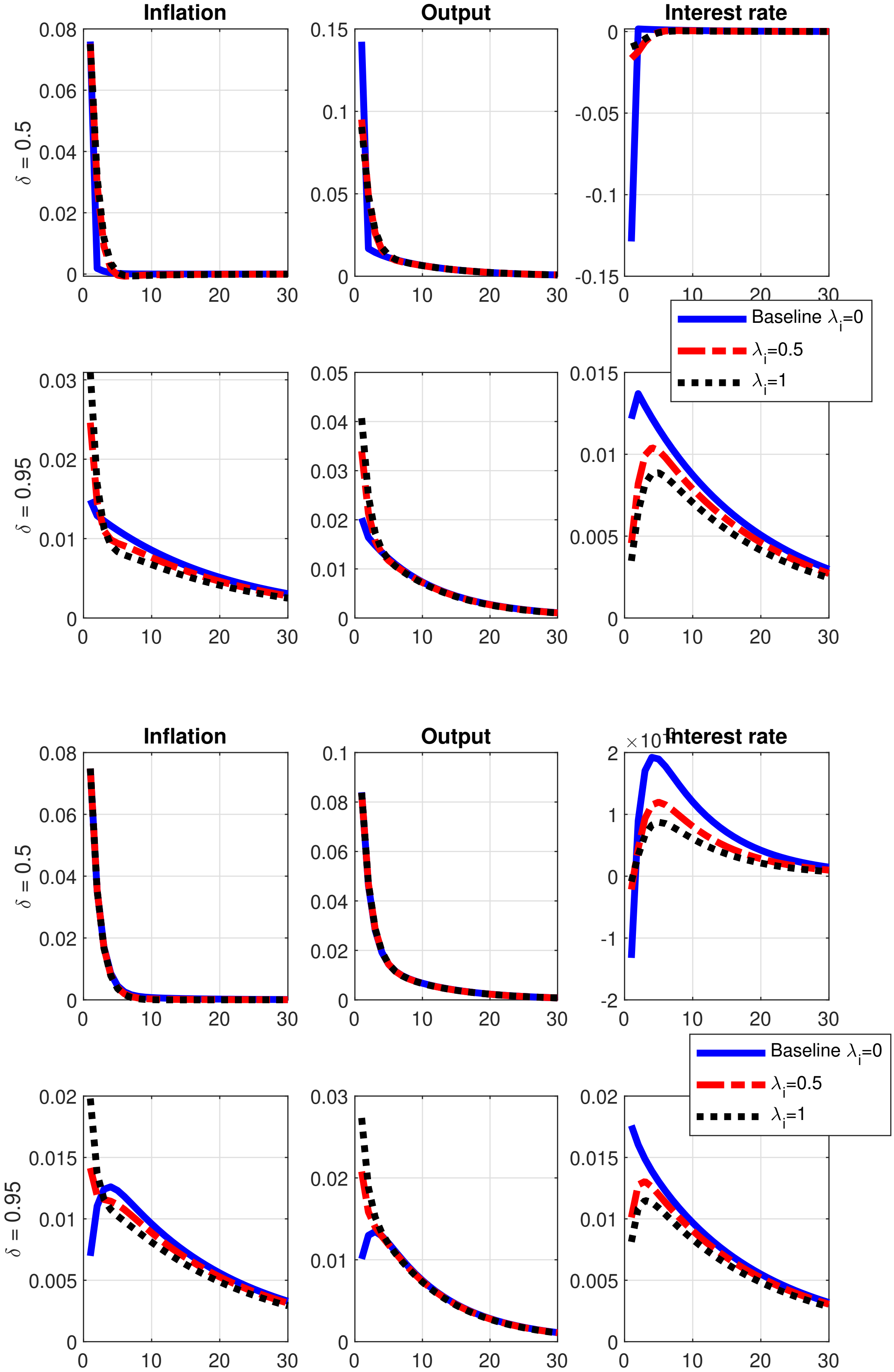
	$\sigma = 1$		$\sigma = 2$	
	$\lambda_i = 0.5$	$\lambda_i = 1$	$\lambda_i = 0.5$	$\lambda_i = 1$
ϕ_π	0.43	0.522	0.4	0.266
ϕ_Y	0	0	0	0
ϕ_i	0.518	0.478	0.547	0.682
ϕ_{Δ_i}	0	0.005	0	0
ϕ_π	0.618	0.458	0.502	0.356
ϕ_Y	0.448	0.333	0.393	0.216
ϕ_i	0.335	0.493	0.447	0.593
ϕ_{Δ_i}	-0.024	-0.035	-0.022	-0.051

Passive FP

	$\sigma = 1$		$\sigma = 2$	
	$\lambda_i = 0.5$	$\lambda_i = 1$	$\lambda_i = 0.5$	$\lambda_i = 1$
ϕ_π	1.65	0.82	0.82	0.41
ϕ_Y	0	0	0	0
ϕ_i	1.83	1.83	1.41	1.41
ϕ_{Δ_i}	1.005	1.005	1.005	1.005
ϕ_π	1.65	0.82	0.82	0.41
ϕ_Y	0.90	0.45	0.45	0.225
ϕ_i	1.83	1.83	1.41	1.41
ϕ_{Δ_i}	1.005	1.005	1.005	1.005

Notes: The tables report model implied Taylor rules $\hat{i}_t = \phi_\pi \hat{\pi}_t + \phi_Y \Delta \hat{Y}_t + \phi_i \hat{i}_{t-1} + \phi_{\Delta_i} \Delta \hat{i}_{t-1}$. The left table assumes that fiscal policy is active, setting $\delta = 0.5$. The middle table also considers active fiscal policy but sets $\delta = 0.95$. The right table reports the coefficients of the Giannoni and Woodford (2003) optimal rule, under passive fiscal policy. In the top panels of each table we assume $\lambda_Y = 0$ and in the bottom panels $\lambda_Y = 0.5$. The values of parameters λ_i, σ corresponding to each of the columns of the tables are stated at the top. The remaining model parameters are assigned the values reported in Table 1.

Figure 15: **Rules vs Ramsey: Steady State Output Target I**



Notes: The figure compares the optimal policy impulse responses of inflation, output, and the nominal interest rate when the loss function features an interest rate smoothing objective. The baseline $\lambda_i = 0$ is shown for comparison using the blue solid line. The red line corresponds to the case $\lambda_i = 0.5$ and the black line to the case $\lambda_i = 1$. We set $\lambda_Y = 0$ in the top sub-figure and $\lambda_Y = 0.5$ in the bottom.

■

E Extensions: Distortionary Taxes and Complete Markets

In this section we present two further extensions of our model. First, we study optimal policy in the model with distortionary taxes and show that our results continue to hold in this case. Second, we consider the a model with complete markets assuming that government debt is state contingent. We characterize optimal monetary policy rules in this case.

E.1 Optimal Policies with Distortionary Taxation

We first present analytical results for the case where taxes are distortionary. Under this assumption the two model equations that need to be changed are the Phillips curve and the government budget constraint. We now have:

$$\hat{\pi}_t = \kappa_1 \hat{Y}_t - \kappa_2 \hat{G}_t + \kappa_3 \hat{\tau}_t + \beta E_t \hat{\pi}_{t+1},$$

where $\kappa_1 \equiv -\frac{(1+\eta)\overline{Y}}{\theta}(\gamma_h + \sigma\frac{\overline{Y}}{\overline{C}}) > 0$, $\kappa_3 \equiv -\frac{(1+\eta)\overline{Y}}{\theta}\frac{\overline{\tau}^d}{(1-\overline{\tau}^d)} \geq 0$, $\kappa_2 \equiv -\frac{(1+\eta)}{\theta\overline{Y}}\sigma\frac{\overline{G}}{\overline{C}} > 0$, and where $\overline{\tau}^d$ denotes the steady state distortionary tax rate.

Moreover, now the surplus of the government becomes a function of output and we have:

$$\overline{s}\hat{S}_t \equiv \left[-\overline{G}\left(\hat{G}_t(1+\sigma\frac{\overline{G}}{\overline{C}}) - \sigma\frac{\overline{Y}}{\overline{C}}\hat{Y}_t + \hat{\xi}_t\right) + \overline{r}\left(\hat{R}_t + \hat{\xi}_t\right)\right]$$

$\overline{r}$ is the steady state revenue of the government, and $\hat{R}_t$ denotes the revenue scaled by marginal utility. We have:

$$\hat{R}_t = \left((1+\gamma_h)\hat{Y}_t + \frac{\hat{\tau}_t}{1-\overline{\tau}^d}\right)$$

We continue assuming that fiscal policy is given by (6).

Optimal policy solves the following system of equations:

$$-\hat{\pi}_t + \Delta\psi_{\pi,t} + \frac{\overline{b}}{1-\beta\delta}\sum_{l=0}^{\infty}\delta^l \Delta\psi_{gov,t-l} = 0 \tag{63}$$

$$-\lambda_Y \hat{Y}_t - \psi_{\pi,t}\kappa_1 + \sigma\frac{\overline{Y}}{\overline{C}}\overline{b}\sum_{l=0}^{\infty}\delta^l \Delta\psi_{gov,t-l} + \omega_Y \psi_{gov,t} = 0 \tag{64}$$

$$\frac{\beta\overline{b}}{1-\beta\delta}\left(\psi_{gov,t} - E_t\psi_{gov,t+1}\right) - \beta\phi_{\tau,b}E_t\left(\kappa_3\psi_{\pi,t+1} - \overline{r}\frac{d\hat{R}_t}{d\hat{\tau}_t}\psi_{gov,t+1}\right) = 0 \tag{65}$$

where now $\omega_Y \equiv \overline{G}\sigma\frac{\overline{Y}}{\overline{C}} + \overline{r}(1+\gamma_h)$.

E.1.1 The two equilibria under distortionary taxes

It is possible to show that this model admits two equilibria under active and passive fiscal policies. However, we now need to separately treat the cases where $\lambda_Y = 0$ and $\lambda_Y > 0$. As we explain below, in the case $\lambda_Y > 0$ we need to modify the objective of the planner slightly assuming that the target is the natural level of output (not the steady state level). Unless we make this assumption the model may not admit an equilibrium in which monetary policy is active.

Case 1: $\lambda_Y = 0$ Assume first that $\lambda_Y = 0$. (65) can be written as:

$$\frac{\beta \bar{b}}{1-\beta\delta}\Bigg(\psi_{gov,t} - E_t\psi_{gov,t+1}\Bigg) - \beta\phi_{\tau,b}E_t\Bigg(\kappa_3\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\bar{b}\sum_{l=0}^{\infty}\delta^l\Delta\psi_{gov,t-l+1} + \frac{\kappa_3\omega_Y}{\kappa_1}\psi_{gov,t+1} - \frac{\overline{r}}{1-\overline{\tau}^d}\psi_{gov,t+1}\Bigg) = 0 \tag{66}$$

and so

$$\widetilde{\eta}_1\Bigg(\psi_{gov,t} - E_t\psi_{gov,t+1}\Bigg) - \beta\phi_{\tau,b}E_t\Bigg(\kappa_3\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\bar{b}\frac{\Delta\psi_{gov,t+1}}{1-\delta L} + \widetilde{\eta}_2\psi_{gov,t+1}\Bigg) = 0$$

where $\widetilde{\eta}_2 < 0$ for the economy to be at the upward sloping part of the Laffer curve.

The above can be rearranged into a second order difference equation:

$$-\Bigg(\widetilde{\eta}_1 + \beta\phi_{\tau,b}(\widetilde{\eta}_2 + \kappa_3\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\bar{b})\Bigg)E_t\psi_{gov,t+1} + \Bigg(\widetilde{\eta}_1(1+\delta) + \beta\phi_{\tau,b}(\widetilde{\eta}_2\delta + \kappa_3\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\bar{b})\Bigg)\psi_{gov,t} - \delta\widetilde{\eta}_1\psi_{gov,t-1} = 0$$

or

$$E_t\psi_{gov,t+1} - \Bigg(1 - (1-\delta)\widetilde{\eta}_2\frac{\beta\phi_{\tau,b}}{\widetilde{\eta}_3} + \frac{\delta\widetilde{\eta}_1}{\widetilde{\eta}_3}\Bigg)\psi_{gov,t} + \frac{\delta\widetilde{\eta}_1}{\widetilde{\eta}_3}\psi_{gov,t-1} = 0$$

where $\widetilde{\eta}_3 = \Bigg(\widetilde{\eta}_1 + \beta\phi_{\tau,b}(\widetilde{\eta}_2 + \kappa_3\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\bar{b})\Bigg)$. It is easy to show that the characteristic polynomial has two roots, one stable and one unstable. The unique non-explosive solution is thus $\psi_{gov,t} = 0$.

We can now derive the threshold value $\phi_{\tau,b}$. From the first order condition of inflation it is easy to show that when $\psi_{gov,t} = 0$, $\hat{\pi}_t = 0$ for all t. With distortionary taxation output evolves according to:

$$\hat{Y}_t = -\frac{\kappa_3}{\kappa_1}\hat{\tau}_t + \frac{\kappa_2}{\kappa_1}\hat{G}_t$$

The Euler equation then gives us:

$$\hat{i}_t = \hat{\xi}_t + \sigma\frac{\overline{Y}}{\overline{C}}E_t\Bigg(-\frac{\kappa_3}{\kappa_1}\Delta\hat{\tau}_{t+1} + \frac{\kappa_2}{\kappa_1}\Delta\hat{G}_{t+1}\Bigg) - \sigma\frac{\overline{G}}{\overline{C}}E_t\Delta\hat{G}_{t+1}$$

Analogously, the price of long term bonds evolves according to:

$$\hat{p}_t = -\hat{\xi}_t - \sigma\frac{\overline{Y}}{\overline{C}}E_t\Bigg(-\frac{\kappa_3}{\kappa_1}\Delta\hat{\tau}_{t+1} + \frac{\kappa_2}{\kappa_1}\Delta\hat{G}_{t+1}\Bigg) + \sigma\frac{\overline{G}}{\overline{C}}E_t\Delta\hat{G}_{t+1} + \beta\delta E_t\hat{p}_{t+1}$$

For simplicity, let us suppress the shocks (This does not change anything with regard to the threshold $\widetilde{\phi}_\tau$). Using the above expression we can then write:

$$\hat{p}_t = \sigma\frac{\overline{Y}}{\overline{C}}\frac{\kappa_3}{\kappa_1}\phi_{\tau,b}(\hat{b}_t - \hat{b}_{t-1}) + \beta\delta E_t\hat{p}_{t+1} \tag{67}$$

Moreover, the consolidated budget constraint (again without shocks) can written as:

$$\beta\overline{b}\Big(\hat{b}_t+\hat{p}_t\Big)=\overline{b}\hat{b}_{t-1}+\beta\delta\overline{b}\hat{p}_t+\overline{r}(1-\beta\delta)\phi_{\tau,b}\Big(\frac{\kappa_3}{\kappa_1}(1+\gamma_h+\sigma\frac{\overline{Y}}{\overline{C}})-\frac{1}{1-\overline{\tau}^d}\Big)\hat{b}_{t-1} \tag{68}$$

(67) and (68) form the system of equations that needs to be resolved.

$$\underbrace{\begin{bmatrix}\beta\delta & \widetilde{\epsilon}\\ 0 & \beta\overline{b}\end{bmatrix}}_{\equiv A}\begin{pmatrix}E_t\hat{p}_{t+1}\\ \hat{b}_t\end{pmatrix}=\underbrace{\begin{bmatrix}1 & \widetilde{\epsilon}\\ \overline{b}\beta(\delta-1) & \overline{b}+\widetilde{\chi}\end{bmatrix}}_{\equiv B}\begin{pmatrix}\hat{p}_t\\ \hat{b}_{t-1}\end{pmatrix}$$

where $\widetilde{\chi}\equiv\overline{R}(1-\beta\delta)\phi_{\tau,b}\Big(\frac{\kappa_3}{\kappa_1}(1+\gamma_h+\sigma\frac{\overline{Y}}{\overline{C}})-\frac{1}{1-\overline{\tau}^d}\Big)$ and $\widetilde{\epsilon}\equiv\sigma\frac{\overline{Y}}{\overline{C}}\frac{\kappa_3}{\kappa_1}\phi_{\tau,b}$.

Then

$$A^{-1}B=\frac{1}{\det(A)}\begin{bmatrix}\beta\overline{b}-\widetilde{\epsilon}\beta\overline{b}(\delta-1) & \widetilde{\epsilon}(\beta-1)\overline{b}-\widetilde{\epsilon}\widetilde{\chi}\\ \beta^2\delta\overline{b}(\delta-1) & \beta\delta(\overline{b}+\widetilde{\chi})\end{bmatrix}$$

The characteristic equation is:

$$\lambda^2-\frac{\lambda}{\det(A)}\Big(\beta\overline{b}+\widetilde{\epsilon}\beta\overline{b}(1-\delta)+\beta\delta(\overline{b}+\widetilde{\chi})\Big)+\frac{1}{\det(A)^2}\Big(\beta^2\delta\overline{b}(\overline{b}+\widetilde{\chi})+\widetilde{\epsilon}\beta^3\delta(1-\delta)\overline{b}^2\Big)$$

The smallest root is:

$$\lambda_1=\frac{1}{2}\frac{1}{\det(A)}\Bigg[\Big(\beta\overline{b}+\widetilde{\epsilon}\beta\overline{b}(1-\delta)+\beta\delta(\overline{b}+\widetilde{\chi})\Big)-$$
$$-\sqrt{\Big(\beta\overline{b}+\widetilde{\epsilon}\beta\overline{b}(1-\delta)+\beta\delta(\overline{b}+\widetilde{\chi})\Big)^2-4\Big(\beta^2\delta\overline{b}(\overline{b}+\widetilde{\chi})+\widetilde{\epsilon}\beta^3\delta(1-\delta)\overline{b}^2\Big)}\Bigg]$$

It is easy to show that $\lambda_1=1$ when $\overline{b}+\widetilde{\chi}=\beta\overline{b}$, or

$$\phi_{\tau,b}=\frac{\overline{b}(1-\beta)}{\overline{r}(1-\beta\delta)\Big(\frac{1}{1-\overline{\tau}^d}-\frac{\kappa_3}{\kappa_1}(1+\gamma_h+\sigma\frac{\overline{Y}}{\overline{C}})\Big)}\equiv\widetilde{\phi}_\tau$$

which is the expression shown in Section 5.3 in text.

Moreover, we can show that λ_1 is monotonically decreasing in $\phi_{\tau,b}$ and $\lambda_1<1$ when $\phi_{\tau,b}>\widetilde{\phi}_\tau$. Finally, the largest root always exceeds 1. Thus the unique stable equilibrium is attained when $\phi_{\tau,b}>\widetilde{\phi}_\tau$.

Case 2: $\lambda_Y>0$ Consider now the case where $\lambda_Y>0$. Now an equilibrium under passive fiscal policy where $\psi_{gov,t}=0$ may not exist. To see this, note that (65) can now be written as:

$$\frac{\beta\overline{b}}{1-\beta\delta}\Big(\psi_{gov,t}-E_t\psi_{gov,t+1}\Big)-\beta\phi_{\tau,b}E_t\Big(-\frac{\kappa_3}{\kappa_1}\lambda_Y\hat{Y}_{t+1}+\kappa_3\frac{\sigma}{\kappa_1}\frac{\overline{Y}}{\overline{C}}\overline{b}\sum_{l=0}^{\infty}\delta^l\Delta\psi_{gov,t-l+1}+\frac{\kappa_3\omega_Y}{\kappa_1}\psi_{gov,t+1}$$
$$-\frac{\overline{r}}{1-\overline{\tau}^d}\psi_{gov,t+1}\Big)=0$$

Following the notation of the previous subsection this can be rearranged into:

$$E_t\psi_{gov,t+1} - \left(1 - (1-\delta)\widetilde{\eta}_2\frac{\beta\phi_{\tau,b}}{\widetilde{\eta}_3} + \frac{\delta\widetilde{\eta}_1}{\widetilde{\eta}_3}\right)\psi_{gov,t} + \frac{\delta\widetilde{\eta}_1}{\widetilde{\eta}_3}\psi_{gov,t-1} = -\frac{\beta\phi_{\tau,b}\kappa_3\lambda_Y}{\kappa_1\widetilde{\eta}_3}\left(E_t\hat{Y}_{t+1} - \delta\hat{Y}_t\right)$$

Notice that the forcing term $E_t\hat{Y}_{t+1} - \delta\hat{Y}_t$ on the RHS of the above equation precludes a solution where $\psi_{gov,t} = 0$. To see this, suppose that indeed $\psi_{gov,t}$ were equal to 0. Then, it should also be that $E_t\hat{Y}_{t+1} - \delta\hat{Y}_t = 0$. From the first order conditions of inflation and output we would get: $\hat{\pi}_t = \frac{\lambda_Y}{\kappa_1}(\hat{Y}_t - \hat{Y}_{t-1})$. Using this and the Phillips curve we can derive a difference equation in aggregate output with forcing term $-\frac{\kappa_3}{\kappa_1}\hat{\tau}_t + \frac{\kappa_2}{\kappa_1}\hat{G}_t$. Aggregate output will generally not equal zero, and $E_t\hat{Y}_{t+1} - \delta\hat{Y}_t$ will not be zero either.

To interpret the above, notice that when taxes are distortionary they become a cost push shock in the Phillips curve which drives the inflation output tradeoff in the equilibrium with active fiscal policy. Thus, the planner will always attempt to use the tax schedule, targeting the path of debt, in order to smooth the shock and thereby smoothing the inflation output tradeoff. This makes the government debt constraint relevant, independent of the value of $\phi_{\tau,b}$. [46] Interestingly, assuming a steady state output target introduces to the model elements of jointly optimal monetary and fiscal policy policies (see e.g. Schmitt-Grohé and Uribe (2004); Lustig et al. (2008); Faraglia et al. (2013); Leeper and Zhou (2021)).

The problem will not arise if we assume that optimal policy seeks to stabilize output around the natural level. Intuitively, under this assumption distortionary taxes will not appear explicitly in Phillips curve and we will once again have a dynamic equation for $\psi_{gov,t}$ which will not feature any forcing term.

To show this explicitly let us assume (for simplicity and wlog) $\sigma = 0$. The Phillips curve can then be written as

$$\hat{\pi}_t = \kappa_1\widetilde{Y}_t + \beta E_t\hat{\pi}_{t+1}$$

where $\widetilde{Y}_t := \hat{Y}_t - \hat{Y}_t^n$ and $\hat{Y}_t^n = -\frac{\overline{\tau}^d}{\gamma_h(1-\overline{\tau}^d)}\hat{\tau}_t$ is the natural level of output.

Moreover, the consolidated budget becomes

$$\begin{aligned}
&\frac{\beta\overline{b}}{1-\beta\delta}\hat{b}_{t,\delta} + \overline{b}\sum_{j=1}^{\infty}\beta^j\delta^{j-1}\left[E_t\left(-\sum_{l=1}^{j}\hat{\pi}_{t+l} + \hat{\xi}_{t+j}\right)\right] \\
&= -\left[-\overline{G}\left(\hat{G}_t + \hat{\xi}_t\right) + \overline{r}\left((1+\gamma_h)\widetilde{Y}_t - (1+\gamma_h)\frac{\overline{\tau}^d}{\gamma_h(1-\overline{\tau}^d)}\hat{\tau}_t + \frac{\hat{\tau}_t}{1-\overline{\tau}^d} + \hat{\xi}_t\right)\right] + \overline{b}\hat{\xi}_t \\
&+\frac{\overline{b}}{1-\beta\delta}(\hat{b}_{t-1,\delta} - \hat{\pi}_t) + \delta\overline{b}\sum_{j=1}^{\infty}\beta^j\delta^{j-1}E_t\left(-\sum_{l=1}^{j}\hat{\pi}_{t+l} + \hat{\xi}_{t+j}\right)
\end{aligned} \tag{69}$$

With a period loss function $-\frac{1}{2}E\left(\hat{\pi}_t^2 + \lambda_Y\widetilde{Y}_t^2\right)$ is trivial to show that the model now admits an equilibrium where $\psi_{gov,t} = 0$. The first order condition for bonds can be written as:

$$\frac{\beta\overline{b}}{1-\beta\delta}\left(\psi_{gov,t} - E_t\psi_{gov,t+1}\right) - \beta\phi_{\tau,b}E_t\,\underbrace{\overline{r}[((1+\gamma_h)\frac{\overline{\tau}^d}{\gamma_h(1-\overline{\tau}^d)} - \frac{1}{1-\overline{\tau}^d})]}_{\widetilde{\eta}_2}\,\psi_{gov,t+1}\Big) = 0$$

[46] In solving this model numerically, we found that increasing $\phi_{\tau,b}$ can bring the model very close to the 3 equation NK model, where debt is not a constraint for monetary policy. Thus, from a practical standpoint, optimal monetary policy effectively becomes active even though this is not possible to show analytically.

where again $\widetilde{\eta}_2 < 0$ otherwise the economy is on the wrong side of the Laffer curve. It is obvious that the solution is $\psi_{gov,t} = 0$ when fiscal policy is passive.

The threshold $\widetilde{\phi}_\tau$ can be found using the budget constraint. Leaving out the shocks we can express this as:

$$\frac{\beta\overline{b}}{1-\beta\delta}\hat{b}_{t,\delta} + \overline{r}((1+\gamma_h)\hat{Y}_t^n + \frac{\hat{\tau}_t}{1-\overline{\tau}^d}) = \frac{\overline{b}}{1-\beta\delta}\hat{b}_{t-1,\delta}$$

and so using the definition of $\hat{Y}_t^n$ above and the fiscal rule we have:

$$\hat{b}_{t,\delta} = \frac{1}{\beta}\left[1 - \overline{r}\frac{(1-\beta\delta)\phi_{\tau,b}}{\overline{b}(1-\overline{\tau}^d)}(1 - \frac{1+\gamma_h}{\gamma_h}\overline{\tau}^d)\right]\hat{b}_{t-1,\delta}$$

Quite evidently we now have

$$\widetilde{\phi}_\tau = \frac{(1-\beta)}{1-\beta\delta}\frac{\overline{b}}{\overline{r}}\frac{(1-\overline{\tau}^d)}{(1-\frac{1+\gamma_h}{\gamma_h}\overline{\tau}^d)}$$

which is the formula shown in text when $\sigma = 0$. It is straightforward to extend the above to the case $\sigma > 0$ and recover the expression for $\widetilde{\phi}_\tau$ in subsection 5.3.

E.1.2 Optimal policy rules with distortionary taxation.

We first derive the optimal interest rate rule for the simple Fisherian model assuming that the planner's objective only features inflation stabilization. Then, we will derive the optimal policies under the alternative versions of the model considered in Section 3. For brevity, we will omit the derivation in the dual mandate case, since it will become evident that our results carry through.

Simple Fisherian policies. With distortionary taxes and assuming $\sigma = \lambda_Y = 0$ optimal inflation is determined by the following condition:

$$\hat{\pi}_t = \frac{\omega_Y}{\kappa_1}\Delta\psi_{gov,t} + \frac{\overline{b}}{1-\beta\delta}\sum_{l=0}^{\infty}\delta^l\Delta\psi_{gov,t-l} = 0$$

where $\omega_Y \equiv \overline{R}(1+\gamma_h)$. We concentrate on the case of active fiscal policy.[47] Using the Euler equation in this model we then have that:

$$\hat{i}_t = \hat{\xi}_t + E_t\hat{\pi}_{t+1} = \hat{\xi}_t + E_t(\frac{\omega_Y}{\kappa_1}\Delta\psi_{gov,t+1} + \frac{\overline{b}}{1-\beta\delta}\sum_{l=0}^{\infty}\delta^l\Delta\psi_{gov,t-l+1}) = \hat{\xi}_t + \delta\frac{\overline{b}}{1-\beta\delta}\sum_{l=0}^{\infty}\delta^l\Delta\psi_{gov,t-l}$$

Thus,

$$\hat{i}_t = \hat{\xi}_t + \delta(\hat{\pi}_t - \frac{\omega_Y}{\kappa_1}\Delta\psi_{gov,t})$$

Therefore, relative to Proposition 2, the optimal rule with distortionary taxes adds the stochastic intercept $-\delta\frac{\omega_Y}{\kappa_1}$. Intuitively, the planner will use inflation between t and $t+1$ to distort output

[47] With passive policy it is simple to show that the rule shown in Proposition 2 continues to apply.

and increase the fiscal revenue of the government (when the debt constraint tightens). We can however show that this effect is not large and a simple rule $\hat{i}_t = \hat{\xi}_t + \delta\hat{\pi}_t$ is nearly optimal.

The canonical model, $\sigma > 0$. Now consider the case where $\sigma > 0$. In this case, optimal inflation is still described by

$$\hat{\pi}_t = \frac{\bar{b}}{1-\beta\delta}\sum_{l=0}^{\infty}\delta^l\Delta\psi_{gov,t-l} + \sigma\frac{\overline{Y}}{\overline{C}\kappa_1}\bar{b}\sum_{l=0}^{\infty}\delta^l\Big(\Delta\psi_{gov,t-l} - \Delta\psi_{gov,t-l-1}\Big) + \frac{\omega_Y}{\kappa_1}\Delta\psi_{gov,t} \tag{70}$$

i.e. the condition we derived in subsection A.2. The difference is that under distortionary taxes $\omega_Y = \overline{G}\sigma\frac{\overline{Y}}{\overline{C}} + \overline{R}(1+\gamma_h)$.

Essentially, all the derivations of subsection A.2 can be repeated here. We will therefore get the same interest rate rule, i.e. equation (41). The only modification concerns the stochastic intercept term.

Output stabilization. Finally, let $\lambda_Y > 0$ and $\sigma = 0$. Optimal inflation obeys the following

$$-\hat{\pi}_t - \frac{\lambda_Y}{\kappa_1^2}(\hat{\pi}_t - \beta E_t\hat{\pi}_{t+1}) + \frac{\lambda_Y}{\kappa_1^2}(\hat{\pi}_{t-1} - \beta E_{t-1}\hat{\pi}_t) + \frac{\overline{R}}{\kappa_1}(1+\gamma_h)\Delta\psi_{gov,t} + \frac{\bar{b}}{1-\beta\delta}\sum_{k=0}^{\infty}\delta^k\Delta\psi_{gov,t-l} = 0$$

We now define:

$$\zeta_t \equiv (\hat{\pi}_t - E_{t-1}\hat{\pi}_t) + \frac{\kappa_1^2}{\beta\lambda_Y}\frac{\bar{b}}{1-\beta\delta}\sum_{k=0}^{\infty}\delta^k\Delta\psi_{gov,t-l} + \frac{\kappa_1^2}{\beta\lambda_Y}\frac{\overline{R}}{\kappa_1}(1+\gamma_h)\Delta\psi_{gov,t} = 0$$

So that inflation again evolves according to:

$$E_t\hat{\pi}_{t+1} - (1 + \frac{1}{\beta} + \frac{\kappa_1^2}{\lambda_Y\beta})\hat{\pi}_t + \frac{1}{\beta}\hat{\pi}_{t-1} = -\zeta_t$$

Solving the second order polynomial we once again obtain:

$$\hat{\pi}_t = \frac{1}{\widetilde{\lambda}_2}E_t\hat{\pi}_{t+1} + \frac{1}{\widetilde{\lambda}_2}\frac{1}{1-\widetilde{\lambda}_1 L}\zeta_t = \frac{1}{\widetilde{\lambda}_2}\frac{1}{1-\widetilde{\lambda}_1 L}\sum_{j\geq 0}\frac{1}{\widetilde{\lambda}_2^j}E_t\zeta_{t+j}$$

Then,

$$\sum_{j\geq 0}\frac{1}{\widetilde{\lambda}_2^j}E_t\zeta_{t+j} = \sum_{j\geq 0}\frac{1}{\widetilde{\lambda}_2^j}E_t\left[(\hat{\pi}_{t+j} - E_{t+j-1}\hat{\pi}_{t+j}) + \widetilde{\kappa}\frac{\overline{R}}{\kappa_1}(1+\gamma_h)\Delta\psi_{gov,t+j} + \widetilde{\kappa}\frac{\bar{b}}{1-\beta\delta}\sum_{k=0}^{\infty}\delta^k\Delta\psi_{gov,t+j-l}\right] =$$
$$\hat{\pi}_t - E_{t-1}\hat{\pi}_t + \widetilde{\kappa}\frac{\overline{R}}{\kappa_1}(1+\gamma_h)\Delta\psi_{gov,t} + \widetilde{\kappa}\frac{\bar{b}}{1-\beta\delta}\frac{1}{1-\frac{\delta}{\widetilde{\lambda}_2}}\frac{1}{1-\delta L}\Delta\psi_{gov,t}$$

and so

$$\hat{\pi}_t = \widetilde{\lambda}_1\hat{\pi}_{t-1} + \frac{1}{\widetilde{\lambda}_2}(\hat{\pi}_t - E_{t-1}\hat{\pi}_t) + \frac{1}{\widetilde{\lambda}_2}\widetilde{\kappa}\frac{\overline{R}}{\kappa_1}(1+\gamma_h)\Delta\psi_{gov,t} + \frac{\widetilde{\kappa}}{\widetilde{\lambda}_2}\frac{\bar{b}}{1-\beta\delta}\frac{1}{1-\frac{\delta}{\widetilde{\lambda}_2}}\frac{1}{1-\delta L}\Delta\psi_{gov,t} \tag{71}$$

We can once again show that

$$E_t\hat{\pi}_{t+1} = \widetilde{\lambda}_1\hat{\pi}_t + \frac{\widetilde{\kappa}}{\widetilde{\lambda}_2}\frac{\overline{b}}{1-\beta\delta}\frac{1}{1-\frac{\delta}{\widetilde{\lambda}_2}}\underbrace{E_t\frac{1}{1-\delta L}\Delta\psi_{gov,t+1}}_{=\frac{\delta}{1-\delta L}\Delta\psi_{gov,t}}$$

and

$$\hat{\pi}_t - E_{t-1}\hat{\pi}_t = \frac{\widetilde{\lambda}_2}{\widetilde{\lambda}_2 - 1}\frac{\widetilde{\kappa}}{\lambda_2}\frac{\overline{b}}{1-\beta\delta}\frac{1}{1-\frac{\delta}{\widetilde{\lambda}_2}}\Delta\psi_{gov,t}$$

The optimal interest rate rule is given by:

$$\hat{i}_t = \hat{\xi}_t + (\widetilde{\lambda}_1 + \delta)\hat{\pi}_t - \delta\widetilde{\lambda}_1\hat{\pi}_{t-1} - \frac{\delta}{\widetilde{\lambda}_2}\widetilde{\kappa}\frac{\overline{R}}{\kappa_1}(1+\gamma_h)\Delta\psi_{gov,t} - \delta\frac{\widetilde{o}}{\widetilde{\lambda}_2 - 1}\Delta\psi_{gov,t}$$

which is of the same form as the rule under lump sum taxes, however, now the stochastic intercept is $-\frac{\delta}{\widetilde{\lambda}_2}\widetilde{\kappa}\frac{\overline{R}}{\kappa_1}(1+\gamma_h)\Delta\psi_{gov,t} - \delta\frac{\widetilde{o}}{\widetilde{\lambda}_2-1}\Delta\psi_{gov,t}$ instead of just $-\delta\frac{\widetilde{o}}{\widetilde{\lambda}_2-1}\Delta\psi_{gov,t}$.

Dual Mandate. It should be evident that in the case of a dual mandate objective, the optimal interest rate rule will be the same as in case of lump sum taxes. The stochastic intercept will again reflect the planner's desire to distort output in t and change the fiscal revenue. In solving the model numerically, we found that this effect is however not significant. For brevity we omit the impulse response graphs.

E.2 Optimal Policy with Complete Markets/when Debt is a Shock Absorber

We now provide a final analytical result, characterizing the optimal interest rate rule under active fiscal policy in a limiting case where $\psi_{gov,t} = 0$. This case is relevant when debt is state contingent (complete markets) and the consolidated budget constraints do not influence the optimal monetary policy even though taxes are constant through time.

Rather than introducing explicitly state contingent government bonds we base our derivations on a equivalent model in which debt acts as a shock absorber, enabling complete markets. Our argument is rooted in the literature on optimal debt management in macroeconomic models[48]. We will show that in the limiting case when shocks that hit the economy do not tighten the intertemporal budget constraint (and so $\psi_{gov,t} = 0$ for t), optimal monetary policy under constant taxes can be again represented as a passive money rule.

To simplify the algebra, we will present our derivations using the canonical New Keynesian model without any output smoothing objective. Therefore $\sigma > 0$ but $\lambda_Y = 0$. Moreover, we will assume that only preference shocks can hit the economy; this assumption further simplifies the notation and the algebra $\overline{G}$=0. We consider $\delta = 1$, the limit when long bonds are consols. In this limit we can show that inflation, output and debt are at steady state for all t.

[48] See for example Angeletos (2002); Buera and Nicolini (2004); Faraglia et al. (2019); Bhandari, Evans, Golosov, and Sargent (2017).

To grasp the intuition behind this last result, notice that when long bonds are consols the consolidated budget (in format analogous to equation (3)) can be written as:

$$\overline{b}\overline{p}_1(\hat{b}_{t,1} + \hat{p}_{t,1}) = (1 + \overline{p}_1)\overline{b}(\hat{b}_{t-1,1} - \hat{\pi}_t) + \overline{b}\overline{p}_1\hat{p}_{t,1} - \overline{R}\hat{R}_t \tag{72}$$

where b_1 now represents the quantity of the consol and $\hat{p}_1$ is the corresponding price. The latter evolves according to:

$$\hat{p}_{t,1} = -\hat{\xi}_t - E_t\left[\hat{\pi}_{t+1} + \sigma(\hat{Y}_t - \hat{Y}_{t+1}) + \beta\hat{p}_{t+1,1}\right].$$

Clearly, the terms $\overline{b}\overline{p}_1\hat{p}_{t,1}$ on the LHS and RHS of (72) cancel out and $\hat{\xi}_t$ can be dropped from this equation. The disturbance $\hat{\xi}_t$ changes the price of newly issued debt, and changes, in proportion, the market value of outstanding debt, which acts as a shock absorber. Under optimal policy we have $\psi_{gov,t} = 0$ and inflation, output do not need to adjust to finance the shock, regardless of the specification of fiscal policy.

The following Proposition derives the optimal monetary policy rule in this model:

Proposition 8 *Consider the case where $\sigma > 0$ and $\lambda_i = \lambda_Y = \overline{G} = 0$. Assume also that $\delta = 1$ (i.e. long bonds are consols). We then have $\psi_{gov,t} = 0$ under both active and passive fiscal policies. Optimal monetary policy sets the nominal rate following a rule $\hat{i}_t = \hat{\xi}_t + \phi_\pi\hat{\pi}_t$ and:*

i) Under passive fiscal policy it sets $\phi_\pi > 1$

ii) Under active fiscal policy it sets $\phi_\pi \leq 1$.

We will prove this Proposition in the next paragraph.

Note that i) is simply a repetition of the result in subsection 3.2.2. ii) states that in the active fiscal case, optimal monetary policy sets the inflation coefficient to any value less than or equal to 1. Because in this model shocks exert no influence on the budget constraint, all that monetary policy needs to do is to support the zero inflation outcome. This can be accomplished with any passive interest rate policy.

Proposition 8 can be extended in several meaningful ways to analogous results when $\lambda_Y > 0$ and when spending shocks can hit the economy.[49] In all these cases we can derive a passive interest rate rule to implement the optimal policy equilibrium.

E.2.1 Proof of Proposition 8

We will first prove that the equilibrium under optimal policies features 0 inflation when $\delta = 1$. Then we will show that this outcome can be implemented with a rule of the form $\hat{i}_t = \hat{\xi}_t + \phi_\pi\hat{\pi}_t + \phi_Y\hat{Y}_t$. This is a more general rule than the one considered in Proposition 8 where we focused on the case $\phi_Y = 0$. The conditions we derive below nest this case as well.

[49]The case $\lambda_Y > 0$ is simple to illustrate. Since in the equilibrium studied above output is always equal to target, any value $\lambda_Y > 0$ would again lead us to Proposition 8. Assuming that spending shocks can hit the economy however, would probably require a change in the modelling of government debt. For example Angeletos (2002) and Buera and Nicolini (2004) consider the case where debt is issued in two zero coupon bonds and show that the portfolio that can absorb fiscal shocks is one where the long bond debt position is several times GDP and short term debt is negative. The returns to this portfolio cannot be replicated with the decaying coupon model we assume here. Finally, when $\lambda_i > 0$ letting $\delta = 1$ would not be enough for debt to fully absorb the demand shocks. Under interest rate smoothing inflation and output will not be zero and it turns out that to insulate the budget constraint from the shock, the portfolio needs to be tilted even more towards long term debt. See below for an analytical derivation supporting this claim.

For clarity, we write here the system of equations that needs to be resolved under the parameter values assumed. We have:

$$\hat{\pi}_t = \kappa_1 \hat{Y}_t + \beta E_t \hat{\pi}_{t+1},$$

$$\hat{i}_t = \hat{\xi}_t + E_t\Big(\hat{\pi}_{t+1} - \sigma\Big(\hat{Y}_t - \hat{Y}_{t+1}\Big)\Big)$$

$$\bar{b}_1 \overline{p}\Big(\hat{b}_{t,1} + \hat{p}_t\Big) = \Big(1 + \overline{p}\Big)\bar{b}_1\Big(\hat{b}_{t-1,1} - \hat{\pi}_t\Big) + \bar{b}_1 \overline{p}\hat{p}_t - \overline{\tau}\hat{\tau}_t$$

$$\hat{p}_t = \Big(-\hat{\xi}_t - E_t\Big(\hat{\pi}_{t+1} + \sigma\Big(\hat{Y}_t - \hat{Y}_{t+1}\Big)\Big)\Big) + \beta \hat{p}_{t+1}$$

where $\overline{p} = \frac{\beta}{1-\beta}$ denotes the steady state of the consol.

Under parameters $\lambda_i = 0$ $\lambda_Y, \sigma > 0$ the first order conditions assuming active fiscal policy give us:

$$-\hat{\pi}_t - \frac{\lambda_Y}{\kappa_1}\Delta \hat{Y}_t + \frac{\sigma}{\kappa_1}\bar{b}_1 \sum_{l=0}^{\infty}\Big(\Delta\psi_{gov,t-l} - \Delta\psi_{gov,t-l-1}\Big) + \omega_Y \Delta\psi_{gov,t} + \frac{\bar{b}_1}{1-\beta}\sum_{l=0}^{\infty}\Delta\psi_{gov,t-l} = 0$$

$$\psi_{gov,t} - E_t\psi_{gov,t+1} = 0$$

We are required to solve the system of equations (E.2.1), (E.2.1), (E.2.1), (E.2.1) and (E.2.1) together with the fiscal rule. [50] (E.2.1) can be written as:

$$\hat{b}_{t,1} = \frac{1}{\beta}\Big(\hat{b}_{t-1,1} - \hat{\pi}_t\Big) - \frac{\overline{\tau}\phi_{\tau,b}}{\bar{b}_1\overline{p}}\hat{b}_{t-1,1} \tag{73}$$

since $\hat{p}_t$ drops from the LHS and RHS of (73) it is also obvious that (E.2.1) can be treated as a residual in this equilibrium system. The complete system is now (E.2.1) (73), (E.2.1) and (E.2.1). Moreover, since ξ_t does not appear anywhere in these equations, it is obvious that the solution is $\hat{\pi}_t = \hat{Y}_t = \hat{b}_{t,1} = \psi_{gov,t} = 0$. It can be shown that this solution is unique.

In the passive fiscal model we know from subsection 3.2.3 that $\hat{\pi}_t = \hat{Y}_t = 0$.

Let us now show that these outcomes can be implemented with an inflation targeting rule. Now the system of equations that we need to resolve is (E.2.1), (73) together with

$$\phi_\pi \hat{\pi}_t + \phi_Y \hat{Y}_t = E_t\Big(\hat{\pi}_{t+1} - \sigma\Big(\hat{Y}_t - \hat{Y}_{t+1}\Big)\Big) \tag{74}$$

$$\hat{p}_t = -\phi_\pi \hat{\pi}_t - \widetilde{\phi}_Y \hat{Y}_t - \hat{\xi}_t + \beta \hat{p}_{t+1} \tag{75}$$

In matrix form we can write this system as:

$$AE_t z_{t+1} = Bz_t + C\hat{\xi}_t \tag{76}$$

[50] (E.2.1) can be satisfied ex post since $\hat{i}_t$ is a 'slack' variable.

where $z_t = \left(\hat{\pi}_t, \hat{Y}_t, \hat{p}_t, \hat{b}_{t-1}\right)$ and

$$A = \begin{bmatrix} \beta & 0 & 0 & 0 \\ 1 & \sigma & 0 & 0 \\ 0 & 0 & \beta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & -\kappa_1 & 0 & 0 \\ \phi_\pi & \widetilde{\phi}_Y + \sigma & 0 & 0 \\ \phi_\pi & \widetilde{\phi}_Y & 1 & 0 \\ -\frac{1}{\beta} & 0 & 0 & \frac{1}{\beta} - \frac{\overline{\tau}\phi_{\tau,b}}{\overline{b}_1\overline{p}} \end{bmatrix}$$

and $C = [0, 0, 1, 0]'$. The dynamics of (76) are governed by the eigenvalues of $A^{-1}B$.

$$A^{-1}B = \begin{bmatrix} \frac{1}{\beta} & -\frac{\kappa_1}{\beta} & 0 & 0 \\ \frac{\phi_\pi}{\sigma} - \frac{1}{\sigma\beta} & \frac{\kappa_1}{\sigma\beta} + \frac{\phi_Y}{\sigma} + 1 & 0 & 0 \\ \frac{\phi_\pi}{\beta} & \frac{\phi_Y}{\beta} & \frac{1}{\beta} & 0 \\ 0 & 0 & \frac{1}{\beta} - \frac{\overline{\tau}\phi_{\tau,b}}{\overline{b}_1\overline{p}} & \end{bmatrix}$$

The characteristic polynomial is:

$$\left(\frac{1}{\beta} - \lambda\right)^2 \left(\frac{\kappa_1}{\sigma\beta} + \frac{\widetilde{\phi}_Y}{\sigma} + 1 - \lambda\right)\left(\frac{1}{\beta} - \frac{\overline{\tau}\phi_{\tau,b}}{\overline{b}_1\overline{p}} - \lambda\right) + \left(\frac{1}{\beta} - \lambda\right)\left(\frac{1}{\beta} - \frac{\overline{\tau}\phi_{\tau,b}}{\overline{b}_1\overline{p}} - \lambda\right)\frac{\kappa_1}{\beta}(\frac{\phi_\pi}{\sigma} - \frac{1}{\sigma\beta}) = 0$$

Rearranging we get:

$$\left(\frac{1}{\beta} - \lambda\right)\left(\frac{1}{\beta} - \frac{\overline{\tau}\phi_{\tau,b}}{\overline{b}_1\overline{p}} - \lambda\right)\left[\left(\frac{1}{\beta} - \lambda\right)\left(\frac{\kappa_1}{\sigma\beta} + \frac{\widetilde{\phi}_Y}{\sigma} + 1 - \lambda\right) + \frac{\kappa_1}{\beta}\frac{\phi_\pi}{\sigma}\right] = 0$$

The solution is unique if we get 3 eigenvalues outside the unit circle. Suppose that fiscal policy is 'passive' so that $\frac{1}{\beta} - \frac{\overline{\tau}\phi_{\tau,b}}{\overline{b}_1\overline{p}} < 1$. Then the 2 eigenvalues that we can obtain from solving the second order polynomial in the square bracket above have to exceed 1. The polynomial is:

$$\lambda^2 - \lambda\Big(\underbrace{\frac{1}{\beta} + \frac{\kappa_1}{\sigma\beta} + \frac{\widetilde{\phi}_Y}{\sigma}}_{\chi_1} + 1\Big) + \underbrace{\frac{1}{\beta}\left(\frac{\widetilde{\phi}_Y}{\sigma} + 1\right) + \frac{\kappa_1}{\beta}\frac{\phi_\pi}{\sigma}}_{\chi_2} = 0$$

Focusing on positive values for parameters ϕ_π, ϕ_Y both roots exceed 1 if:

$$\frac{(\chi_1 + 1) - \sqrt{(\chi_1 + 1)^2 - 4\chi_2}}{2} > 1$$

or

$$(\chi_1 - 1) > \sqrt{(\chi_1 + 1)^2 - 4\chi_2} \rightarrow (\chi_1 - 1)^2 > (\chi_1 + 1)^2 - 4\chi_2 \rightarrow 4\chi_2 > (\chi_1 + 1)^2 - (\chi_1 - 1)^2 = 4\chi_1$$

The condition $\chi_2 > \chi_1$ holds when

$$\frac{1}{\beta}\left(\frac{\widetilde{\phi}_Y}{\sigma} + 1\right) + \frac{\kappa_1}{\beta}\frac{\phi_\pi}{\sigma} > \frac{1}{\beta} + \frac{\kappa_1}{\sigma\beta} + \frac{\phi_Y}{\sigma}$$

or

$$\frac{\phi_Y(1-\beta)}{\kappa_1} + \phi_\pi > 1 \tag{77}$$

In the case where fiscal policy is active it is easy to check that a unique equilibrium requires the opposite inequality to hold in (77). Finally, it is easy to verify that in either of these cases inflation, debt and output are always at steady state. ■